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2.2: Methods to Solve Hydraulic Networks - Manual Solution

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2.2. Methods to Solve Hydraulic Networks – Manual Solution

The solution to a hydraulic network is the set of flows that satisfy continuity and that satisfy energy. Recall that mass continuity must be satisfied at each junction node. Furthermore, energy must be balanced around each loop and across each path. It is critical that both mass be conserved and that energy be conserved. A set of pipeline flows that satisfy continuity yet do not satisfy energy are not the correct set of flows. Likewise, a set of flows that satisfy energy yet do not satisfy continuity are not the correct set of flows. The correct set of flows satisfy both continuity and energy.

It is not a terribly difficult matter to manually find a set of flows that satisfy continuity. All we need to insure is that the sum of the flows into a junction node minus the sum of the flows leaving the junction node is equal to the demand at the junction node. Developing a system of flows that satisfy energy can be a difficult undertaking. We will see later in this chapter that methods have been developed to systematically converge to a set of flows that satisfy continuity and that satisfy energy. However, before proceeding to these methods, let's gain an appreciation of their power by attempting to solve a simple hydraulic network in a manual fashion.

2.2.1. Manual Solution – Example #1

Consider the network shown in Figure 12 with the corresponding pipe characteristics given in Table 13.

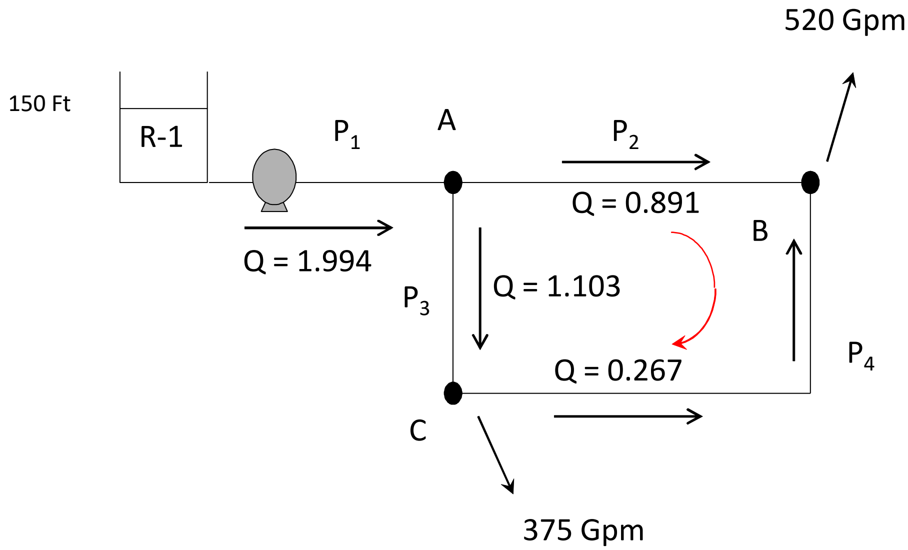

Figure 12 – Initial Flow Direction (Iteration #0)

The first thing we need to do is establish a set of flows that satisfy continuity. We know that the total system demand is 520 + 375 = 895 Gpm. Clearly therefore an initial flow for pipe P_1 = 895 Gpm is suitable. If 895 Gpm enter Node A then we can develop any combinations of flows in pipes P_2 and P_3 that equal 895 Gpm since the demand at Node A is zero. Let's assume that the flow in pipe P_2 = 400 Gpm. Therefore, by continuity, the flow in pipe P_3 = 495 Gpm.

$$\sum Q_{in,A} - \sum Q_{out,A} = 0 = 895 - (400 + 495) = 0\ Gpm$$

If we know the flow in pipe P_2 = 400 Gpm and we know the demand at node B = 520 Gpm, then from continuity we know that the flow in pipe P_4 = -120 Gpm.

$$\sum Q_{in,B} - \sum Q_{out,B} = 520 = 400 - (-120) = 52\ Gpm$$

If we know the flow in pipe P_3 = 495 Gpm and we know the demand at node C = 375 Gpm, then from continuity we know that the flow in pipe P_4 = 120 Gpm. Notice that continuity provides consistency with the flowrate in pipe P_4.

$$\sum Q_{in,C} - \sum Q_{out,C} = 375 = 495 - 120 = 375\ Gpm$$

We will see later that some energy balancing methods require an initial guess that satisfy continuity while others do not.

Table 13

Pipe Label	Length (Ft)	Diameter (In)	C-Factor	Minor Loss Coeff	Initial Guess for Discharge (Gpm)
P_1	800	12	120	10	895.00
P_2	1,200	10	120	0	400.00
P_3	1,500	8	120	0	495.00
P_4	2,200	6	120	0	120.00

Now that we have a set of flows that satisfy continuity, we need to see if energy around the loop is conserved. We do so by computing the net head change for each pipe, F(Q), and then adding the net head change terms together taking into consideration the flow directions.

Notice that there is a single loop in the system shown in Figure 12 and this loop consists of pipes P_2, P_3 and P_4. We assume a clockwise sign convention. This is shown by the red arrow in Figure 12. Pipes P_3 and P_4 have initial flow directions that oppose the sign convention, therefore we will assign the flows as negative values.

Table 14 provides a summary of the pipe resistance coefficient, K_P, the minor loss resistance coefficient, K_M, and the net head change, F(Q), for all pipes in the loop. Notice that pipe P_1 is not included in the analysis to find the correct flows because the flow in pipe P_1 will always be 895 gpm as this pipe serves as the only connection to our source of water. Also shown in the table is the initial flow in each pipe with the sign on the flow term indicating its direction with respect to the assumed sign convention. Also provided below are sample calculations for these terms.

Table 14 – Flows and Net Head Change for Loop (Iteration #0)

Pipe Label	K_P	K_M	Q (Gpm)	Q (Cfs)	F(Q)
P_2	1.945	0.000	400.00	0.891	1.571
P_3	7.209	0.000	-495.00	-1.103	-8.644
P_4	42.920	0.000	-120.00	-0.267	-3.720

$$K_{P,1} = \frac{4.73L}{C^{1.852}D^{4.87}} = \frac{4.73(800)}{120^{1.852}\left({}^{12}/_{12}\right)^{4.87}} = 0.534$$

$$A_1 = \frac{\pi}{4}\mathrm{D}^2 = \frac{\pi}{4}\left(\frac{12}{12}\right)^2 = 0.785$$

$$K_{M,1} = \frac{\Sigma K_L}{2gA^2} = \frac{10}{2g(0.785)^2} = 0.252$$

To convert flow from gallons per minute (Gpm) to cubic feet per second (Cfs) we have:

$$Q\left(\frac{ft^3}{Sec}\right) = Q\,\frac{Gal}{Min} * \frac{Min}{60\ Sec} * \frac{ft^3}{7.48\ Gal} = \frac{1\ Cfs}{448.84\ Gpm}$$

$$Q_1 = 895\ Gpm * \frac{1\ Cfs}{448.84\ Gpm} = 1.994\ Cfs$$

From Eq. (23) the net head change in any pipe can be found from:

$$F(Q)_i = K_{P,i} Q_i |Q|_i^{0.852} + K_{M,i} Q_i |Q|_i - E(|Q|)_i$$

Pipe P_2 the net head change is:

$$F(Q)_2 = 1.945(0.891)|0.891|^{0.852} + 0 * (0.891)|0.891| - 0$$
$$= 1.571\ Ft$$

Pipe P_3 the net head change is:

$$F(Q)_3 = 7.209(-1.103)|1.103|^{0.852} + 0 * (-1.103)|1.103| - 0$$
$$= -8.644\ Ft$$

Pipe P_4 the net head change is:

$$F(Q)_4 = 42.920(-0.267)|0.267|^{0.852} + 0 * (-0.267)|0.267| - 0$$
$$= -3.720\ Ft$$

We now compute the sum of the net head changes around the loop and, if energy is conserved, the sum of the net head change will be zero for the loop. Recall that for a path connecting two tanks or reservoirs, the net head change will equal the HGL difference between the two tanks/reservoirs.

For the loop consisting of pipes P_2, P_3 and P_4 the net head change is:

$$\sum_{i=1}^{NL} F(Q)_i = F(Q)_2 + F(Q)_3 + F(Q)_4$$
$$= 1.571 + (-8.644) + (-3.720) = -10.793\ Ft$$

Notice that the sum of the net head change is not equal to zero for the loop. This indicates that the flows given are not the correct set of flows even though they satisfy continuity. What we must now do is correct the flows so we conserve energy. Notice that the net head change (head loss) in pipe P_2 is too small and that for pipes P_3 and P_4 the net head change (head loss) is too large. This indicates that we must increase the flow in pipe P_2 and decrease the flow in pipes P_3 and P_4.

If we are smart and change flows by the same amount in each pipe in the loop, then we can maintain continuity. In other words, if we increase the flow in pipe P_2 by ΔQ and decrease the flow in pipes P_3 and P_4 by the same amount, we maintain continuity at the nodes.

