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METHODS FOR SOLVING HYDRAULIC NETWORKS

Don Chase, Ph.D., P.E.

November 2023

First Edition



Methods for Solving Hydraulic Networks

Don Chase, Ph.D., P.E.



Greetings!

Thank you for taking time to download and read some or all of this document. This document represents a more organized version of my course notes in this subject. This subject consists of material developed while a faculty member in the Department of Civil and Environmental Engineering at the University of Dayton. Also the content of this document may contain material developed as part of a short course for professional engineers working in the Water Resources area of civil engineering. While every effort has been taken to produce a document that is free of errors and typos, I am human after all. So please contact me at dchase1@udayton.edu and let me know of any errors that you find. You may also contact me with comments or suggestions on how to improve the content of this document.

Thanks!

Don Chase

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