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Ancillary Service Capacity Optimization for Both Electric Power Suppliers and Independent System Operator

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Abstract

Ancillary Services (AS) in electric power industry are critical to support the transmission of energy from generators to load demands while maintaining reliable operation of transmission systems in accordance with good utility practice. The ancillary services are procured by the independent system operator (ISO) through a process called the market clearing process which can be modeled by the partial equilibrium from the ends of ISO. There are two capacity optimization problems for both Market participants (MP) and Independent System Operator (ISO). For a market participant, the firm needs to determine the capacity allocation plan for various AS to pursue operating revenue under various uncertainties which can never be accurately estimated. We thereby employ a heuristic named "resource reservation" to suggest two types of bids, the regular and the must-win for a market participant to pursue higher expected revenue and satisfactory performance in terms of revenue under the worst case scenario. Meanwhile, the ISO, needs to determine the total amount of capacity required to guarantee the overall reliability of the transmission system. Our numerical experiment is based on our industrial partner' operational data and the simulation result suggests that our proposed methods would greatly

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outperform the deterministic methods in terms of the profitability for a market participant and the ISO's entire system's reliability.

Keywords: Electricity Markets, Ancillary Services, Resource Reservation, Chance Constrained Optimization

1 Introduction

1.1 ancillary services market for the power system reliability

In the electricity industry, ancillary service is critical to support the transmission of generated energy to loads, while maintaining reliable operation of the Transmission Service Providers (TSPs) in the transmission system. Ancillary services (AS) are commonly recognized in the industry as a collection of secondary services offered to ensure the reliability and availability of energy. According to the Federal Energy Regulatory Commission (FERC), the ancillary Services are defined as "services necessary to support the transmission of electric power from seller to purchaser given the obligations of control areas and transmitting utilities within those control areas to maintain reliable operations of the interconnected transmission system" in its Notice of Proposed Rulemaking (NOPR) (see [5]).

Ancillary services have been an important topic since 1995 when the FERC issued a major rule on open-access nondiscriminatory transmission service. Ancillary services are now openly traded at a financial settlement market for efficient acquisition and pricing. More importantly, the ancillary services market is beneficial to the power system for the following reasons: 1) the market provides transparent economic signals to govern the provision of these services; 2) the market reconciles operating practices with market incentives so that participating parties are compensated for providing reliability; 3) ancillary services market would also reduce cost to meet reliability requirements; and 4) correctly pricing energy and ancillary service under shortage conditions is important for resource adequacy in an Energy-only market [12]. Hence, ancillary services have quickly become irreplaceable functions to ensure the reliability and availability of energy to consumers.

1.2 Ancillary services acquired

The procurement of ancillary services is either cost-based or market-based. The cost-based procurement is to purchase services offered at pre-determined regulated costs and the market-based procurement is to purchase the services provided at market rates, granted by state or federal authorities. There are four ancillary services: regulation reserve (denoted in short as "regulation" in this paper), spinning reserve ("spinning"), non-spinning reserve ("non-spinning"), and replacement reserve ("replacement"), which are procured through market clearing processes. Before introducing the market clearing process, we briefly introduce these four major ancillary services.

Regulation is the use of online generation of well-configured units that can change output quickly to track the minimal fluctuations, e.g. $1 \sim 5$ seconds in customer loads and unintended fluctuations (see [6]). Regulation helps maintain interconnection frequency, minimize differences between actual and scheduled power flows between control areas, and match generation to load within the control area.

Spinning is the use of online generating equipment which is synchronized to the grid to increase output responding immediately to changes in power balance, so that it can be fully available within ten minutes to correct for generation/load imbalances caused by generation or transmission outages (see [6]).

Non-Spinning can be synchronized and ramped to a specified output level within 30 minutes and operates at a specified output level for at least one hour. Non-Spinning reserve may also be provided from unloaded capacity which meets the 30-minute response requirements, and is reserved exclusively to be used for this service.

Replacement is the service by which the ISO needs to make sure it has enough online capacity for a well-functioning balancing market on an hourly basis (see [14] for details).

Other ancillary services We must remark that there are two other services, voltage support and black start. These services are not required through a market clearing process, and therefore, are excluded from the scope of this paper.

1.3 Market clearing process

Ancillary services are procured by the market clearing process which determines quantities and prices of ancillary services where the quantity supplied equals to quantity demanded. In the past, ISO used to procure ancillary services in the *sequential* market in which ancillary services market bids by certain supplier include the overall available capacity for ancillary services, bid prices and *fixed* capacities by service types. This system has been proved less efficient, when closely related services, such as regulation and spinning are procured in separate markets, and in such cases inefficiencies can occur (see [2]). Some cases are observed, where higher valued ancillary services are priced lower (see [1] and [8]). Some ISO, e.g. the Electric Reliability Council of Texas (ERCOT), has changed the ancillary markets to avoid this unwanted consequence.

The current experience is called *simultaneous co-optimization*, which determines energy and ancillary service schedules *at the same time*, based on an evaluation of all the trade-offs involved in resource scheduling. There are several benefits for implementing the co-optimization. First, the co-optimization will minimize total cost of energy and ancillary services; ensure that all energy and ancillary services requirements are satisfied; consider trade-offs between a unit producing energy or providing ancillary services; and a market participant will have incentive to submit offers that reflect their actual marginal costs. We use example 1.3 to illustrate the co-optimization.

Example 1 (Example for the co-optimization market)**.** *Consider that a public agency needs power* *and ancillary service. Today, he needs 50MW electricity and 5MW ancillary services. There are two market participants, A and B with the following capacities and prices respectively. If he purchases*

MP	Capacity (MW)	Electricity ($/MW)	Ancillary service ($/MW)
A	40	$1	$2
B	20	$3	$6

them in a sequential order of electricity then ancillary service, the total cost is

$$\$1 \times 40 + \$3 \times 10 + \$6 \times 5 = \$100 \tag{1}$$

If he purchases them in a co-optimization market, then the total cost becomes

$$\$1 \times 35 + \$3 \times 15 + \$2 \times 5 = \$90 \tag{2}$$

Thus, the co-optimization market will reduce the overall cost of the public agency.