Let's assume that $\Delta Q = 0.50$ Cfs for the loop. Notice that the flow in pipe P_1 is not changed since this pipe is not contained in the loop. The update procedure for pipes P_2, P_3 and P_4 is:

$$Q_{2,New} = Q_{2,Old} + \Delta Q = 0.891 + 0.50 = 1.391\ Cfs$$

$$Q_{3,New} = Q_{3,Old} + \Delta Q = -1.103 + 0.50 = -0.603\ Cfs$$

$$Q_{4,New} = Q_{4,Old} + \Delta Q = -0.267 + 0.50 = 0.233\ Cfs$$

Table 15 - Flows and Net Head Change for Loop (Iteration #1)

Pipe Label	K_P	K_M	Q (Cfs)	F(Q)
P_2	1.945	0.000	1.391	3.584
P_3	7.209	0.000	-0.603	-2.825
P_4	42.920	0.000	0.233	2.891

Notice that the sign on pipe P_4 has changed. This indicates that the flow direction in pipe P_4 has changed. Figure 13 shows the updated flows and their direction. Table 15 provides the updated net head change in each pipe.

Now we repeat the procedure to find the net head change around the loop.

$$\sum_{i=1}^{NL} F(Q)_i = F(Q)_2 + F(Q)_3 + F(Q)_4 = 3.584 + (-2.825) + 2.891$$
$$= 3.650\ Ft$$

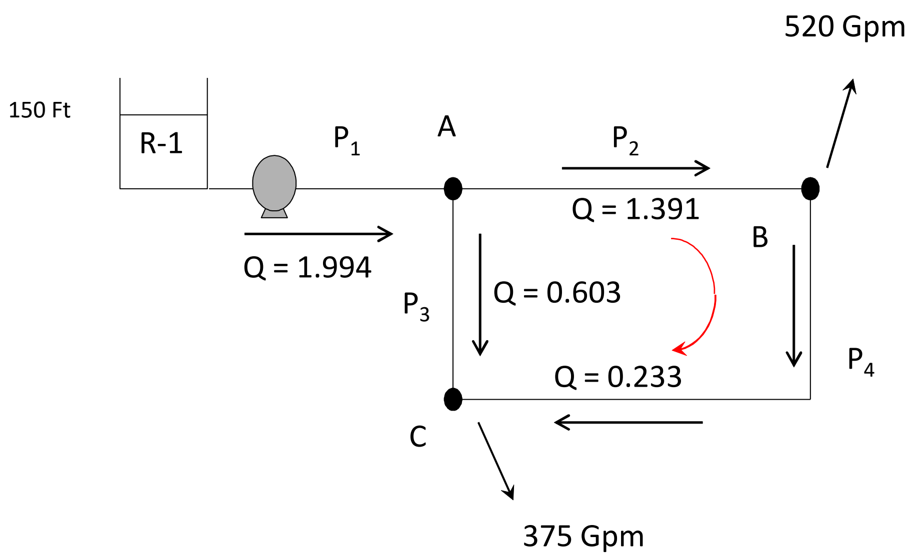

Figure 13 – Initial Flow Direction (Iteration #1)

Again the net head change around the loop is not zero, although the new flows do cause the net head change around the loop to be closer to zero. It appears that the flow in pipe P_2 and P_4 is a bit too large, so let's decrease the flow in these pipes by ΔQ and increase the flow in pipe P_3 by the same amount. Let's let ΔQ = -0.15 Cfs.

$$Q_{2,New} = Q_{2,Old} + \Delta Q = 1.391 + (-0.15) = 1.241\ Cfs$$

$$Q_{3,New} = Q_{3,Old} + \Delta Q = -0.603 + (-0.15) = -0.753\ Cfs$$

$$Q_{4,New} = Q_{4,Old} + \Delta Q = 0.233 + (-0.15) = 0.083\ Cfs$$

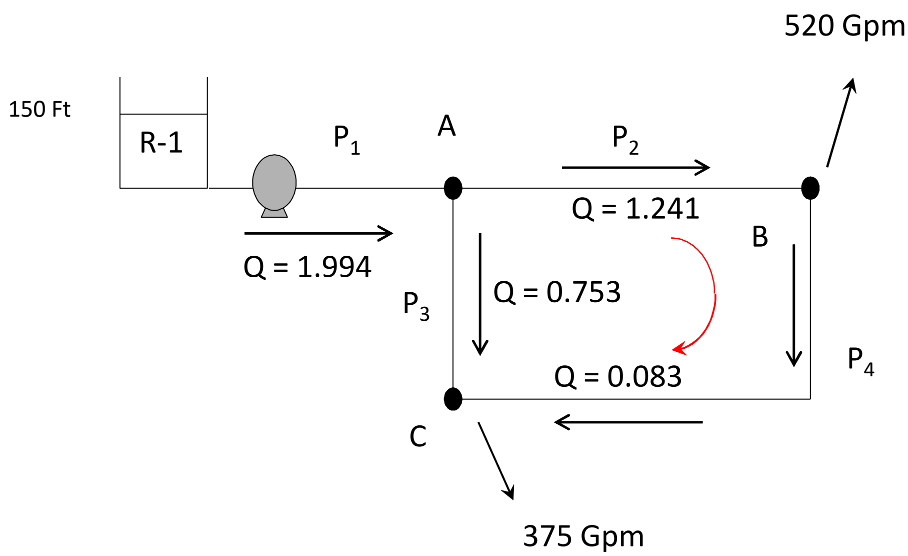

Figure 14 – Initial Flow Direction (Iteration #2)

Table 16 - Flows and Net Head Change for Loop (Iteration #2)

Pipe Label	K_P	K_M	Q (Cfs)	F(Q)
P_2	1.945	0.000	1.241	2.901
P_3	7.209	0.000	-0.753	-4.263
P_4	42.920	0.000	0.083	0.427

Now we repeat the procedure to find the net head change around the loop.

$$\sum_{i=1}^{NL} F(Q)_i = F(Q)_2 + F(Q)_3 + F(Q)_4 = 2.901 + (-4.263) + 0.427$$
$$= -0.935\ Ft$$

We are getting closer! However, the net head change around the loop still does not equal zero. It appears that the net head change in pipes P_2 and P_4 is a bit too low and that the net head change in pipe P_3 is a bit too large. So let's shift some of the flow away from P_3 and towards pipes P_2 and P_4. Let ΔQ = 0.05 Cfs.

$$Q_{2,New} = Q_{2,Old} + \Delta Q = 1.241 + 0.05 = 1.291\ Cfs$$

$$Q_{3,New} = Q_{3,Old} + \Delta Q = -0.753 + 0.05 = -0.703\ Cfs$$

$$Q_{4,New} = Q_{4,Old} + \Delta Q = 0.083 + 0.05 = 0.133\ Cfs$$

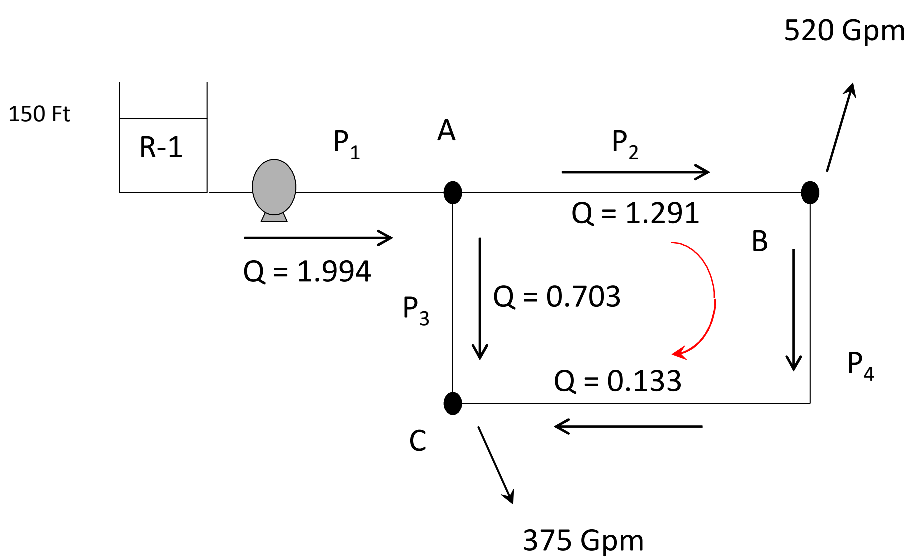

Figure 15 – Initial Flow Direction (Iteration #3)

Table 17 - Flows and Net Head Change for Loop (Iteration #3)

Pipe Label	K_P	K_M	Q (Cfs)	F(Q)
P_2	1.945	0.000	1.291	3.121
P_3	7.209	0.000	-0.703	-3.753
P_4	42.920	0.000	0.133	1.023

Now we repeat the procedure to find the net head change around the loop.

