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Research exercise: Exploring the Sinc-Collocation Method for Solving the Integro-Differential Equation

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Abstract

In this project we study the Sinc approximation method to solve a family of integral differential equations. First we will apply the Sinc-collocation method to solve the second order Fredholm integro-differential equation. Numerical results and examples demonstrate the reliability and efficiency of this method. Secondly, various types of integro-differential equations are solved by Sinc-collocation technique and the numerical results are compared, to explore the stability of this method.

Introduction and Objective

There has been developed a variety of numerical methods based on the Sinc approximations in the past thirty years. In recent years, Sinc method has been used to obtaining approximate solutions of a wide class of differential and integral equations. One method called Galerkin-type scheme converges at fast rate and easy to apply. This work is motivated by the desire to obtain numerical solution to boundary value problems for second-order Fredholm integro-differential equations via Sinc-Collocation method.

Methodology

- ✓ Review some basic facts about the Sinc approximation that are necessary for the formulation of the discrete linear system and understand the functions which exhibit Kronecker delta behavior on the grid points x_k.
- ✓ Derive the discrete system from the problem using the Sinc-Collocation method.
- ✓ The Toeplitz matrix of δ function are well defined.
- ✓ After getting the linear system of 2N+1 equations for the 2N+1unknown coefficients, we approximate the solution by solving this linear system by Q-R method.

We will consider the numerical solution of a class of linear Fredholm integro-differential boundary value problems in the form

$$\sum_{i=0}^{n}\mu_i(x)u^{(i)}(x) = f(x)+\lambda\int_a^b K(x,t)u(t)dt,\ x\in[a,b], length(x)=2N+1 \quad (1)$$

We assume that $u(x)$, the solution of (1), is approximated by the finite expansion of Sinc basis functions

$$u_m(x)=\sum_{j=-N}^{N}u_j S(j,h)\circ\phi(x), m=2N+1 \quad (2)$$

If the assumed approximate solution of the equation (1) is (2), the discrete Sinc-Collocation system for the determination of the unknown coefficients is given by

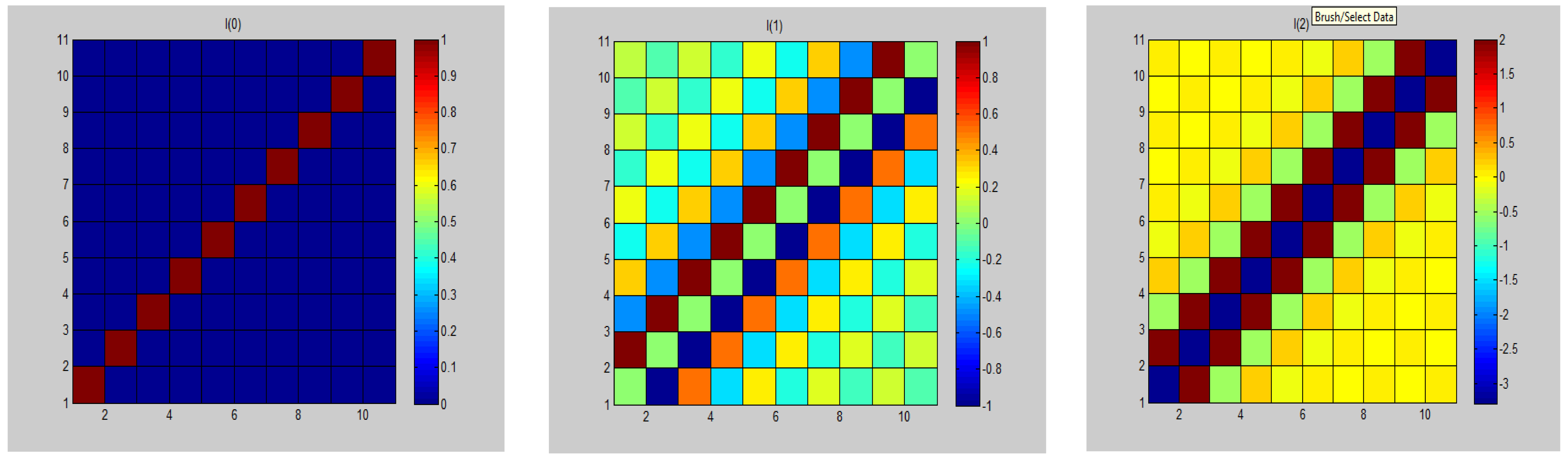
$$\sum_{j=-N}^{N}[\sum_{i=0}^{n}g_i(x_k)\frac{\delta_{kj}^{(i)}}{h^i}-h\lambda\frac{K(x_k,t_j)}{\phi'(t_j)}]u_j = f_k, k=-N,-N+1...,N \quad (3)$$