The prices of ancillary services determined by the market are called *market clearing price* (MCP) and we will use this term throughout this paper. It is also called *equilibrium price* in other literature. Suppliers who bid to provide ancillary services must bear various technical operating characteristics. Each bid consists of a capacity price ($/MW) and quantity (MW). The amount of awards of each ancillary service increases with their capacity price. All markets are cleared and suppliers are given the MCP and how much the capacity is accepted (see [17] and references therein). For each ancillary service type, the MCP is the value when demand meets aggregated system-wide offer of a specific ancillary service type. Since there are hundreds of suppliers and the MCPs are determined by all the bids submitted, individual bid will not significantly change the service type MCP. The MCP is systemically discussed in the literature (see [4, 15, 16] and references therein).

For the ancillary services market, the MCP is determined by the partial equilibrium (PE), which is a type of economic equilibrium, where the clearance on the market of some specific goods is obtained independently from prices and quantities demanded and supplied in other markets. In the ancillary services market, the ancillary services' demand and supply curves are well isolated from other alternatives. The reason is obvious that there is no much alternative available to replace the ancillary services, although, markets do not operate in a vacuum. Factors other than prices and capacities interact in complex ways to affect the procurements of the ancillary services. The political and cultural systems, electricity distribution, and innovative techniques place the contribution of the equilibrium analysis in a broader context. In particular, in a world of uncertainty and conflicting interests, analysis does not automatically translate into decision making. The insights derived from the partial equilibrium must be integrated with these broader considerations. The emphasis is on developing an approach to understanding and assessing the problems and using the analysis as an aid to decision making.

We present the PE analysis as the follow. Under the abundant conditions, the supply curve sets the price and the demand curve determines the amount of ancillary service supplied (see Figure 1 left) where the term *OR requirement* means the necessary capacity of operating reserves.

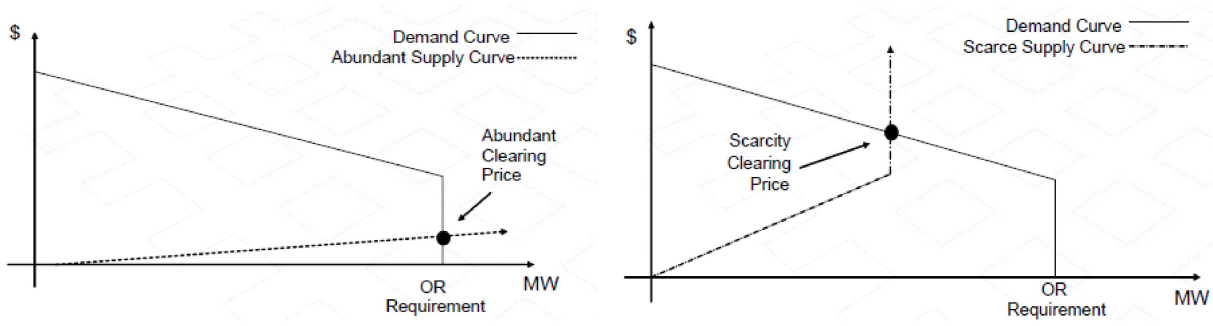

Figure 1: Partial equilibrium under abundant (left) and scarcity (right) conditions

Likewise, under the scarcity conditions, the demand curve sets the price and the supply curve determines the amount supplied (see Figure 1 right). Our partner, ERCOT, has the operating data to confirm the existences of supply curves. For example, the supply curves for the non-spinning service on 12:00am-1:00am, November 10, 2010 are presented to illustrate both abundant (left) and scarcity (right) conditions respectively in Figure 2. Solving PE model at the ISO level

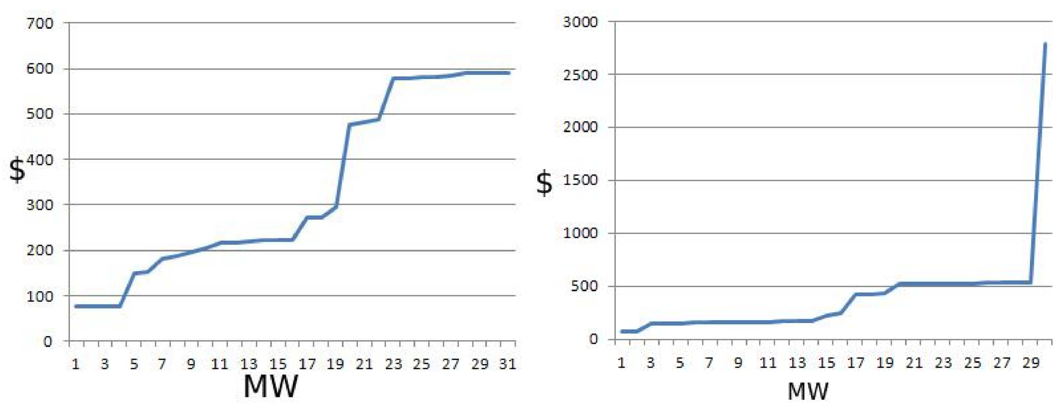

Figure 2: ERCOT's sample supply curves

would address the market clearing price, and might provide some results on the distribution of the prices. However, we will not use the PE model for individual market participant's ancillary services capacity allocation decision because the MCP is determined at the ancillary services market where the individual market participant's allocation decision is less likely to impact the market clearing process significantly.

All awarded offers would be paid at MCP, due to the commonly adapted paid-by-clearing mechanism. Under this system, the price of ancillary service is equal to the opportunity cost of non-operating generators (see [2]). This mechanism is formulated by federal regulations, in order to regulate the ancillary services market and to encourage suppliers to lower down their generators' variable cost. Given an energy price and the suppliers' variable cost, the opportunity of not operating the generator becomes the cost of ancillary services. Thus, suppliers with a low variable cost will have a lower cost of ancillary services. In a competitive market environment, there is no reason to place greater bid price because all the winning bids would be paid at MCP regardless the bid price previously placed. When a bid price greater than the cost is placed, the supplier has to take the risk of losing the contract. To avoid these risks, a market participant usually place bids at their generators' opportunity costs.