$$\sum_{i=1}^{NL} F(Q)_i = F(Q)_2 + F(Q)_3 + F(Q)_4 = 3.121 + (-3.753) + 1.023$$

$$= 0.391\,Ft$$

We continue to get closer to ΣF(Q) = 0, but we are still not quite there. So again we must repeat the procedure. It appears now that the flow in pipes P_2 and P_4 is a bit too high and that flow in pipe P_3 is a bit too low. So let ΔQ = -0.015 Cfs.

$$Q_{2,New} = Q_{2,Old} + \Delta Q = 1.291 + (-0.015) = 1.276\,Cfs$$

$$Q_{3,New} = Q_{3,Old} + \Delta Q = -0.703 + (-0.015) = -0.718\,Cfs$$

$$Q_{4,New} = Q_{4,Old} + \Delta Q = 0.133 + (-0.015) = 0.118\,Cfs$$

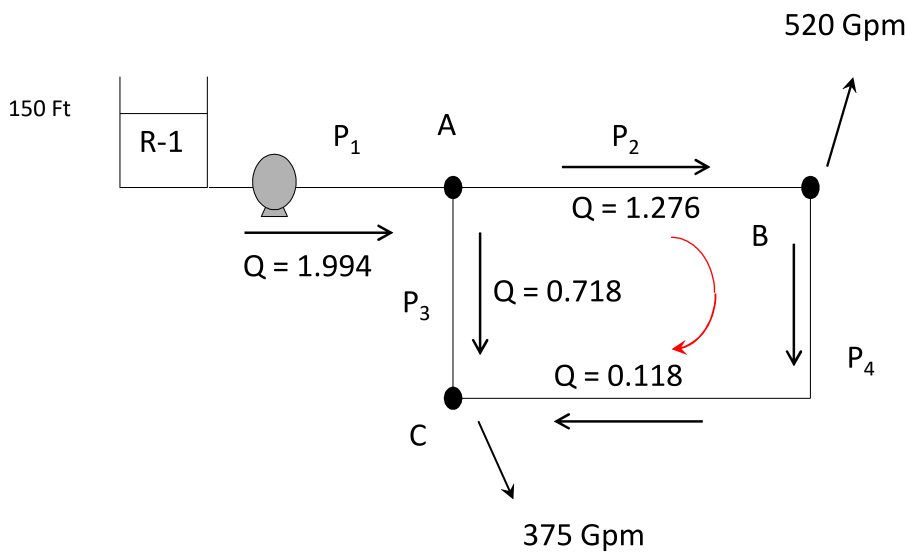

Figure 16 – Initial Flow Direction (Iteration #4)

Table 18 - Flows and Net Head Change for Loop (Iteration #4)

Pipe Label	K_P	K_M	Q (Cfs)	F(Q)
P_2	1.945	0.000	1.276	3.055
P_3	7.209	0.000	-0.718	-3.903
P_4	42.920	0.000	0.118	0.820

Now we repeat the procedure to find the net head change around the loop.

$$\sum_{i=1}^{NL} F(Q)_i = F(Q)_2 + F(Q)_3 + F(Q)_4 = 3.055 + (-3.903) + 0.820$$
$$= -0.028\ Ft$$

Wow – we are really close! Let's perform one more iteration. Notice that the flow correction factors that we have chosen (ΔQ) are converging towards zero. That's exactly what we wish to happen. It appears that the flow in pipe P_3 is just a bit too high and that the flow in pipes P_2 and P_4 are just a bit too low. Thus let $\Delta Q = 0.001$ Cfs.

$$Q_{2,New} = Q_{2,Old} + \Delta Q = 1.276 + 0.001 = 1.277\ Cfs$$

$$Q_{3,New} = Q_{3,Old} + \Delta Q = -0.718 + 0.001 = -0.717\ Cfs$$

$$Q_{4,New} = Q_{4,Old} + \Delta Q = 0.118 + 0.001 = 0.119\ Cfs$$

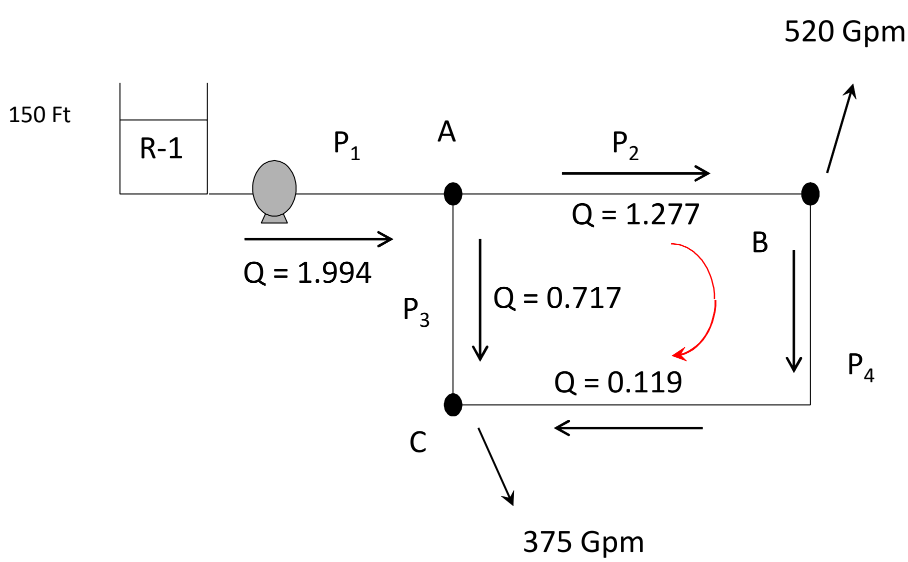

Figure 17 – Initial Flow Direction (Iteration #5)

Table 19 - Flows and Net Head Change for Loop (Iteration #5)

Pipe Label	K_P	K_M	Q (Cfs)	F(Q)
P_2	1.945	0.000	1.277	3.059
P_3	7.209	0.000	-0.717	-3.893
P_4	42.920	0.000	0.119	0.833

Now we repeat the procedure to find the net head change around the loop.

$$\sum_{i=1}^{NL} F(Q)_i = F(Q)_2 + F(Q)_3 + F(Q)_4 = 3.059 + (-3.893) + 0.833$$
$$= -0.001\ Ft$$

Note that the set of flows presented in Table 19 produce net head changes in each pipe such that the sum of the net head changes for all pipes in the loop is, for all practical purpose, zero. We have now found a set of flows that satisfy both continuity and energy and thus represent the correct set of flows for this system.

Table 20 summarizes the flow correction factors and the net head change around the loop for the different iterations used in this analysis. Notice the convergence towards $\Delta Q = 0$ and the sum of the head changes around the loop $\Sigma F(Q)=0$.

Table 20

Iteration	ΔQ	ΣF(Q)
0	N/A	-10.793
1	0.500	3.650
2	-0.150	-0.935
3	0.050	0.391
4	-0.015	-0.028
5	0.001	-0.001

2.2.2. Manual Solution – Example #2

Now let's apply the manual solution method to a more complex system containing two loops. This example will illustrate how we can apply multiple flow correction factors to pipes that are common to two or more loops.

Consider the eight-pipe system shown below with the flow arrows indicating the flow directions associated with the initial guess of flows. During the course of the calculations we may, and most likely will, experience flow direction changes in one or more pipelines. Pipeline and node data are presented in Table 21 and Table 22 respectively. Also presented is a table of head-discharge values for the pump in Table 39.

Table 21 – Pipeline Data

Pipe Label	Length (Ft)	Diameter (In)	C-Factor	Minor Loss Coeff
P_1	800	12	120	10
P_2	1200	10	120	0
P_3	1500	8	120	0
P_4	2200	6	120	0
P_5	1500	6	110	0
P_6	2000	8	120	0
P_7	3100	6	110	0
P_8	1200	8	120	0

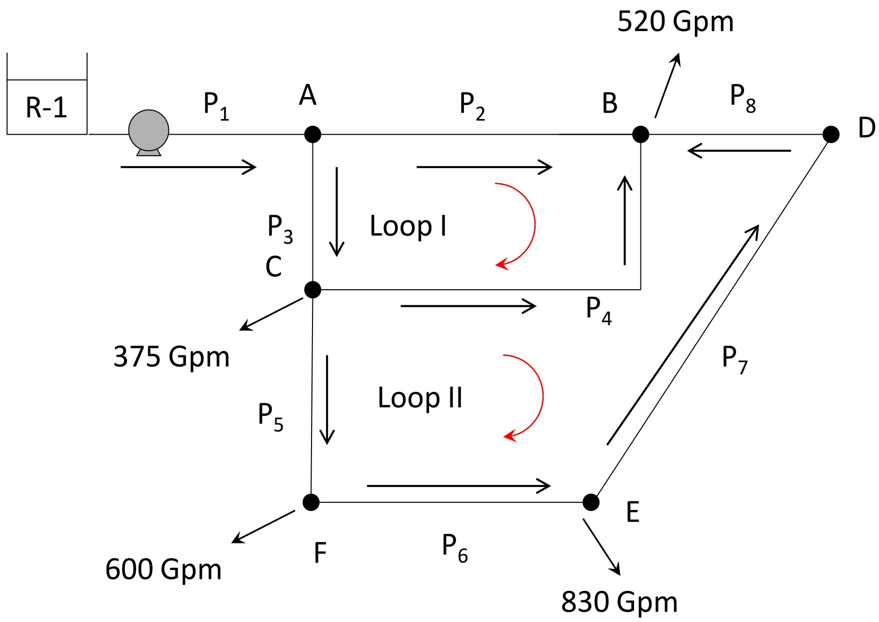

Figure 18 – Flows for Iteration 0 (Initial Flows)

Table 22 – Node Data

Node Label	Demand (Gpm)	Demand (Cfs)	Elevation
R-1	N/A	N/A	990
A	0	0	1020
B	520	1.159	1030
C	375	0.835	1030
D	0	0	1100
E	830	1.849	1080
F	600	1.337	1055

Table 23 – Pump Data

Flow (Gpm)	Pump Head (Ft)
0	350
1500	330
2800	290

As with the first example, the first step is to develop a set of flows that satisfy continuity at each junction node. Let's use the pipeline flows shown in Table 24. Let's also see how these flow satisfy continuity.

