Negative Index in Chiral Metamaterials under Conductive Loss and First-Order Material Dispersion Using Lorentzian, Condon and Drude Models

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Abstract: The aim of this paper is to explore the emergence of negative index (NIM) in chiral materials with conductive loss using standard dispersive models (Lorentzian, Condon and Drude) for the material parameters. Expanding to first-order, results are derived for phase and group velocities, and corresponding indices, thereby finding appropriate sideband ranges. As expected, in the NIM region the phase index is negative and group index is positive, it is also found that NIM may occur both for positive and negative sidebands.

OCIS codes: (160.1585) chiral media; (160.3918) metamaterials; (260.2030) dispersion

1. General constitutive relations for isotropic chiral media

In metamaterials the relationships between the fields (the electric field $\tilde{E}$ and the electric field displacement $\tilde{D}$ and also the magnetic field $\tilde{B}$ and the inductive magnetic field $\tilde{H}$) are not parallel, and electric and magnetic vectors are coupled [1]. In particular, the following form is convenient in solving many wave propagation problems in a reciprocal chiral medium [2]:

$$\tilde{D} = \tilde{\varepsilon}\tilde{E} - j\tilde{\kappa}\sqrt{\tilde{\mu}\tilde{\varepsilon}}\tilde{H}, \tag{1}$$

$$\tilde{B} = \tilde{\mu}\tilde{H} + j\tilde{\kappa}\sqrt{\tilde{\mu}\tilde{\varepsilon}}\tilde{E}\ , \tag{2}$$

where $\tilde{\kappa}$ is the chirality parameter, $\tilde{\varepsilon}$ is the electric permittivity, and $\tilde{\mu}$ is the magnetic permeability.

2. Derivation of dispersive and lossy chiral wavevectors and phase and group velocities via spectral analysis

Starting with expansions of the material parameters up to the first order, one may express the frequency-dependent chirality parameter $\tilde{\kappa}_p$, electric permittivity $\tilde{\varepsilon}_p$, the magnetic permeability $\tilde{\mu}_p$, and the conductivity $\tilde{\sigma}_p$ as first-order Taylor expansions assuming monochromatic, sinusoidal phasor analyses to be valid [3,4].

$$\begin{bmatrix} \tilde{\varepsilon}_p(\Omega) \\ \tilde{\mu}_p(\Omega) \\ \tilde{\alpha}_p(\Omega) \\ \tilde{\sigma}_p(\Omega) \end{bmatrix} = \begin{bmatrix} \tilde{\varepsilon}_p(\Omega) \\ \tilde{\mu}_p(\Omega) \\ \omega\sqrt{\mu_0\varepsilon_0}\,\tilde{\kappa}_p(\Omega) \\ \tilde{\sigma}_p(\Omega) \end{bmatrix} = \begin{bmatrix} \tilde{\varepsilon}_{p0} + \Omega\tilde{\varepsilon}_{po} \\ \tilde{\mu}_{p0} + \Omega\tilde{\mu}'_{po} \\ \omega\sqrt{\mu_0\varepsilon_0}\,(\tilde{\kappa}_{po} + \Omega\tilde{\kappa}'_{p0}) \\ \tilde{\sigma}_{p0} + \Omega\tilde{\sigma}'_{p0} \end{bmatrix}, \tag{3}$$

where $\tilde{\varepsilon}_p(\Omega), \tilde{\mu}_p(\Omega), \tilde{\kappa}_p(\Omega), \tilde{\sigma}_p(\Omega)$ are the material parameters of interest expressed as functions of the sideband frequency (Ω) of the propagating field, and $\tilde{\alpha}_p$ is the *chiral wavenumber* which has the dimension rad m^{-1}. More details about calculating the chiral wavevectors in a material with conductive losses may be found in Ref. [3].

2.1 Wavevector solution under conductive losses and plane wave propagation

We have introduced complex conductivity into the electromagnetic problem thereby implying that the material has (here) only conductive losses; it is found that in this case the chiral wavevector becomes complex as follow:

$$\tilde{k}_{z3} = -\omega\tilde{\kappa}\sqrt{\tilde{\varepsilon}\tilde{\mu}} + \sqrt{\tilde{\mu}\omega(\omega\tilde{\varepsilon} - j\tilde{\sigma})}\ , \tag{4}$$