1.4 Optimization models for the market clearing process

There are two optimization problems for both market participants and ISO. For a market participant, the firm usually has a fixed amount of generator capacity for ancillary services. The firm needs to determine the capacity allocation plan to pursue better profit *prior to* the market clearing process. Thus, this is formulated as resource allocation problem under uncertainty by stochastic programming with recourse. However, stochastic programming model with recourse requires the possession of the distributional information which is usually, unfortunately, unavailable or inaccurate at the best. Although there are seasonal trends from the historical data, it is extremely difficult to estimate such demands accurately. We thereby propose a heuristic named "resource reservation" to pursue higher expected revenue and satisfactory performance in terms of revenue under the worst case scenario. Meanwhile, the ISO needs to determine the total amount of ancillary services to ensure satisfactory system reliability at a certain probabilistic level. Thus, the model for ISO is formulated as chance constrained optimization. Both problems have different objectives. The market participant capacity allocation plan is aimed to maximize the firm profitability and the ISO total ancillary services amount is to pursue better system reliability.

The current practices on these optimization problems are rather deterministic. For the market participant's capacity allocation plan, the firm will simply allocate all the capacities to the ancillary service with the most *promising* projected prices. Similarly, the ISO estimates the required total ancillary service amounts from historical data with seasonal adjustments. Both solutions totally ignore the randomnesses during the MCP and they are heavily relying on the projections, which could be incomplete, erroneous, or wrong. Under the *less satisfactory* projections, the resulting optimal solution would be more likely to be problematic. As a solution, some optimization techniques which attempt to tackle the various uncertainties have been developed since the 1950s (see [13] and references therein). In many cases, the stochastic optimization models have shown the advantage over the deterministic models both numerically and theoretically.

The obstacle on the way of deploying stochastic models is primarily computational concerns. Comparing with the simple and computationally efficient deterministic models, the stochastic models, however, seem to be complicated in formulation, and demand more time and computational expenses. In this paper, we model both problems with strong uncertainties as convex optimization problems and thus, we employ computationally efficient solvers to obtain optimal solution within a timely manner. We organize the remaining part of the paper as follows. We propose the stochastic models for the market participant's capacity allocation planning problem in Section 2 with our novel "resource reservation" method. We propose the ISO's total ancillary services amount problem in Section 3 and solve it by an approximation scheme. We show the merit of proposed stochastic models by the real operational data from ERCOT in Section 4 and the results suggest that our proposed method would yield better revenue for a market participant and more reliability for ISO. We conclude our research in Section 5.

2 MP's ancillary service capacity allocation planning problem

The purpose of proposed models is to determine bid quantities for multiple ancillary services from a capacity-limited supplier, who is a price-taker. The bid prices for these services are determined by the opportunity cost of not operating generator, and thus the bid prices are largely fixed. When the supplier wins the bid, the service will yield revenue at MCP. If we know market clearing prices for each ancillary service type, it is trivial that the MCP will allocate all the capacity to the most highly paid services. In reality, neither MCPs nor their distributional information have been available until the auction is closed. Thus, before formulating the model by stochastic programming, we need to address the issue that how to properly model the market clearing price in Subsection 2.1.

2.1 Market clearing price modeling

In order to quantitatively model the market clearing price, we acquire real-world data from ERCOT. ERCOT market first opened in July 2001. In September 2005, ERCOT modified the procuring process to simultaneously procure all the ancillary services in the day-ahead manner, with the objective to minimize the total ancillary services procurement costs.

We notice that the MCPs are constantly changing in both numerical values and distributions. In Figure 3, we present the MCP's histogram, quantile comparison plot against normal and χ^2

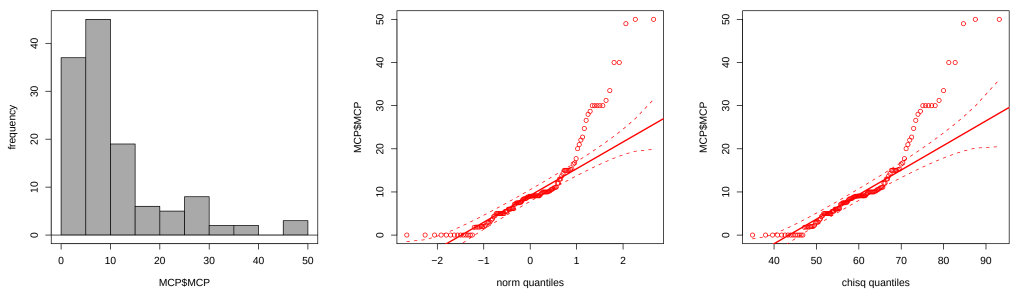

Figure 3: March 2010 Market Clearing Price at ERCOT

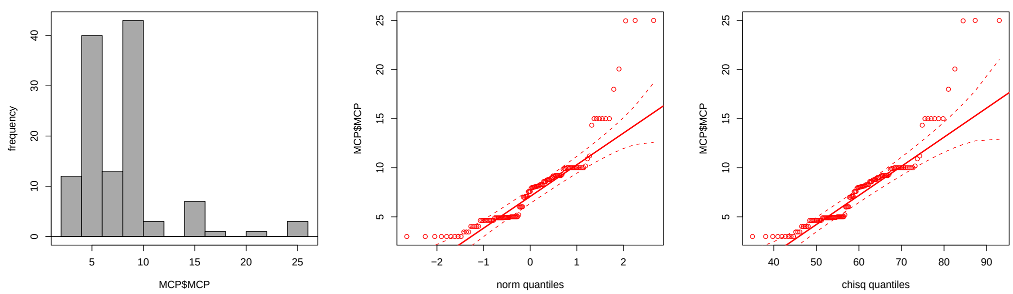

Figure 4: April 2010 Market Clearing Price at ERCOT

distributions. We repeat the plotting in Figure 4 on ERCOT's April MCP data. It can be observed that the histograms are substantially different. We also tried other commonly used distributions such as exponential and Weibull with the similar conclusion that the MCPs are constantly changing in terms of numerical values and best-fit distributions and none of known distributions can be properly assumed as model inputs.