Table 24

Pipeline	Flow (Gpm)	Flow (Cfs)
P_1	2325	5.180
P_2	270	0.602
P_3	2055	4.578
P_4	100	0.223
P_5	1580	3.520
P_6	980	2.183
P_7	150	0.334
P_8	150	0.334

Recall that continuity is applied at each junction node and states the sum of pipeline flows into the node minus the sum of pipeline flows leaving the node is equal to the demand at the node. For simplicity, we will work entirely in flow units of Cfs for this example.

For Node A:

$$\sum Q_{in,A} - \sum Q_{out,A} = 0 = 5.180 - (0.602 + 4.578) = 0\ Cfs$$

For Node B:

$$\sum Q_{in,B} - \sum Q_{out,B} = 1.159 = (0.602 + 0.223 + 0.334) - 0 = 1.159\ Cfs$$

For Node C:

$$\sum Q_{in,C} - \sum Q_{out,C} = 0.835 = 4.578 - (0.223 + 3.520) = 0.835\ Cfs$$

For Node D:

$\sum Q_{in,D} - \sum Q_{out,D} = 0 = 0.334 - 0.334 = 0 \; Cfs$

For Node E:

$\sum Q_{in,E} - \sum Q_{out,E} = 1.849 = 2.183 - 0.334 = 1.849 \; Cfs$

For Node F:

$\sum Q_{in,F} - \sum Q_{out,F} = 1.337 = 3.520 - 2.183 = 1.337 \; Cfs$

Notice that continuity is satisfied at each junction node, therefore we have a suitable set of flows at least with regard to continuity. Next we need to check if energy is satisfied. Energy must be satisfied for both Loop I and Loop II. We examine Loop I first.

<u>Loop I – Iteration 0</u>

Pipe P_2 the net head change is:

$F(Q)_2 = \; 1.945(0.602)|0.602|^{0.852} + 0 * (0.602)|0.602| - 0 = 0.760 \; Ft$

Pipe P_3 the net head change is:

$F(Q)_3 = \; 7.209(-4.578)| - 4.578|^{0.852} + 0 * (-4.578)|-4.578| - 0 = -120.627 \; Ft$

Pipe P_4 the net head change is:

$F(Q)_4 = \; 42.920(-0.223)| - 0.223|^{0.852} + 0 * (-0.223)|-0.223| - 0 = -2.665 \; Ft$

The net head change around Loop I is:

$$\sum_{i=1}^{NL} F(Q)_i = F(Q)_2 + F(Q)_3 + F(Q)_4 = 0.760 + (-120.627) + (-2.665) = -122.532\ Ft$$

Table 25 – Loop I (Iteration 0)

Pipe Label	K_P	K_M	Q (Cfs)	F(Q)
P_2	1.945	0.000	0.602	0.760
P_3	7.209	0.000	-4.578	-120.627
P_4	42.920	0.000	-0.223	-2.665
			ΣF(Q)	-122.532

Table 25 provides a summary of the net head change for each pipe contained in Loop I. Clearly the net head change for Loop I is not close to zero (ΣF(Q) = -122.532). It is clear that the flow in pipe P_3 is too high so let's shift some of the flow in pipe P_3 to pipes P_2 and P_4. We do so by assuming a flow correction factor ΔQ and apply this same flow correction factor to each pipe in the loop.
Since we are performing a manual solution our selection of ΔQ is rather arbitrary. All we know is that we want the flow in pipe P_3 to decrease and the flow in pipe P_2 and P_4 to increase. So let ΔQ = 0.800 Cfs. We now update the flows for pipes contained in Loop I.

$$Q_{2,New} = Q_{2,Old} + \Delta Q = 0.602 + 0.800 = 1.402\ Cfs$$

$$Q_{3,New} = Q_{3,Old} + \Delta Q = -4.578 + 0.800 = -3.778\ Cfs$$

$$Q_{4,New} = Q_{4,Old} + \Delta Q = -0.223 + 0.800 = 0.577\ Cfs$$

Notice that there is a sign change on the flow in pipe P_4. This indicates that the flow direction in this pipe has changed. Figure 19 below shows the old and new flow directions for Loop I.

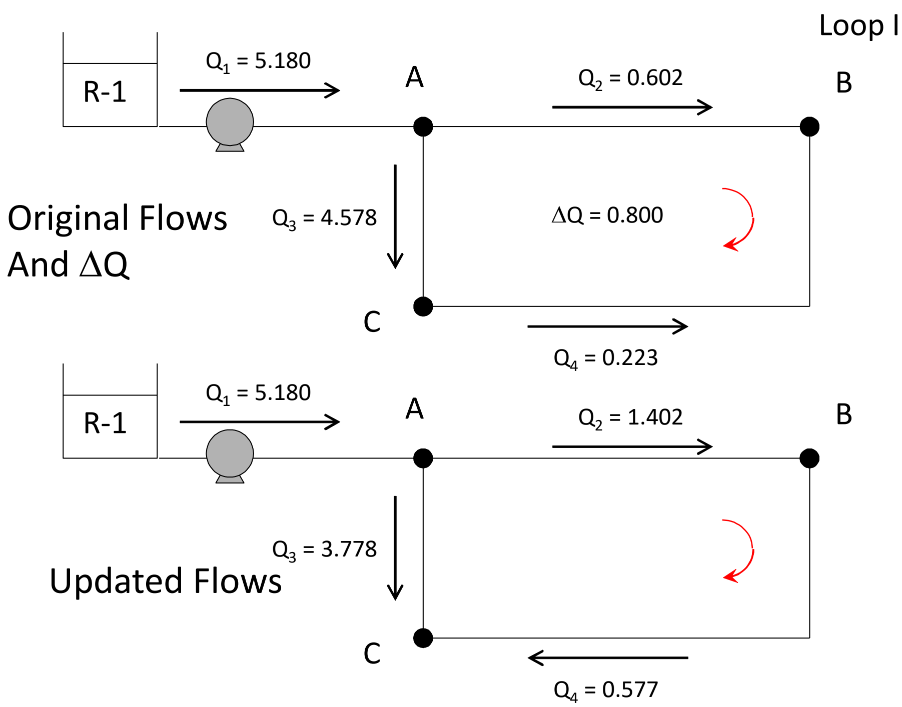

Figure 19 – Flow correction factor applied to Loop I (Iteration #1)

Loop II – Iteration 0

We now turn our attention to Loop II. Let's compute the net head change in each pipe and the net head change across the loop. Notice that we are going to use the updated value of the flow in pipe P_4 (Q=0.577 Cfs). Figure 20 shows the flow directions associated with the flows in Loop II.

