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2.6: Node Equations

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2.6. Node Equations

The methodologies presented in the sections above are examples of loop equations. With loop equations the unknown is the flowrate in each pipe. Flows are found by solving the loop and path (energy) equations expressed in terms of pipeline flows.

For example, $\Sigma F(Q)_i = 0$ for a loop where $F(Q)_i$ is the net head change across pipe i expressed in terms of the flow rate through the pipe. The Hazen-Williams and Darcy-Weisbach head loss equations are convenient tools that express head change as a function of flow rate.

Once pipeline flows are found, then the head at each node can be computed starting at a known head – such as a tank or reservoir – and then applying the energy equation in a sequential manner one node at a time until all nodal heads are found.

Head equations express pipeline flows in terms of the nodal heads. Methods based on head equations solve for the head at each node by satisfying continuity in terms of nodal heads. Once heads are found, then pipeline flows can be determined. Once pipeline flows are found, then continuity can be examined to determine if it is satisfied.

Recall that the net head change for a pipe is equal to the difference in head between the upstream and downstream end of the pipe. Also recall that using the Hazen-Williams head loss equation, the net head change for a pipe that has no pump nor any minor losses can be expressed as:

$$F(Q)_i = K_{P,i}Q_i^{1.852} + K_{m,i}Q_i^2 - E(Q)_i = H_{Up,i} - H_{Down,i} = \Delta H_i$$

For a pipeline that has no minor losses nor pumps we can solve for the pipeline flow in terms of the nodal head as:

$$Q_i = \left(\frac{H_{Up,i} - H_{Down,i}}{K_{P,i}}\right)^{1/1.852}$$

Eq. (71)

Recall that in order for a hydraulic network to be balanced, continuity and energy must be satisfied. We write a continuity equation for each node except with the node equations the pipeline flows are expressed in terms of head loss.

We know that for any junction node in the system the following continuity equation applies:

$$\sum_i Q_{in,j} - \sum_i Q_{out,j} - D_j = 0 \quad j = 1,2, \dots J$$

Eq. (1)

If we express the pipeline flows in terms of the heads at the upstream and downstream end of a pipe using the Hazen-Williams head loss formula, then the continuity equation becomes:

$$\sum_i \left(\frac{H_{up,i} - H_{down,i}}{K_{p,i}}\right)_j^{1/1.852} - D_j = 0 \quad j = 1,2, \dots J$$

Where: $H_{up,i}$ – head at the upstream end of pipe i
$H_{down,i}$ – head at the downstream end of pipe i
$K_{p,i}$ – pipe resistance coefficient for pipe i
D_j – demand at junction node j

For pipes that contain minor losses or contain pumps the difficulty level associated with finding the pipeline flow increases. For pipe i the net head change is:

$$F(Q)_i = K_{P,i}Q_i^{1.852} + K_{m,i}Q_i^2 - E(Q)_i = H_{Up,i} - H_{Down,i} = \Delta H_i$$

If we include a pump head-discharge relationship having the form shown in Eq. (72) below into the net head change equation we have Eq. (73).

$$E(Q)_i = H_o - cQ_i^N$$

Eq. (72)

Notice that Eq. (73) cannot be solved explicitly for discharge. In cases where 1) the Hazen-Williams head loss equation is used and 2) a minor loss term exists and/or 3) a pump exists, then we must use a root solving technique to find the value of Q that satisfies the net head change equation. If there are no minor losses or pumps in the pipe, then we can use Eq. (71) to find the pipeline flow.

$$F(Q)_i = K_{P,i}Q_i|Q|_i^{0.852} + K_{m,i}Q_i|Q|_i - \left(H_o - c|Q|_i^N\right) = \Delta H_i$$

Eq. (73)

The objective when using head equations is to find the head at each junction node that produces pipeline flows that satisfy continuity. We will see that it is not a particularly easy task to find the nodal heads manually. Rather we seek a systematic way of finding the nodal heads. Just as we sought a systematic way of finding pipe flows using the loop equations, i.e. flow correction factors, we will develop a systematic way of finding head correction factors to assist in our solution effort. First, though, we will perform a manual solution to illustrate how the solution to the node equations works.