Table 1 shows hourly summary statistics of price dispersion characteristics for the ancillary services' MCP. We use the following abbreviations: SD - Standard deviation; RT - Real-time market; DA - Day-ahead market; and CoV - Coefficient of Variation. Both variations and ranges reflect the strong uncertainty involved. The MCP differences on the real time market and the day ahead market are strongly uncertain as well. We illustrate the operational data for the regulation service MCP differences in Figure 5.

Clearly, all the above evidences suggest that we will inevitably encounter immense difficulty when estimating the general distributional information for the MCP on both the day-ahead and the real-time markets. Thus, assessing the distribution of random variables in the ancillary services capacity allocation becomes a lofty goal but difficult to achieve. Under this circumstance, we realize

	Max	Average	Minimum	SD	CoV
Regulation-DA	\$45.38	\$11.04	\$2.75	\$5.13	46.50%
Spinning-DA	\$24.97	\$2.96	\$0.38	\$3.28	110.52%
Non-Spinning-DA	\$9.00	\$1.08	\$0.38	\$0.86	79.94%
Regulation-RT	\$125.78	\$12.51	\$2.34	\$12.46	99.65%
Spinning-RT	\$87.45	\$2.26	\$0.18	\$6.37	281.31%
Non-Spinning-RT	\$48.41	\$1.02	\$0.18	\$2.54	249.28%

Table 1: Hourly MCP summary statistics for September, 2010

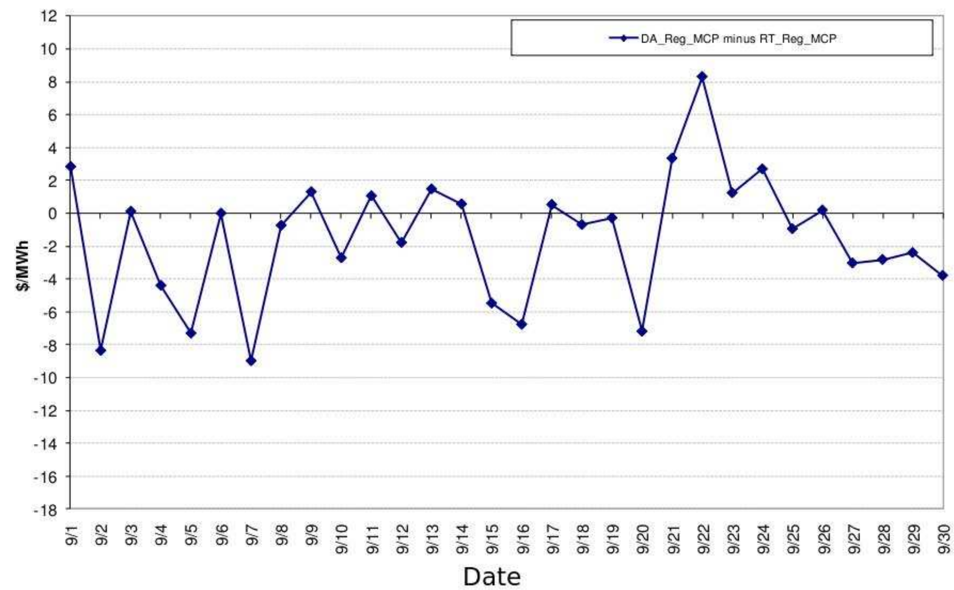

Figure 5: Day ahead market MCP differences

that the jump diffusion techniques may be suitable to build optimization models for the ancillary services capacity allocation. Nevertheless, the current practice at the ISOs are primarily based on the linear/nonlinear optimization and PE analysis, and we would like to study the jump diffusion based optimization as our future topics. Based on the above observation, we conclude that MCPs are constantly changing from time to time and should not be assumed any specific distribution.

2.2 Stochastic programming formulation

Consider a market participant providing n ancillary services from m generators by distinct locations with a fixed capacity $c := [c_1, \ldots, c_m]'$. Since the market clearing prices are constantly changing and highly unpredictable, the market participant usually place K pre-determined bid prices $p_{ijk}, i =$

$1, \ldots, n, j = 1, \ldots, m, k = 1, \ldots, K$. The decision variables are the amount of capacities allocated to ancillary services at distinct locations with distinct bid prices, x_{ijk}. The demands on these ancillary services are random variables ζ_{ijk}. Thus, the ancillary service capacity allocation is

$$\max_x \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{K} p_{ijk} x_{ijk}$$

$$\text{subject to} \sum_{i=1}^{n} \sum_{k=1}^{K} x_{ijk} \le c_j, i = 1, \ldots, n, j = 1, \ldots, m, k = 1, \ldots, K \tag{3}$$

$$x_{ijk} \le \zeta_{ijk} \tag{4}$$

which can be equivalently re-formulated as a nonlinear optimization:

$$\phi(c) := \max_x \{\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{K} p_{ijk} \mathbb{E}[\min(x_{ijk}, \zeta_{ijk})] : \sum_{i=1}^{n} \sum_{k=1}^{K} x_{ijk} \le c_j, j = 1, \ldots, m\} \tag{5}$$

and $\phi(\cdot) : \mathbb{R}^m \to \mathbb{R}$ is the optimal function.

The model (5) will provide a bidding capacity allocation plan for ancillary services before the MCPs are revealed. The solution is the fixed amount of capacity to multiple services and implementing this solution directly will be sub-optimal, because the result will "backfire" the revenue. The ancillary service market is an unanimously paid (so called "paid by clearing") market. In this market, all winning offers are paid at MCP. This has become a regulation code by North American Electric Reliability Corporation (NERC). For instance, when finalized MCP is between bid prices $p_{ij(\kappa-1)}$ and $p_{ij\kappa}$, the allocation, x_{ijk}, will be unsold and the capacity of $\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{\kappa-1} x_{ijk}$ will be procured at MCP. The optimal solution is to allocate as much capacity as possible to the ancillary service with the greatest MCP. Nevertheless, implementing the solution from (5) may lead to lower revenue.