Loop II

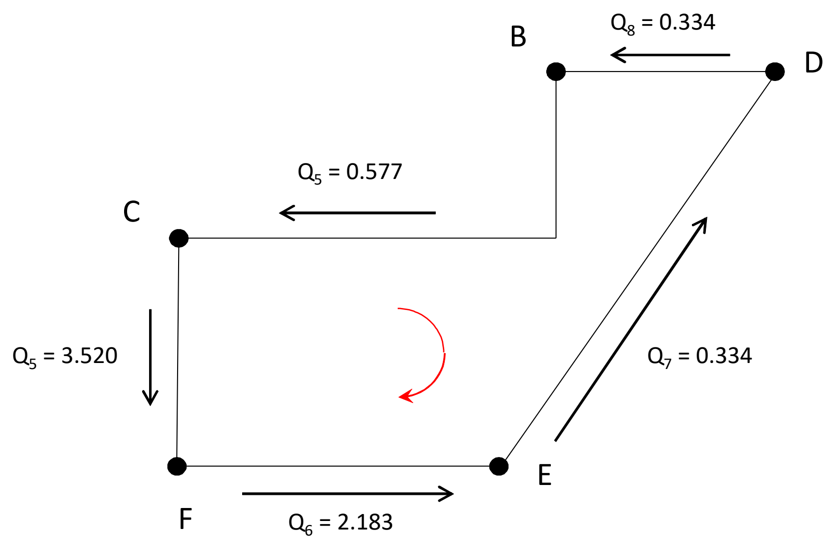

Figure 20 – Initial Flows for Loop II with updated Q_4 (Iteration #0)

Pipe P_4 the net head change is:

$F(Q)_4 = 42.920(-0.577)|-0.577|^{0.852} + 0 * (0.577)|-0.577| - 0 = -15.501\ Ft$

Pipe P_5 the net head change is:

$F(Q)_5 = 34.380(-3.520)|-3.520|^{0.852} + 0 * (-3.520)|-3.520| - 0 = -353.595\ Ft$

Pipe P_6 the net head change is:

$F(Q)_6 = 9.612(-2.183)|-2.183|^{0.852} + 0 * (-2.183)|-2.183| - 0 = -40.807\ Ft$

Pipe P_7 the net head change is:

$F(Q)_7 = 71.053(-0.334)|-0.334|^{0.852} + 0 * (-0.334)|-0.334| - 0 = -9.323\ Ft$

Pipe P_8 the net head change is:

$F(Q)_8 = 5.767(-0.334)|-0.334|^{0.852} + 0 * (-0.334)|-0.334| - 0 = -0.757\ Ft$

The net head change around Loop II is:

$\sum_{i=1}^{NL} F(Q)_i = F(Q)_4 + F(Q)_5 + F(Q)_6 + F(Q)_7 + F(Q)_8 = -15.501 - 353.595 - 40.807 - 9.323 - 0.757 = -419.983\ Ft$

Table 26 – Loop II (Iteration 0)

Pipe Label	K_P	K_M	Q (Cfs)	F(Q)
P_4	42.920	0.000	-0.577	-15.501
P_5	34.380	0.000	-3.520	-353.595
P_6	9.612	0.000	-2.183	-40.807
P_7	71.053	0.000	-0.334	-9.323
P_8	5.767	0.000	-0.334	-0.757
			ΣF(Q)	-419.983

An examination of the net head change values in Table 26 indicate that the head loss in pipe P_5 dominates the net head change around the loop, and thus the flow in this pipe is too large. So let's decrease the flow in pipe P_5 by setting ΔQ=1.000 Cfs. Figure 21 shows the updated flow directions and their values.

$Q_{4,New} = Q_{4,Old} + \Delta Q = -0.577 + 1.000 = 0.423\ Cfs$

$Q_{5,New} = Q_{5,Old} + \Delta Q = -3.520 + 1.000 = -2.520\ Cfs$

$Q_{6,New} = Q_{6,Old} + \Delta Q = -2.183 + 1.000 = -1.183\ Cfs$

$$Q_{7,New} = Q_{7,Old} + \Delta Q = -0.334 + 1.000 = 0.666\ Cfs$$

$$Q_{8,New} = Q_{8,Old} + \Delta Q = -0.334 + 1.000 = 0.666\ Cfs$$

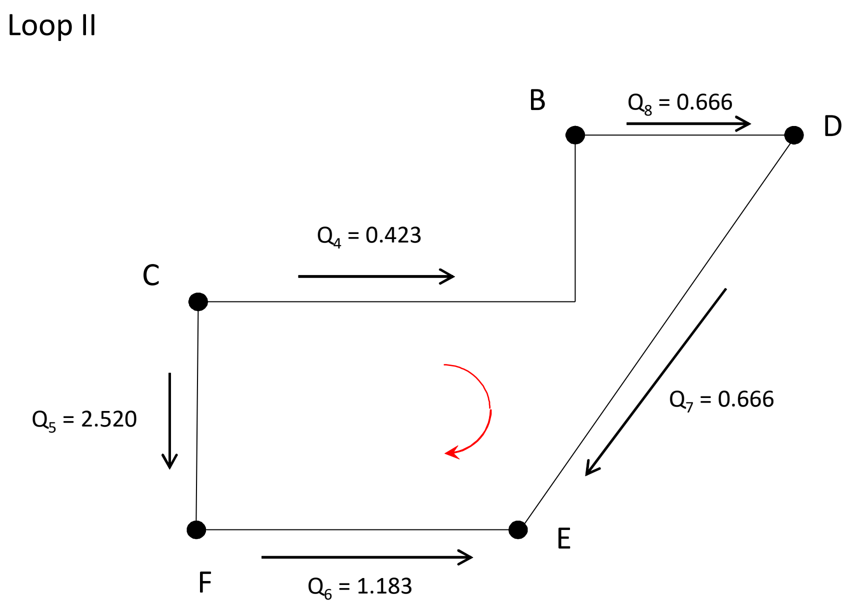

Figure 21 – Loop II with updated flows (Iteration #1)

Loop I – Iteration 1

We now repeat the process with Loop I using the most recent set of pipeline flows. Again our objective is to find the set of flows that satisfy continuity and energy. As long as we apply flow correction factors in the manner that we have been doing, then we will maintain continuity – assuming, of course, that we start with a set of flows that satisfy continuity.

Pipe P_2 the net head change is:

$F(Q)_2 = 1.945(1.402)|1.402|^{0.852} + 0 * (1.402)|1.402| - 0 = 3.637\ Ft$

Pipe P_3 the net head change is:

$F(Q)_3 = 7.209(-3.778)|-3.778|^{0.852} + 0 * (-3.778)|-3.778| - 0 = -84.521\ Ft$

Pipe P_4 the net head change is:

$F(Q)_4 = 42.920(-0.423)|-0.423|^{0.852} + 0 * (-0.423)|-0.423| - 0 = -8.722\ Ft$

The net head change around Loop I is:

$\sum_{i=1}^{NL} F(Q)_i = F(Q)_2 + F(Q)_3 + F(Q)_4 = 3.637 + (-84.521) + (-8.722) = -89.606\ Ft$

Table 27 – Loop I (Iteration 1)

Pipe Label	K_P	K_M	Q (Cfs)	F(Q)
P_2	1.945	0.000	1.402	3.637
P_3	7.209	0.000	-3.778	-84.521
P_4	42.920	0.000	-0.423	-8.722
			ΣF(Q)	-89.606

Table 27 provides a summary of the net head change for each pipe contained in Loop I. Again the net head change for Loop I is not close to zero (ΣF(Q) = -89.606). Again it is clear that the flow in pipe P_3 is too high so let's shift some of the flow in pipe P_3 to pipes P_2 and P_4. Let ΔQ=0.800 Cfs.

$Q_{2,New} = Q_{2,Old} + \Delta Q = 1.402 + 0.800 = 2.202\ Cfs$

$$Q_{3,New} = Q_{3,Old} + \Delta Q = -3.778 + 0.800 = -2.978\ Cfs$$

$$Q_{4,New} = Q_{4,Old} + \Delta Q = -0.423 + 0.800 = 0.377\ Cfs$$

Notice that there is a sign change on the flow in pipe P_4. This indicates that the flow direction in this pipe has changed. Figure 22 below shows the old and new flow directions for Loop I.

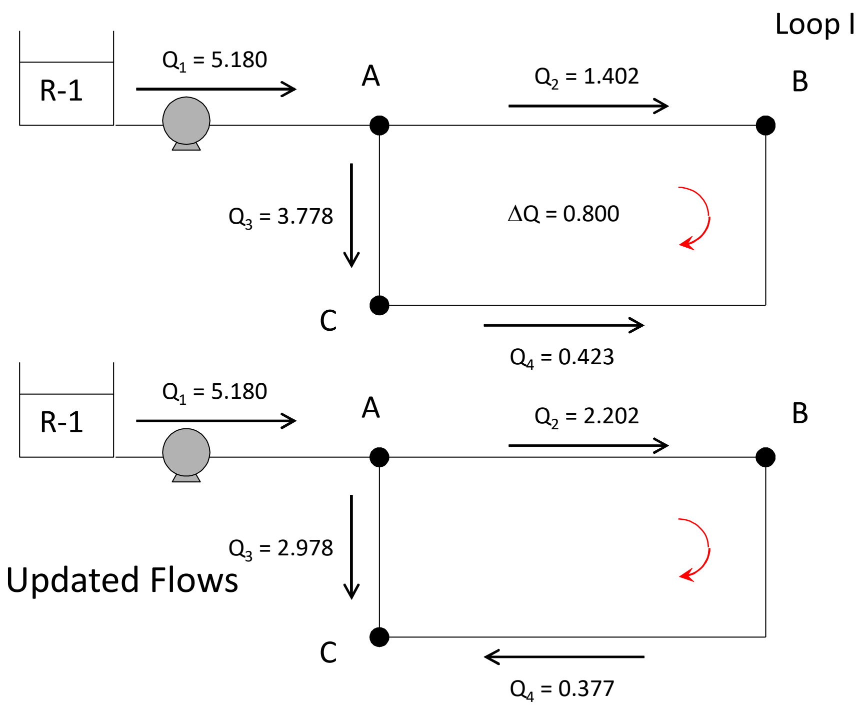

Figure 22 – Flow correction factor applied to Loop I (Iteration #1)

Loop II – Iteration 1

We now turn our attention to Loop II. Let's compute the net head change in each pipe and the net head change across the loop. Notice that we are going to use the updated value of the flow in pipe P_4 (Q=0.377 Cfs). Figure 23 shows the flow directions associated with the flows in Loop II.

