Fall 2005

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University of Dayton. Department of Mathematics

Abstracts of the Colloquium Talks: Fall 2005
Department of Mathematics

Two Methods for a Linear Half-Order Fractional Difference Equation with Constant Coefficients
Dr. Paul Eloe
Abstract: We define a linear half-order fractional difference equation with constant coefficients. We develop two methods of solution. In the first method, we define a corresponding characteristic polynomial and employ the roots of the polynomial. The second method is an analogue of the Laplace transform method for ordinary differential equations. Sufficient preliminary material in fractional finite calculus is presented to make the talk self-contained. The material generalizes corresponding material in a first course in ordinary differential equations, and hence the talk is accessible to undergraduate students.

The non-real zeros of P and a conjecture of Gauss
Dr. Stephanie Edwards
Abstract: Let P be a real polynomial. An old conjecture of Gauss' state that the number of non-real zeros of P is not exceeded by the number of real critical points of the logarithmic derivative of P. It has been thought that this conjecture can be affirmed using Gamma-lines. We will discuss Gamma-lines and show the short comings of the technique.

Cycles and Convexity in Bipartite Tournaments
Dr. Darren Parker
Abstract: Let $T = (V, E)$ be a directed graph. In the case that T is a strong tournament with n vertices, J.W. Moon proved that every vertex is part of a k-cycle for $3 \leq k \leq n$. We seek to prove analogous results in bipartite tournaments. We look for generalizations from the perspective of convexity. A subset C of V is convex if, whenever v and w are in C, x in V with $(v, x), (x, w)$ in E, we have x in C. One of the natural convex invariants is the rank $d(T)$. It was (indirectly) shown by Varlet that $d(T) = 2$ when T is a tournament. Thus, we consider bipartite tournaments of rank 2. We show that every vertex in a bipartite tournament of rank 2 is in a 2k-cycle for $2 \leq k \leq \min(|P1|, |P2|)$, where the Pi are the partite sets of T and $k \leq 6$. We then ponder what the analogue of "strong" is in bipartite tournaments.

Ordered Sets and Models of Topological Spaces
Dr. Joe Mashburn
Abstract: In the late 1960's and early 1970's mathematicians and computer scientists began using ordered sets, along with a special relation, as models for information theory. These ordered sets are called domains. This involved a special topology called the Scott topology. It is not a very desirable topology, since it has few of the properties that topologists consider important. More recently, it has been discovered that ordered sets with this topology can be used to develop models for much nicer topological spaces. In fact, every complete metric space can be modeled by one of these ordered sets, as can every locally compact Hausdorff space. In this talk we will define a domain and discover some of its basic properties. We will show how these are used to model topological spaces and, if time permits, show some examples.

Quenching Phenoma Due to a Concentrated Nonlinear Source
Dr. C.Y. Chan

Abstract: Let q, a, T, and b be any real numbers such that $q \geq 0, a > 0, T > 0$, and $0 < b < a$. We consider the following degenerate semilinear parabolic first initial-boundary value problem with a concentrated nonlinear source situated at b:

$$x^q U_t - U_{xx} = \delta(x - b) f\big(u(x,t)\big) in\ (0,a) \times (0,T]$$

$$u(x,0) = 0\ on\ [0,a]\ , u(0,t) = u(a,t) = 0\ for\ 0 < t \leq T$$

where $\delta(x)$ is the Dirac delta function, f is a given function such that $lim_{u \to c^-} f(u) = \infty$ for some positive constant c, and $f(u)$ and $f'0\ (u)$ are positive for $0 \leq u < c$. A solution u of the problem is said to quench if there exists some t_q such that $\max\{u(x,t) : 0 \leq x \leq a\} \to c^-$ as $t \to t_q$. It is shown that the problem has a unique continuous solution u before quenching occurs, u is a strictly increasing function of t for $0 < x < a$, and if quenching occurs, then it occurs only at the point b. The problem is shown to have a unique a^* such that a unique global solution u exists for $a \leq a^*$, and quenching occurs in a finite time for $a > a^*$; this a^* is the same as that for $q = 0$. A formula for computing a^* is given, and no quenching in infinite time is deduced.