We need to apply a post optimization treatment, named *nesting*, which suggests that the ancillary services with higher MCP will have higher priority in acquiring the fixed generator capacity. Under the current ERCOT market clearing process, the market participants can integrate the nesting operation into their capacity allocation. We illustrate this mechanism by the following example. Consider a supplier, which provides three ancillary services, i.e. regulation, spinning, and non-spinning with total capacity of 100MW. Regulation, spinning, and non-spinning services' nominal MCPs are ($\$25, \$20, \$15$) and the optimal solution of model (15) is $(30, 40, 30)$MW respectively. *Without* nesting, the supplier will place bids for three services as the following,

- Regulation bid, 30MW at \$10 per MW.
- Spinning bid, 40MW at \$10 per MW.

- Non-spinning bid, 30MW at \$10 per MW.

When implementing the above algorithm, the supplier will place three flexible bids.

- Regulation bid, $30 \sim 100$MW at \$10 per MW.
- Spinning bid, $40 \sim 70$MW at \$10 per MW.
- Non-spinning bid, up to 30MW at \$10 per MW.

When the realized MCPs are $\$25, \$20, \$15$, nesting will not affect the revenue generated by both methods. However, when the realized MCPs are $\$15, \$25, \$18$, the revenue when nesting is excluded will be

$$\$15 \times 30 + \$25 \times 40 + \$18 \times 30 = 1990$$

and the revenue *with* nesting becomes

$$\$15 \times 30 + \$25 \times 70 + \$18 \times 0 = 2200 > 1990$$

In Section 4, we show that the nesting mechanism will outperform the non-nesting method by the real-world data based simulations.

This model is a typical stochastic programming which can be handily solved as long as the distributional information of ζ is available. However, the estimation of ζ could be very rough if it is not worse. The underlying distribution of model (5) would be substantially different in reality and consequently, the obtained solution becomes problematic. Thus, this problem becomes stochastic programming with stability issues and it is usually a difficult problem. Thanks to the "paid by clearing" of the market clearing process, it is possible for us to develop a heuristic to place bids without ζ's distribution information.

2.3 Resource reservation heuristic

We will present a method "resource reservation" (RR) as a practical heuristic for the stability issue of the resource allocation problem. Since this method can also be applied to other resource allocation problems and we need to cite theoretical results from past literature to justify this heuristic, we adopt the standard formulation of stochastic programming as follows:

$$\begin{aligned} \min\ & f_0(x) + \mathbb{E}[g(x, \xi)] \\ \text{subject to } & Ax \leq b \end{aligned} \tag{6}$$

for the sake of simplifying notation. For the above model, the random variable x is on $(\Omega, \mathcal{F}, \mathbb{P})$ where $\mathbb{P}$ is the underlying, but rough estimation of distributional information of ξ. $A \in \mathbb{R}^{m\times n}$, and functions $g : \mathbb{R}^n \times \Omega \to \mathbb{R}$, $f_0(x) : \mathbb{R}^n \to \mathbb{R}$ are convex with respect to x. We are interested in the model performance under the real distribution of ξ is another measure, $\mathbb{Q}$. For the problem in this paper, we can assume that $f_0(x), g(x, \xi)$ are defined on a compact set and all the constraints $Ax \leq b$ are deterministic and nonempty. Let us define

1. $\nu(\mathbb{P}) := \inf\{f_0(x) + \mathbb{E}[g(x, \xi)] : Ax \leq b\}$.
2. $X^*_\epsilon(\mathbb{P}) := \{Ax \leq b : f_0(x) + \mathbb{E}[g(x, \xi)] \leq \nu(\mathbb{P}) + \epsilon\}$.
3. $X^*(\mathbb{P}) := X^*_0(\mathbb{P}) = \{Ax \leq b : f_0(x) + \mathbb{E}[g(x, \xi)] = \nu(\mathbb{P})\}$

where $\epsilon > 0$.

The stability of stochastic programming, in general, is rather difficult. The current research is about the continuity of the objective function and the Lipschitz property on the optimal value function. For the discussion of continuity, the distance has to be selected such that it allows to estimate differences of objective and constraint function values, and it is optimum adapted to the model. In the literature, a distances with ζ-structure that are given uniform distances of expectations:

$$d_{\mathcal{F}}(\mathbb{P}, \mathbb{Q}) := \sup_{f \in \mathcal{F}} \left| \mathbb{E}_{\mathbb{P}}[f(\xi)] - \mathbb{E}_{\mathbb{Q}}[f(\xi)] \right|. \tag{7}$$

In practice, one possible useful quantitative stability result is with respect to weak convergence of probability measures.

Theorem 1. *Let the set $\{x : Ax \leq b\}$ be non-empty. Let a sequence of probability measures $\{\mathbb{P}_n\}$ is weakly convergent to $\mathbb{Q}$ and*

$$\lim_{n \to \infty} d_{\mathcal{F}}(\mathbb{P}_n, \mathbb{Q}) = 0. \tag{8}$$

Then the sequence $\nu(\mathbb{P}_n)$ converges to $\nu(\mathbb{P})$ and

$$\lim_{n \to \infty} \sup_{x \in X^*(\mathbb{P}_n)} [\inf_{y \in X^*(\mathbb{Q})} \{||x - y||\}] = 0 \tag{9}$$

Proof. The proof is in [11]. □

The above theorem plays a central role in justifying the stochastic programming part of our heuristic. Essentially, the resource reservation can be summarized into to points: first, we need to adaptively incorporate available information to revise the previous rough estimations on the distributional information of ξ. Thus, we need to revisit previously placed bids. Second, we need to control the worst case performance. Since our estimation is never accurate, there is a chance that our plan may lead to poor performance in terms of the total revenue. Therefore, we propose to reserve a certain amount of capacities to ensure the market participant's satisfactory performance in terms of revenue.