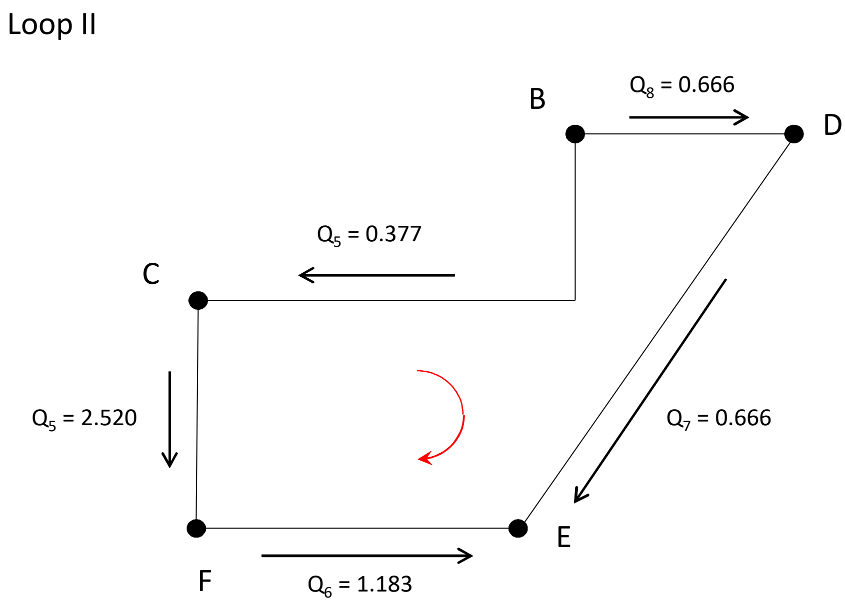

Figure 23 – Initial Flows for Loop II with updated Q_4 (Iteration #1)

Pipe P_4 the net head change is:

$$F(Q)_4 = 42.920(-0.377)|-0.377|^{0.852} + 0 * (0.377)|-0.377| - 0 = -7.048\ Ft$$

Pipe P_5 the net head change is:

$$F(Q)_5 = 34.380(-2.520)|-2.520|^{0.852} + 0 * (-2.520)|-2.520| - 0 = -190.416\ Ft$$

Pipe P_6 the net head change is:

$$F(Q)_6 = 9.612(-1.183)|-1.183|^{0.852} + 0 * (-1.183)|-1.183| - 0 = -13.121\ Ft$$

Pipe P_7 the net head change is:

$$F(Q)_7 = 71.053(0.666)|0.666|^{0.852} + 0 * (0.666)|0.666| - 0 = 33.470\ Ft$$

Pipe P_8 the net head change is:

$$F(Q)_8 = 5.767(0.666)|0.666|^{0.852} + 0 * (0.666)|0.666| - 0 = 2.717\ Ft$$

The net head change around Loop II is:

$$\sum_{i=1}^{NL} F(Q)_i = F(Q)_4 + F(Q)_5 + F(Q)_6 + F(Q)_7 + F(Q)_8 = -7.048 - 190.416 - 13.121 + 33.470 + 2.717 = -174.398\ Ft$$

Table 28 – Loop II (Iteration 1)

Pipe Label	K_P	K_M	Q (Cfs)	F(Q)
P_4	42.920	0.000	-0.377	-7.048
P_5	34.380	0.000	-2.520	-190.416
P_6	9.612	0.000	-1.183	-13.121
P_7	71.053	0.000	0.666	33.470
P_8	5.767	0.000	0.666	2.717
			ΣF(Q)	-174.398

An examination of the net head change values in Table 28 indicate that the head loss in pipe P_5 still dominates the net head change around the loop, and thus the flow in this pipe is still too large. So let's decrease the flow in pipe P_5 by setting ΔQ=0.700 Cfs. Figure 24 shows the updated flow directions and their values.

$$Q_{4,New} = Q_{4,Old} + \Delta Q = -0.377 + 0.700 = 0.323\ Cfs$$

$$Q_{5,New} = Q_{5,Old} + \Delta Q = -2.520 + 0.700 = -1.820\ Cfs$$

$$Q_{6,New} = Q_{6,Old} + \Delta Q = -1.183 + 0.700 = -0.483\ Cfs$$

$$Q_{7,New} = Q_{7,Old} + \Delta Q = 0.666 + 0.700 = 1.366\ Cfs$$

$$Q_{8,New} = Q_{8,Old} + \Delta Q = 0.666 + 0.700 = 1.366\ Cfs$$

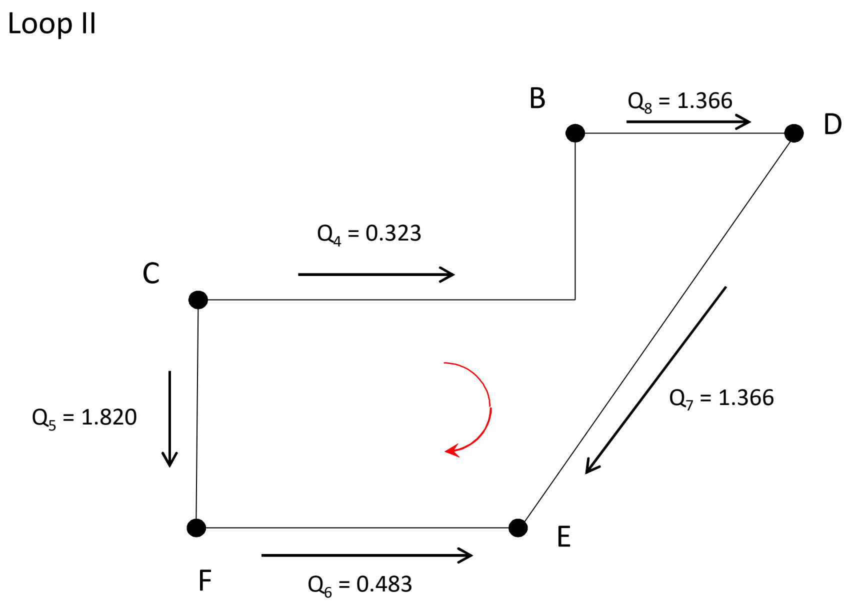

Figure 24 – Loop II with updated flows (Iteration #1)

Loop I – Iteration 2

We now repeat the process with Loop I using the most recent set of pipeline flows. Again our objective is to find the set of flows that satisfy continuity and energy. As long as we apply flow correction factors in the manner that we have been doing, then we will maintain continuity – assuming, of course, that we start with a set of flows that satisfy continuity.

Pipe P_2 the net head change is:

$$F(Q)_2 = 1.945(2.202)|2.202|^{0.852} + 0 * (2.202)|2.202| - 0 = 8.393\ Ft$$

Pipe P_3 the net head change is:

$$F(Q)_3 = 7.209(-2.978)|-2.978|^{0.852} + 0 * (-2.978)|-2.978| - 0 = -54.398\ Ft$$

Pipe P_4 the net head change is:

$$F(Q)_4 = 42.920(-0.323)|-0.323|^{0.852} + 0 * (-0.323)|-0.323| - 0 = -5.293\ Ft$$

The net head change around Loop I is:

$$\sum_{i=1}^{NL} F(Q)_i = F(Q)_2 + F(Q)_3 + F(Q)_4 = 8.393 + (-54.398) + (-5.293) = -51.298\ Ft$$

Table 29 – Loop I (Iteration 2)

Pipe Label	K_P	K_M	Q (Cfs)	F(Q)
P_2	1.945	0.000	2.202	8.393
P_3	7.209	0.000	-2.978	-54.398
P_4	42.920	0.000	-0.323	-5.293
			ΣF(Q)	-51.298

Table 29 provides a summary of the net head change for each pipe contained in Loop I. Again the net head change for Loop I does not equal zero (ΣF(Q) = -51.298) though we are heading in the right direction. Again it appears that the flow in pipe P_3 is too high so let's shift some of the flow in pipe P_3 to pipes P_2 and P_4. Let ΔQ=1.000 Cfs. Figure 25 below shows the old and new flow directions for Loop I.