In order to have a linear system of m equations for the m unknown coefficients, (3) Takes the matrix form

$$AU = f, U=[u_{-N},u_{-N+1}..u_N]^T \quad (4)$$

$$A=\sum_{i=0}^{n}\frac{1}{h^i}I(i)D(g_i)-h\lambda\frac{K(x_k,t_j)}{\phi'(t_j)} \quad (5)$$

When $N=5$, $I(i)=\delta_{kj}^{(i)}$ is defined as



Example

In the case of

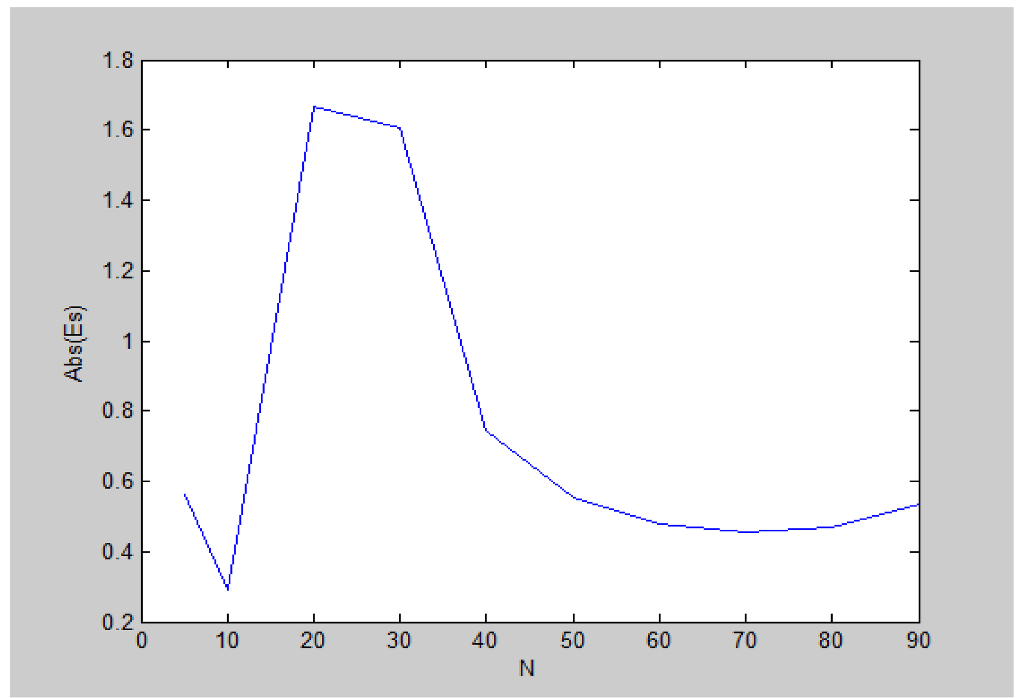
$\mu_0(x)=1/x^3, \mu_1(x)=1/x^2, \mu_2(x)=1,$

equation (1) becomes

$$u''+\frac{1}{x^2}u'+\frac{1}{x^3}u = f(x)+\int_0^1 K(x,t)u(t)dt, 0<x<1$$

$$f(x)=\frac{24x^3-x^2-3x+2}{x^2}, K(x,t)=-\pi^3(6x+\frac{1}{4})\sin(\pi t)$$

The maximum absolute error is report in the table from N=5 to N=90



Conclusion and future work

- ✓ In this work we have explored the work proposed by Adel Mohamed in the [1].
- ✓ There are improvements needed to get the accuracy promised by the Sinc Collocation methods.
- ✓ Applying the methods to more general class of the integro-differential equations and equations with delays.

References

[1] Adel Mohsen, A Sinc-Collocation method for the linear Fredholm integro-differential equations. angew. Math. Phys. **58**(2007), 380-390.