Consider $\mathbb{P}_1$ the first estimation on ξ for model (6). As more information revealed, the estimation on ξ becomes more meaningful. Thus, we assume that the sequence of underlying probability measures $\{\mathbb{P}_n\}$ which are the estimations of ξ with continuous "learning", converges weakly to the real distribution $\mathbb{Q}$. By theorem 1, the sequence of optimal values $\{\nu(\mathbb{P}_n)\}$ will converges to $\nu(\mathbb{Q})$, i.e.,

$$\mathbb{P}_n \to_w \mathbb{P} \text{ and } \nu(\mathbb{P}_n) \to \nu(\mathbb{Q}) \tag{10}$$

and

$$\lim_{n\to\infty} \sup_{x\in X^*(\mathbb{P}_n)} [\inf_{y\in X^*(\mathbb{Q})} \{||x-y||\}] = 0 \tag{11}$$

as $n \to \infty$. Usually, the model (5) under $\mathbb{P}_1$ is solved several days earlier and the bidding plan will be revisited multiple times prior to the formal placement to incorporate available information as much as possible. Although the estimation of ξ has been updated towards the true distribution, the uncertainty during the daily market is still considerably significant. On the other hand, when bids are placed at ancillary services market, the market participant can not further revise them. Thus, the placed bids should also control the performance in terms of revenue under the worst case scenarios.

To control the performance in terms of revenue under the worst case, a robust optimization (RO) model is suitable because RO method does not *without* assuming ξ's distributional information. In some articles, RO model is described as a quantitative approach to control loss under the worst scenario and a RO model might seem overly conservative because of the term, *the worst scenario*. We need to elaborate that *worst scenario* does not mean the parameters are all "bad" as they could be. RO models take into account an uncertainty set, which reflects the fact that the extremely worst values will not simultaneously occur.

Rather than modeling the demands on pre-determined bid prices, we model the MCP as unknown-but-bounded random variables, i.e. $p_{ij} > 0$ such that $p_{ij} \in [p^*_{ij} - \Delta p_{ij}, p^*_{ij} + \Delta p_{ij}]$ where p^*_{ij} is the nominal or promising MCP of the i^{th} ancillary service at jth location and $\Delta p_{ij} > 0$ is the range of corresponding price range. We assume p^*_{ij} are distinct and we define $p := [p_{11}; \ldots; p_{ij}; \ldots; p_{mn}]$ be a $mn \times 1$ dimensional MCP vector. The realized MCP is modeled as

$$p_{ij} = p^*_{ij} + u_{ij}\Delta p_{ij}, i = 1, \ldots, n, j = 1, \ldots, m, \text{ where } ||u|| \leq \theta \tag{12}$$

where $u := [u_{11}; \ldots; u_{ij}; \ldots; u_{mn}]$ and $|| \cdot ||$ is the norm of u. Using ℓ_2 norm, the set for possible MCPs is an ellipsoid,

$$\mathcal{U}^\theta = \left\{ p \middle| \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{(p_{ij} - p^*_{ij})^2}{\theta^2 \Delta p^2_{ij}} \leq 1 \right\} \tag{13}$$

The ancillary capacity allocation problem is

$$\max_{[y_{ij};]} \left\{ \sum_{i=1}^{n}\sum_{j=1}^{m} p_{ij}y_{ij} : \sum_{i=1}^{n} y_{ij} \leq c_j, j=1,\ldots,m, p \in \mathcal{U}^{\theta} \right\} \tag{14}$$

Theorem 2. *Model* (14) *with general ellipsoidal uncertainty set can be converted into a conic quadratic program* (15) *for some* $\theta \geq 0$.

$$\min \left\{ \theta \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m} \Delta p_{ij}^2 y_{ij}^2} - \sum_{i=1}^{n}\sum_{j=1}^{m} p_{ij}^* y_{ij} \middle| \sum_{i=1}^{n} y_{ij} \leq c_j, j=1,\ldots,m \right\} \tag{15}$$

Proof. We can re-write the model (14) into

$$\min_{y} \left\{ t | -\sum_{i=1}^{n}\sum_{j=1}^{m} p_{ij}y_{ij} \leq t,\ \sum_{i=1}^{n} y_{ij} \leq c_j, j=1,\ldots,m \right\} \tag{16}$$

The constraint $-\sum_{i=1}^{n}\sum_{j=1}^{m} p_{ij}y_{ij} \leq t$ for $p \in \mathcal{U}^{\theta}$ is

$$\begin{aligned}
& -\sum_{i=1}^{n}\sum_{j=1}^{m} p_{ij}y_{ij} = -\sum_{i=1}^{n}\sum_{j=1}^{m} p_{ij}^* y_{ij} - \sum_{i=1}^{n}\sum_{j=1}^{m} u_{ij}\Delta p_{ij} y_{ij} \leq t \\
\Longleftrightarrow & \max_{||u|| \leq \theta} -\sum_{i=1}^{n}\sum_{j=1}^{m} u_{ij}\Delta p_{ij} y_{ij} \leq t + \sum_{i=1}^{n}\sum_{j=1}^{m} p_{ij}^* y_{ij} \\
\Longleftrightarrow & \theta \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m} \Delta p_{ij}^2 y_{ij}^2} \leq t + \sum_{i=1}^{n}\sum_{j=1}^{m} p_{ij}^* y_{ij} \\
\Longleftrightarrow & \theta \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m} \Delta p_{ij}^2 y_{ij}^2} - \sum_{i=1}^{n}\sum_{j=1}^{m} p_{ij}^* y_{ij} \leq t
\end{aligned} \tag{17}$$

We remove t and we reach our conclusion by formulating model (15). □

RO model is designed to provide a robust solution under some less favorable scenarios which are modeled by the uncertainty set. When $\theta = 0$, RO model becomes a linear programming with nominal parameters. When $\theta = 1$, $\mathcal{U}^{\theta}$ becomes the largest volume ellipsoid contained in $\mathcal{B} := \{p||p_{ij} - p_{ij}^*| \leq \Delta p_{ij}, i = 1,\ldots,n\}$ and when $\theta = \sqrt{n}$, $\mathcal{U}^{\theta}$ becomes the smallest volume ellipsoid contains $\mathcal{B}$. Thus, the value of θ would be understood as a trade-off between less risk averse and more risk averse. A greater θ will lead to a larger ellipsoid or feasible region which leads to more conservative decision. Since we are allocating $n = 3 \sim 6$ ancillary services, we thus set $\theta \in [0, 2.5]$ ($\sqrt{3} = 1.732, \sqrt{6} \approx 2.449$) under various situations for the ancillary service capacity allocation.