$$Q_{2,New} = Q_{2,Old} + \Delta Q = 2.202 + 1.000 = 3.202\ Cfs$$

$$Q_{3,New} = Q_{3,Old} + \Delta Q = -2.978 + 1.000 = -1.978\ Cfs$$

$$Q_{4,New} = Q_{4,Old} + \Delta Q = -0.323 + 1.000 = 0.677\ Cfs$$

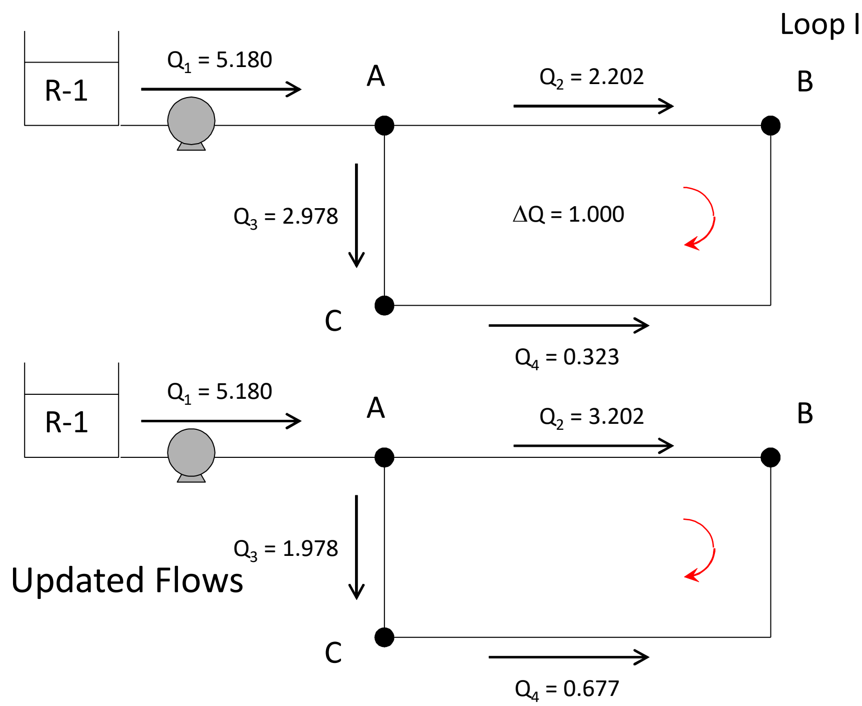

Figure 25 – Flow correction factor applied to Loop I (Iteration #2)

<u>Loop II – Iteration 2</u>

We now turn our attention to Loop II. Let's compute the net head change in each pipe and the net head change across the loop. Figure 26 shows the flow directions associated with the flows in Loop II.

Loop II

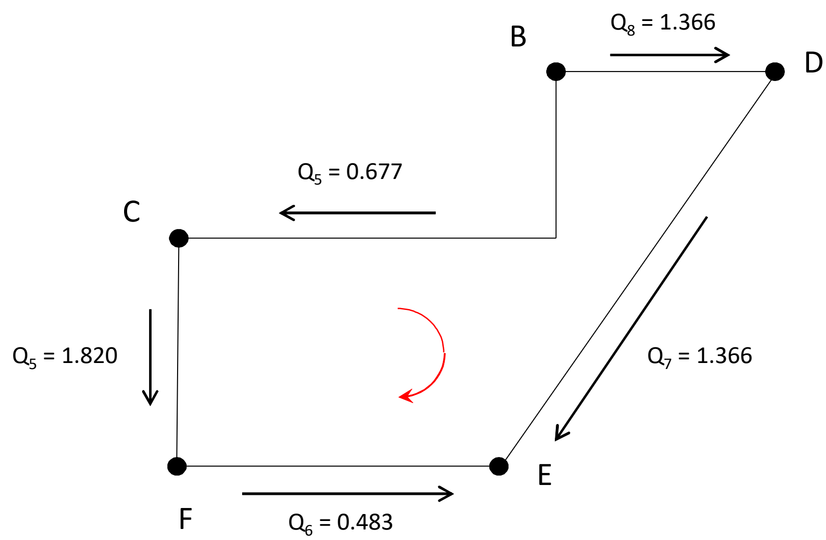

Figure 26 – Initial Flows for Loop II with updated Q_4 (Iteration #2)

Pipe P_4 the net head change is:

$$F(Q)_4 = 42.920(-0.677)|-0.677|^{0.852} + 0 * (0.677)|-0.677| - 0 = -20.840\ Ft$$

Pipe P_5 the net head change is:

$$F(Q)_5 = 34.380(-1.820)|-1.820|^{0.852} + 0 * (-1.820)|-1.820| - 0 = -104.222\ Ft$$

Pipe P_6 the net head change is:

$$F(Q)_6 = 9.612(-0.483)|-0.483|^{0.852} + 0 * (-0.483)|-0.483| - 0 = -2.497\ Ft$$

Pipe P_7 the net head change is:

$F(Q)_7 = 71.053(1.366)|1.366|^{0.852} + 0 * (1.366)|1.366| - 0 = 126.600\ Ft$

Pipe P_8 the net head change is:

$F(Q)_8 = 5.767(1.366)|1.366|^{0.852} + 0 * (1.366)|1.366| - 0 = 10.276\ Ft$

The net head change around Loop II is:

$\sum_{i=1}^{NL} F(Q)_i = F(Q)_4 + F(Q)_5 + F(Q)_6 + F(Q)_7 + F(Q)_8 = -20.840 - 104.222 - 2.497 + 126.600 + 10.276 = 9.317\ Ft$

Table 30 – Loop II (Iteration 2)

Pipe Label	K_P	K_M	Q (Cfs)	F(Q)
P_4	42.920	0.000	-0.677	-20.840
P_5	34.380	0.000	-1.820	-104.222
P_6	9.612	0.000	-0.483	-2.497
P_7	71.053	0.000	1.366	126.600
P_8	5.767	0.000	1.366	10.276
			ΣF(Q)	9.317

An examination of the net head change values in Table 30 indicate that perhaps we should lower the flow in pipe P_7 just a bit. However, we will change the discharge in pipe P_4 and that will have an effect on the net head change in Loop I. So we must take care to select a flow correction factor that optimizes the overall energy balance. That is, we should select a flow correction factor that acts to lower the net head change around Loop I and Loop II at the same time.

So let's decrease the flow in pipe P_7 by setting $\Delta Q=-0.050$ Cfs. Figure 27 shows the updated flow directions and their values.

$$Q_{4,New} = Q_{4,Old} + \Delta Q = -0.677 + (-0.050) = -0.727\ Cfs$$

$$Q_{5,New} = Q_{5,Old} + \Delta Q = -1.820 + (-0.050) = -1.870\ Cfs$$

$$Q_{6,New} = Q_{6,Old} + \Delta Q = -0.483 + (-0.050) = -0.533\ Cfs$$

$$Q_{7,New} = Q_{7,Old} + \Delta Q = 1.366 + (-0.050) = 1.316\ Cfs$$

$$Q_{8,New} = Q_{8,Old} + \Delta Q = 1.366 + (-0.050) = 1.316\ Cfs$$

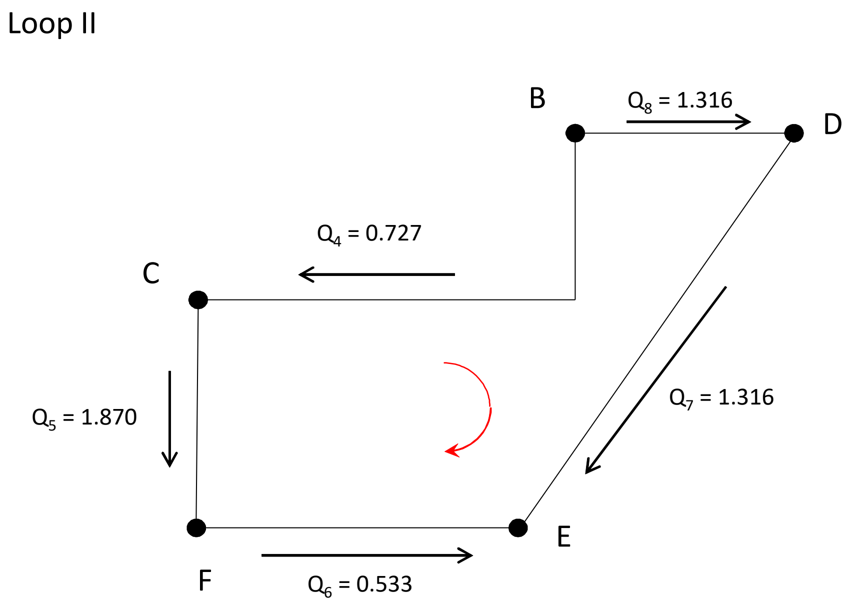

Figure 27 – Loop II with updated flows (Iteration #2)

<u>Loop I – Iteration 3</u>

We now repeat the process with Loop I using the most recent set of pipeline flows. Again our objective is to find the set of flows that satisfy continuity and energy. As long as we apply flow correction factors in the manner that we have been doing, then we will maintain continuity – assuming, of course, that we start with a set of flows that satisfy continuity.