where the conductivity is equal to $\tilde{\sigma} = \tilde{\sigma}_{p0} + \Omega\tilde{\sigma}'_{p0}$, $\tilde{\sigma}_{p0} = \tilde{\sigma}_{p0r} + j\tilde{\sigma}_{p0i}$, $\tilde{\sigma}'_{p0} = \tilde{\sigma}'_{p0r} + j\tilde{\sigma}'_{p0i}$. After significant algebra, the complex wavevector (choosing the 3rd of three possible solutions) in a lossy metamaterial as described can be expressed to first order as follows:

$$\tilde{k}_{z3} = \omega\sqrt{\tilde{\mu}_{p0}\tilde{\varepsilon}_{p0}}\left\{\begin{array}{l}\left\{-\tilde{\kappa}_{po} + \dfrac{\tilde{\sigma}_{p0i}}{2\omega_0\tilde{\varepsilon}_{p0}} + 1 + \Omega\left(-\dfrac{\tilde{\kappa}_{po}}{2}(\dfrac{\tilde{\mu}'_{po}}{\tilde{\mu}_{p0}} + \dfrac{\tilde{\varepsilon}'_{po}}{\tilde{\varepsilon}_{p0}}) - \tilde{\kappa}'_{p0} + \dfrac{\tilde{\mu}'_{po}}{2\tilde{\mu}_{p0}} + \dfrac{\tilde{\varepsilon}'_{po}}{2\tilde{\varepsilon}_{p0}} + \dfrac{\tilde{\sigma}'_{p0i}}{2\omega_0\tilde{\varepsilon}_{p0}} + \dfrac{\tilde{\mu}'_{po}\tilde{\sigma}_{p0i}}{2\omega_0\tilde{\mu}_{p0}\tilde{\varepsilon}_{p0}}\right)\right\} + \\ j\left\{-\dfrac{\tilde{\sigma}_{p0r}}{2\omega_0\tilde{\varepsilon}_{p0}} + \Omega[-\dfrac{\tilde{\sigma}'_{p0r}}{2\omega_0\tilde{\varepsilon}_{p0}} - \dfrac{\tilde{\mu}'_{po}\tilde{\sigma}_{p0r}}{2\omega_0\tilde{\mu}_{p0}\tilde{\varepsilon}_{p0}}]\right\}\end{array}\right\}. \quad (5)$$

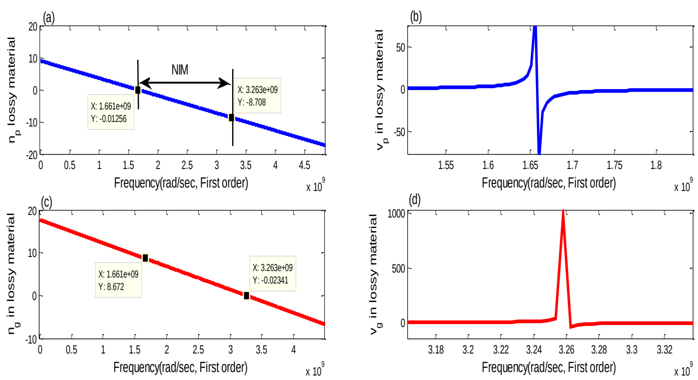
We know that the propagation constant $\gamma = j\tilde{k}_{z3} = j(\alpha_k + j\beta_k)$, from which one may extract the effective attenuation and phase constants of the system ($\alpha_{eff} = -\beta_k$ and $\beta_{eff} = \alpha_k$ respectively). Thus, one obtains the normalized phase and group velocities as:

$$v_{p3N} = \frac{\omega}{c\beta} = \frac{1}{n_{p3}} = \frac{1}{\tilde{\varepsilon}_{pro}\left\{-\tilde{\kappa}_{pro} + \dfrac{\tilde{\sigma}_{p0i}}{2\omega_0\varepsilon_0\tilde{\varepsilon}_{pro}} + 1 + \Omega\left(-\tilde{\kappa}_{pro}(\dfrac{\tilde{\varepsilon}'_{pro}}{\tilde{\varepsilon}_{pro}}) - \tilde{\kappa}'_{pro} + \dfrac{\tilde{\varepsilon}'_{pro}}{\tilde{\varepsilon}_{pro}} + \dfrac{\tilde{\sigma}'_{p0i}}{2\omega_0\varepsilon_0\tilde{\varepsilon}_{pro}} + \dfrac{\tilde{\mu}'_{pro}\tilde{\sigma}_{p0i}}{2\omega_0\tilde{\mu}_{pro}\varepsilon_0\tilde{\varepsilon}_{pro}}\right)\right\}}\hat{a}_z. \quad (6)$$