A correct formulation of a multi-dimensional quenching problem due to a concentrated nonlinear source is discussed. It is shown that the problem has a unique continuous solution u before quenching occurs. If u exists for a finite time, then it quenches everywhere on the concentrated source only. The talk should be of interest to a general audience.

Consistency of Bootstrapping Sample Quantiles Under Dependence

Dr. Shuxia Sun

Abstract: In this talk, we investigate consistency properties of block bootstrap approximations for sample quantiles of weakly dependent data. Under mild weak dependence conditions and mild smoothness conditions on the one-dimensional marginal distribution function, we show that the moving block bootstrap method provides a valid approximation to the distribution of normalized sample quantile in the almost sure sense. Strong consistency of the block bootstrap estimator of the asymptotic variance of the sample quantile is also established under similar conditions. Results from a small simulation study will be presented.

Size-Structured Population Models: Existence-Uniqueness and Long Time Behavior

Dr. Azmy S. Ackleh

Abstract: In this talk we present two population models. The first one is a quasilinear hyperbolic initial-boundary value problem that models the evolution of n size-structured subpopulations. We discuss existence-uniqueness of global weak solutions. We prove that if reproduction is closed (i.e., offspring of one ecotype belongs to the same ecotype) then competitive exclusion between these ecotypes will occur. We give conditions on the model parameters which determine the fittest ecotype that wins the competition. Furthermore, we show that if reproduction is open then coexistence between the n ecotypes is possible.

The second model we present is a hierarchically size-structured population model with growth, mortality and reproduction rates that depend on a function of the population density (environment).

This model is also described by a quasilinear hyperbolic initial-boundary value problem. But unlike the first one, it is shown that the introduction of a hierarchy in the population leads to singular solutions. In particular, we will present a numerical example to show that if the growth rate is not always a decreasing function of the environment (e.g., a growth which exhibits the Allee effect) the emergence of a structure in the form of a Dirac delta measure is possible. Therefore, we discuss the existence and uniqueness of measure-valued solutions to this model.

Option Pricing in a Regime Switching Model Using the Fast Fourier Transform

Dr. Ruihua Liu

Abstract: Fast Fourier Transform (FFT) has been increasingly used in financial engineering to numerically determine derivative values. It is applicable to problems for which the characteristic functions of the underlying asset processes can be obtained analytically. In this talk I will present the FFT scheme to option valuation where the underlying asset price follows a regime-switching Geometric Brownian Motion (GBM). We present that the Fourier transform of the option price is obtained in terms of the joint characteristic function of the sojourn times of the Markov chain, and this joint characteristic function is given in explicit form for two-state (m=2) Markov chains, and in terms of solution of a system of m-dimensional differential equations for m-state case. Then we present how FFT is employed to calculate the option values and show numerical results.

If time permits, I would also like present the second part of our work, which is concerned with a near-optimal FFT scheme for large state space of the underlying Markov chain. Aiming at reducing computational complexity, we used a two-time-scale structure for the Markov chain. A limit price process with reduced state space and the associated option pricing formula are defined. We prove that the Fourier transform of the original option values converges to the Fourier transform of the limit option values as the time-scale parameter approaches to zero, which in turn implies the convergence of the option values. As a result, the FFT to the limit problem can be used to approximate the true option values. The computation time required can be significantly reduced. Numerical results are provided.

A Permutation Encoding of RNA Secondary Structure

Dr. Jennifer Galovich

Abstract: This talk will describe several combinatorial descriptions of RNA secondary structure topologies. I will review some of the older models and describe in detail a new model invented by my student Bob Willenbring which uses permutations. Among other things, this new model relates classical statistics on permutations to information on the secondary structures.