Suppose model (15)'s solution be $y^* \in \mathbb{R}^{mn}$ and model (5)'s solution be $x^* \in \mathbb{R}^{mnK}$. y^* has a unique meaning which is the necessary amount of capacities, of course with certain θ, to ensure a satisfactory performance in terms of revenue. x^* is the solution to maximize the expected revenue. Neither of these two solutions should be placed as bids for ancillary service. Given the fact that the estimation on ζ could be incomplete, then placing ancillary service bids by x^* could be questionable. Likewise, the solution from RO model is rather conservative. Although under the worst case, the performance in terms of revenue will outperform other approaches, its performance by average could be poor. Since ancillary service bids are daily events for market participants, the decision makers are more concerned about the performance in terms of revenue *by average*.

In order to obtain a balanced bidding plan for market participants, we present our heuristic named "resource reservation" which is based on the idea of reserving a certain amount of capacities. In the ancillary service capacity allocation problem, the resource reservation heuristic requires the market participant to place two bids: the regular bids and the must-win bids. The purpose of regular bids is to pursue maximized expected revenue and the must-win bids are to ensure the performance in terms of revenue under the worst scenario.

We determine the capacities for the regular bids by the following: let

$$z_{ij} := \min \left\{ \sum_{k=1}^{K} x^*_{ijk},\ y^*_{ij} \right\} \in \mathbb{R}^m \tag{18}$$

and

$$\sum_{i=1}^{n} z_{ij}, j = 1, \ldots, m \tag{19}$$

would be the capacity being allocated to the regular bids. Usually, the market participant's bid prices are pre-determined p_{ijk}.

$$c_j - \sum_{i=1}^{n} z_{ij} \tag{20}$$

would be the capacity being allocated to the must-win bids. The market participant will place the must-win bids with significantly low bid prices to ensure winning the auction. Fortunately, due to the paid by clearing process, the capacity for the must-win bids will be paid at the finalized MCPs. The allocations for n ancillary services for both regular and must-win bids are calculated by solving $\phi(\sum_{i=1}^{n} z_{ij})$ and $\phi(c_j - \sum_{i=1}^{n} z_{ij})$ respectively.

The resource reservation method for the ancillary service capacity allocation heuristic can be described as follows,

Step 1. Initializing the resource reservation heuristic several days prior to the market opening and continuously update the estimation of demands, i.e., ζ, to pursue more accurate estimation by adaptively incorporating available information.

Step 2. Solve $\phi(c)$ for x^* and solve (16) for y^*.

Step 3. Calculate $z_{ij} = \min \left\{ \sum_{k=1}^{K} x^*_{ijk},\ y^*_{ij} \right\}$.

Step 4. Solve model (5) with available capacities at m distinct locations, $\sum_{i=1}^{n} z_{ij}$, i.e., calculating $\phi(\sum_{i=1}^{n} z_{ij})$ for the regular bids.

Step 5. Applying nesting for the regular bids obtained in step 4. Nesting is an operation which market participant allows higher MCP ancillary service to use the capacity previously allocated to other less promising ancillary service. Consider a capacity allocation $x_{ij}, i = 1, \ldots, n$ with expected MCPs $p^*_{1j} \leq \ldots \leq p^*_{ij} \leq \ldots \leq p^*_{nj}$. Then, a bid with nesting is to allow ℓth ancillary service to use a capacity of $\sum_{i=1}^{\ell} x_{ij}$.

Step 6. Solve model (5) with available capacities at m distinct locations, $\sum_{i=1}^{n} z_{ij}$, i.e., calculating $\phi(c_j - \sum_{i=1}^{n} z_{ij})$ for the must-win bids.

Step 7. Applying nesting for the must-win bids obtained in step 6.

Step 8. Place bids at the ancillary services market.

The resource reservation is rather an idea that reserving well determined amount of resources will be an effective way to deal with uncontrollable uncertainty, perturbation, and poor estimations. Given the difficulty to pursue a theoretical analysis of this heuristic, we work closely with our power industry partner to test this heuristic with real data. The numerical result suggests that this heuristic will considerably improve the market participant revenue's by average performance and bidding performance under worst case scenario. We present results in section 4.

3 ISO's total AS amount problem

In the previous section, we assume that the individual market participants have a fixed amount capacity, c, for allocation, with the goal of better profitability. The Independent System Operator (ISO), however, is non-profit organization directly regulated by state agencies. The goal of ISO is to ensure the overall reliability of the power transmission. The ISO has responsibility to advise market participants to provide service capacity to meet ancillary services demands. Let the set $I, |I| = n$ be the set of market participants providing ancillary services services and the set D be the set of ancillary services demands with $|D| = m$. The market participants have limited service capacity M_i for $i \in I$ to j^{th} ancillary service demand. There is a fixed cost c_{ij} for providing ancillary services from market participant $i \in I$ to ancillary service demand $j \in D$. The fixed cost c_{ij} is usually the opportunity cost of market participant and it equals to the ancillary service bid price at the current ancillary services market. The ancillary services demands are highly random and are represented by a random vector $\xi \in \mathbb{R}^m_+$. The decision variables are the capacity from market participant i for the ancillary service demand j, i.e. x_{ij}. Therefore, in order to pursue a system wise minimum cost and a reliability of $1-\alpha, \alpha \in [0, 0.5]$, we impose the chance constraint in the following model,

$$\min_x c'x$$

$$\text{subject to: } \mathbb{P}\{\sum_{i\in I} x_{ij} \geq \xi_i, j = 1, \ldots, m\} \geq 1-\alpha, x_{ij} \geq 0 \quad (21)$$

$$\sum_{j\in D} x_{ij} \leq M_i, i \in I \quad (22)$$

The objective is to minimize the overall ancillary services costs and satisfy the reliability constraints and capacity constraints. Model (21) is usually called the probabilistic programming, and chance constrained optimization. For the sake of consistency, we will use the term *chance constrained optimization* in the remaining part of the paper.