$$\tilde{v}_{g3N} = \frac{1}{c\dfrac{\partial\tilde{\beta}_{z3}}{\partial\omega}} = \frac{1}{n_{g3}} = 1/\left[\tilde{\varepsilon}_{pro}\left\{\begin{array}{l}-\tilde{\kappa}_{pro} + \dfrac{\tilde{\sigma}_{p0i}}{2\omega_0\varepsilon_0\tilde{\varepsilon}_{pro}} + 1 + \omega_0\left(\begin{array}{c}-\tilde{\kappa}_{pro}(\dfrac{\tilde{\varepsilon}'_{pro}}{\tilde{\varepsilon}_{pro}}) - \tilde{\kappa}'_{pro} + \dfrac{\tilde{\varepsilon}'_{pro}}{\tilde{\varepsilon}_{pro}} + \\ \dfrac{\tilde{\sigma}'_{p0i}}{2\omega_0\varepsilon_0\tilde{\varepsilon}_{pro}} + \dfrac{\tilde{\mu}'_{pro}\tilde{\sigma}_{p0i}}{2\omega_0\tilde{\mu}_{pro}\varepsilon_0\tilde{\varepsilon}_{pro}}\end{array}\right) + 2\Omega\{-\tilde{\kappa}_{pro}(\dfrac{\tilde{\varepsilon}'_{pro}}{\tilde{\varepsilon}_{pro}}) \\ -\tilde{\kappa}'_{pro} + \dfrac{\tilde{\varepsilon}'_{pro}}{\tilde{\varepsilon}_{pro}} + \dfrac{\tilde{\sigma}'_{p0i}}{2\omega_0\varepsilon_0\tilde{\varepsilon}_{pro}} + \dfrac{\tilde{\mu}'_{pro}\tilde{\sigma}_{p0i}}{2\omega_0\tilde{\mu}_{pro}\varepsilon_0\tilde{\varepsilon}_{pro}}\}\end{array}\right\}\right]\hat{a}_z. \quad (7)$$

2.2. Numerical results and plots

The phase and group velocities and likewise the indices have been calculated and plotted using Lorentzian, Condon and Drude models to derive the related materials parameters. Fig.1 shows the NIM region in the range of frequencies from 1.66×10^9 to 3.23×10^9 r/ and the n_{p3}= -0.0125 and $n_{g3=+}$8.67.



$$\tilde{\varepsilon}_{pro} = 1 + \frac{\omega_p^2}{\omega_c^2}(1 + \frac{\omega_0^2}{\omega_c^2}) \quad \tilde{\varepsilon}'_{pro} = \frac{2\omega_p^2\omega_0}{\omega_c^4}.$$

$\omega_0 = 1.6\times10^{13}$,

$\omega_c = 2\times10^{12}$,

$\omega_p = 1\times10^7$, $\beta = \omega_p/\omega_c$, $\beta = \omega_p/\omega_c$

$$\tilde{\kappa}'_{pro} = \frac{-\alpha}{\omega_c}[(1+\beta^2) + 3\omega_n^2 + 6\beta^2\omega_n^2 + 5\beta^2\omega_n^4]$$

$$\tilde{\kappa}_{pro} = -\alpha\omega_n[(1+\omega_n^{\,2}) + \beta^2(1+2\omega_n^{\,2})] . \quad \tau = 0.4\times10^{-11}.$$

Fig. 1. Phase and phase index in lossy material (a) np_{3N} under first order at sideband frequency (Ω); (b) vp_{3N} under first order at (Ω) (ω=Ω+ω0); (c) ng_{3N} under first order at (Ω) ; (d) vg3N under first order at (Ω) .

It is known that the polarization state of a plane wave may be described by the locus of the total the $\tilde{E}$-field vector at a given point in space *as a function of time*. To evaluate the polarization state of the plane wave in a *lossy* material we assume the electric field phasor $\tilde{E}$(z) is propagating in the +z direction. Hence, for a transverse EM field,

$$\tilde{E}(z) = E_x\hat{a}_x + E_y\hat{a}_y = \left(E_{x0}\ \hat{a}_x + E_{y0}\ \hat{a}_y\right)e^{-\gamma z}, \quad (8a)$$

such that, with $\gamma = j\,k_{z3} = j(\alpha_k + j\,\beta_k) = -\beta_k + j\,\alpha_k$, eq.(8a) becomes

$$\tilde{E}(z) = \left(E_{x0}\ \hat{a}_x + E_{y0}\ \hat{a}_y\right)e^{-(-\beta_k + j\alpha_k)z} = \left(E_{x0}\ \hat{a}_x + E_{y0}\ \hat{a}_y\right)e^{-(\alpha_{eff} + j\beta_{eff})z}, \quad (8b)$$