Pipe P_2 the net head change is:

$$F(Q)_2 = 1.945(3.202)|3.202|^{0.852} + 0 * (3.202)|3.202| - 0 = 16.790\ Ft$$

Pipe P_3 the net head change is:

$$F(Q)_3 = 7.209(-1.978)|-1.978|^{0.852} + 0 * (-1.978)|-1.978| - 0 = -25.497\ Ft$$

Pipe P_4 the net head change is:

$$F(Q)_4 = 42.920(0.727)|0.727|^{0.852} + 0 * (0.727)|0.727| - 0 = 23.780\ Ft$$

The net head change around Loop I is:

$$\sum_{i=1}^{NL} F(Q)_i = F(Q)_2 + F(Q)_3 + F(Q)_4 = 16.790 + (-25.497) + 23.780 = 15.073\ Ft$$

Table 31 – Loop I (Iteration 3)

Pipe Label	K_P	K_M	Q (Cfs)	F(Q)
P_2	1.945	0.000	3.202	16.790
P_3	7.209	0.000	-1.978	-25.497
P_4	42.920	0.000	0.727	23.780
			ΣF(Q)	15.073

Table 31 provides a summary of the net head change for each pipe contained in Loop I. Again the net head change for Loop I does not equal zero (ΣF(Q) = 15.073) though we are heading in the right direction.

This time it appears that the flow in pipes P_2 and P_4 is a little too high. So we will reduce the flow in this pipes while simultaneously increasing the flow in pipe P_3 by the same amunt. Let ΔQ=-0.172 Cfs. Figure 28 below shows the old and new flow directions for Loop I.

$$Q_{2,New} = Q_{2,Old} + \Delta Q = 3.202 + (-0.172) = 3.030\ Cfs$$

$$Q_{3,New} = Q_{3,Old} + \Delta Q = -1.978 + (-0.172) = -2.150\ Cfs$$

$$Q_{4,New} = Q_{4,Old} + \Delta Q = 0.727 + (-0.172) = 0.555\ Cfs$$

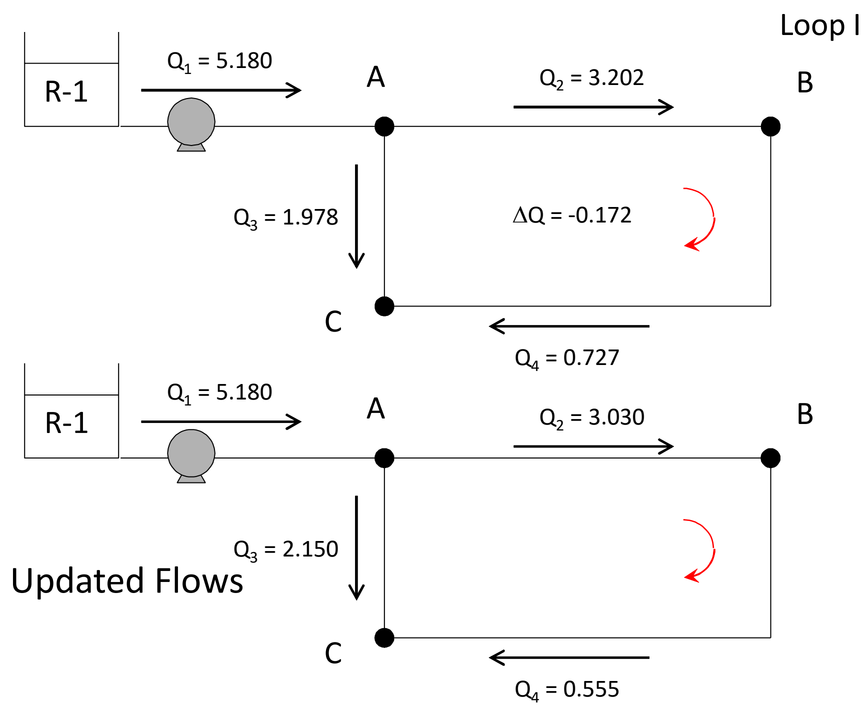

Figure 28 – Flow correction factor applied to Loop I (Iteration #3)

<u>Loop II – Iteration 3</u>

We now turn our attention to Loop II. Let's compute the net head change in each pipe and the net head change across the loop. Figure 29 shows the flow directions associated with the flows in Loop II.

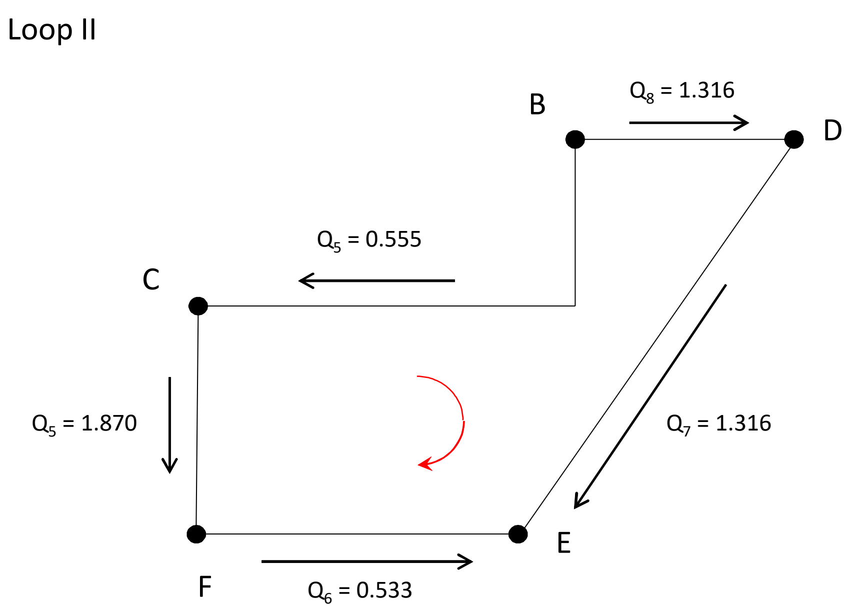

Figure 29 – Initial Flows for Loop II with updated Q_4 (Iteration #3)

Pipe P_4 the net head change is:

$$F(Q)_4 = 42.920(-0.555)|-0.555|^{0.852} + 0 * (0.555)|-0.555| - 0 = -14.424\ Ft$$

Pipe P_5 the net head change is:

$$F(Q)_5 = 34.380(-1.870)|-1.870|^{0.852} + 0 * (-1.870)|-1.870| - 0 = -109.587\ Ft$$

Pipe P_6 the net head change is:

$F(Q)_6 = 9.612(-0.533)|-0.533|^{0.852} + 0 * (-0.533)|-0.533| - 0 = -2.997\ Ft$

Pipe P_7 the net head change is:

$F(Q)_7 = 71.053(1.316)|1.316|^{0.852} + 0 * (1.316)|1.316| - 0 = 118.152\ Ft$

Pipe P_8 the net head change is:

$F(Q)_8 = 5.767(1.316)|1.316|^{0.852} + 0 * (1.316)|1.316| - 0 = 9.590\ Ft$

The net head change around Loop II is:

$\sum_{i=1}^{NL} F(Q)_i = F(Q)_4 + F(Q)_5 + F(Q)_6 + F(Q)_7 + F(Q)_8 = -14.424 - 109.587 - 2.997 + 118.152 + 9.590 = 0.734\ Ft$

Table 32 – Loop II (Iteration 3)

Pipe Label	K_P	K_M	Q (Cfs)	F(Q)
P_4	42.920	0.000	-0.555	-14.424
P_5	34.380	0.000	-1.870	-109.587
P_6	9.612	0.000	-0.533	-2.997
P_7	71.053	0.000	1.316	118.152
P_8	5.767	0.000	1.316	9.590
			ΣF(Q)	0.734

An examination of the net head change values in Table 32 indicate that the flows have converged for Loop II since the net head change around the loop is sufficiently close to zero (ΣF(Q) = 0.734). Now we just need to confirm that the net head change around Loop I is sufficiently close to zero. If it is, then we have found the correct set of flows for this system.

Table 33 shows that the net head change around Loop I for the current set of flows is sufficiently close to zero that we may assume that we have converged to the correct set of flows (ΣF(Q)=-0.172). Incidentally, it is a coincidence that the net head change around Loop I for Iteration 3 has the same value as the flow correction factor for Loop I.

Table 33 – Loop I (Iteration 3)

Pipe Label	K_P	K_M	Q (Cfs)	F(Q)
P_2	1.945	0.000	3.030	15.158
P_3	7.209	0.000	-2.150	-29.754
P_4	42.920	0.000	0.555	14.424
			ΣF(Q)	-0.172

Clearly a manual solution method such as the one used here is not efficient and will be terribly difficult to implement particularly for large complex hydraulic networks. We seek a systematic way to find flow correction factors so that we converge to a solution. The following sections describe methods that are designed to do exactly this.

Table 34 – Summary of Flow Correction Factors

	ΣF(Q)		ΔQ	
Iteration	Loop I	Loop II	Loop I	Loop II
0	-122.532	-419.983	0.800	1.000
1	-89.606	-174.398	0.800	0.700
2	-51.298	9.317	1.000	-0.050
3	15.073	0.734	-0.172	0.000
4	-0.172	0.734	0.000	0.000