We may rewrite the electric field in the time domain as follows:

$$\tilde{E} = \left(E_{x0}\cos(\omega t - \beta_{z3} z)\,\hat{a}_x - E_{x0}\sin(\omega t - \beta_{z3} z)\,\hat{a}_y\right)e^{-\alpha z} \qquad (9)$$

Therefore, setting $t = 0$ (instead of the usual z=0) leads to

$$\hat{\mathcal{E}}_x(x,y,z;0) = \left(E_0 \cos(\beta_{z3} z)\right)e^{-\alpha z}, \qquad (10a)$$

$$\hat{\mathcal{E}}_y(x,y,z;0) = +\left(E_0 \sin(\beta_{z3} z)\right)e^{-\alpha z}, \qquad (10b)$$

where $\beta_3 = \dfrac{\omega}{\tilde{v}_{p3}} = \dfrac{\omega}{c\tilde{v}_{p3N}} = \dfrac{\omega n_{p3}}{c} = \dfrac{(\Omega+\omega_0)n_{p3}}{c}$.

Case A: Electric field and polarization in the NIM region ($n_{p3} < 0$)

Since loss is present, the electric field will decay along the propagation path when passing through the material. The plots are generated by choosing a sideband around mid-NIM ($\Omega = 2.5 \times 10^9$ r/s), where the phase index was shown earlier to be about -4.531 (see Fig.1(c)). The plot indicates a decaying oscillation whose envelope is controlled by the effective attenuation constant α, and the oscillation frequency is determined by the effective phase constant $\beta_{3.}$. As a result, we obtain a spiral polarization state which converges towards zero as $z \to \infty$. Moreover, it may be readily shown that in the NIM region ($n_{p3} < 0$), the field is actually right *spiral polarized* (RSP), and the polarization spiral rotates counter clockwise (CCW), as shown in Fig.2.

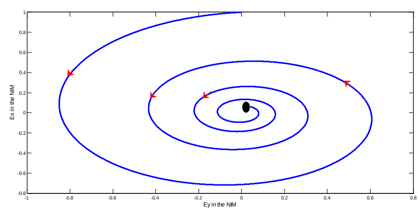


Fig. 2. Spiral spatial polarization in the lossy chiral material in the NIM region

Case B: Electric field and polarization outside the NIM region ($n_{p3} > 0$)

The plots are generated by choosing a sideband outside the NIM ($\Omega = 1.5 \times 10^8$ r/s), where the phase index is about 8.19 (see Fig.1(c)). It may be shown that the electric field in the non-NIM region when ($n_{p3} > 0$) (n_{p3}=8.19, $\Omega = 1.5 \times 10^8$ r/s) again exhibits right spiral polarization (RSP), and the (spatial) polarization spiral rotates clockwise (CW) with propagation distance, as shown in Fig.3. The wave in this case propagates along +Z because of the positive β_{z3}.

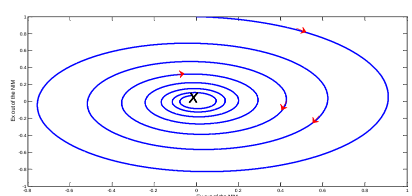


Fig. 3. Spiral spatial polarization in the lossy chiral material out of the NIM region

4. Conclusion

Negative index under dispersive models for material parameters in a lossy isotropic chiral medium was examined under first-order dispersion. It was found that for the parameters chosen, NIM occurs under positive sideband frequencies in the far-RF radio band; also, due to the decay induced by loss, the spatial field polarization undergoes spiral decay in the RSP sense.

4. References

[1] A. Lakhtakia, V. K. Varadan, and V. V. Varadan, *Time-Harmonic Electromagnetic Fields in Chiral Media*, Springer-Verlag, New York (1989).

[2] A. Serdyukov, I. Semchenko, *Electromagnetic of Bi-Anisotropic Materials Theory and Applications*, University of East Anglia, UK,(2001).

[3] M.R. Chatterjee and T.A. Algadey, "Emergence of negative index in a lossy chiral metamaterial under first-order material dispersion," OSA Annual Meeting Technical Digest, paper NM2C.5, 1-3 (2015).

[4] M.R. Chatterjee and T.A. Algadey, "Investigation of electromagnetic velocities and negative refraction in a chiral metamaterial with second-order material dispersion using spectral analyses and dispersive models," Opt. Eng. 54(3), 037108-11 (2015).