Model (21) is rather a difficult problem in general for many reasons. First, it is numerically difficult to check whether or not a given point is in the feasible region rather than Monte Carlo simulation which could be expensive when $\alpha = 0.01$ or less. Second, the feasible region is, generally speaking, not convex and resulting convergence could only be a local optimal. We need to remark that the second difficulty may not occur under further assumptions. When the distribution is *logarithmically concave distributions* and F is a set of affine functions, the feasible set of model (21) is convex (see [10]). The recent research removes the probability measure by explicit, differentiable if possible, constraints. For example, when the F function can be re-written into $Ax \geq \xi$ where $A \in \mathbb{R}^{d\times n}$ is deterministic matrix. Thus the probabilistic constraints

$$\mathbb{P}\{Ax \geq \xi\} \geq 1-\alpha \text{ becomes } F_\xi(Ax) \geq 1-\alpha \quad (23)$$

The cumulative probability function F_ξ of logarithmically concave distributions will never be concave/convex. However, the log composition function efficiently transforms a non-concave function into a convex function be the following proposition.

Proposition 1. *If the distribution F_ξ is logarithmically concave, then* $\log(F_\xi)$ *is concave.*

The proof is provided in [9]. Thus, model (21) with necessary assumptions, i.e. logarithmically concave distribution, affine constraints, and right hand side only random vectors, is equivalent to the following model

$$\min_{x \in X} f(x)$$
$$\text{subject to } \log(1-\alpha) - \log(F_\xi(Ax)) \leq 0 \tag{24}$$

where $A \in \mathbb{R}^{d \times n}$ and if we define a $g(x) : \mathbb{R}^n \to \mathbb{R}$

$$g(x) := \log(1-\alpha) - \log(F_\xi(Ax)) \tag{25}$$

and g is a convex function in respect to $x \in \mathbb{R}^{mn}$.

Model (5) is a convex optimization problem with computable objective and constraints and as such it can be efficiently solved (see [7]). Many optimization methods combined with Monte-Carlo techniques have been developed to take this advantage. In this paper, we apply a newly developed method (see [3]) to evaluate the gradient of $g(x)$ by the polynomial approximation approach and apply the feasible direction (namely gradient mapping) on the model (25). Although we need to adopt Monte Carlo to assess the value of cumulative function, the author proved that the obtained optimal solution will converge to the true optimal solution with probability by an argument of epigraph convergence. Numerically speaking, the proposed method can effectively solve mildly large scale optimization with chance constraints.

The model (21) provides an alternative to the current market clearing process. By the current experience, market participant's service capacity for ancillary services is determined by market participants themselves and ISO will host and regulate the trading. Although ISO's goal is to maintain the power transmission reliability, the current market clearing process does not effectively ensure that the procured capacity will meet the uncertain demands. The proposed solution, however, will solve this problem for ISO by suggesting service capacities to market participants. The proposed service capacity for market participants, if implemented, will inevitably generate meaningful impacts on the ancillary services market by proactively ensuring the overall system reliability. We present the supportive numerical results in Subsection 4.2.

4 Numerical experiment

4.1 Numerical experiments for the MP's capacity allocation plan

In this section, we will compare the proposed method with an existing method (also called "Current Method" in this paper). The data we obtained is from the ERCOT ancillary service MCP in public domain for March and April 2010. The experiment subjects are:

- **Current method:** Market Participant submits bids with full capacity in ancillary services market from regulation, spinning, non-spinning. If the market participant's bid wins, all/partial capacity will be subtracted from the available capacity.
- **Stochastic programming based method:** Market Participant will solve the model (5) and revisit the previous allocation until the market opening. The bid prices are usually the opportunity cost of generators.
- **Resource reservation without nesting:** Market Participant will apply the resource reservation heuristic in section 2.3 to place the regular and the must-win bids. The suggested capacities for these bids are calculated by (5) and the market participant does not apply nesting to the bids.
- **Resource reservation with nesting:** Market Participant will apply the resource reservation heuristic in section 2.3 to place the regular and the must-win bids. The suggested capacities for these bids are calculated by (5) and the market participant apply nesting to the bids to pursue higher revenue and to control risk.

We use all the previous data to determine the range of MCPs and we will obtain the optimal solution without any heuristics involved because model (15) is a differentiable, convex optimization on a closed and convex set. Thus model (15) can be solved within several milliseconds on an average desktop (about 100G floating point operations per second). In the simulation part, we model the MCP by continuous uniform distribution on $[p^* - \Delta p_i, p_i^* + \Delta p_i]$ and we use the same pseudo random number generated by Matlab R2009a to obtain a fair comparison. The numerical experiments are conducted on a Dual-Core Xeon Debian/GNU/Linux workstation with 12G memory. The software is the Matlab R2009a with CVX developed by Stanford University.

4.1.1 ERCOT AS market, March data

A market participant in the ERCOT tries to allocate a pre-determined capacity for three major ancillary services, regulation (URS), spinning (RRS) and non-spinning (NSRS) at one location, i.e., $m = 1$. Thus, we remove the index j in this experiment. Another major ancillary service, replacement, is procured differently at ERCOT and we thereby exclude it from our numerical experiments.

Result in Table 2 shows that adjusting θ will change the feasible region accordingly. Once the feasible region is enlarged, the decision will be more conservative. When $\theta = 0.5$, resource reservation method does the same as model (18) which allocates all the capacity to the most promising service. Since, resource reservation method is designed to control the resource allocation

service Name	p_i^*	Δp_i	$\theta = 0.5$, Capacity %	$\theta = 1$, Capacity %	$\theta = 1.5$, Capacity %
Regulation (URS)	\$11.03	\$4.38	100%	64.5%	45.0%
Spinning (RRS)	\$7.51	\$2.37	0%	35.5%	55.0%
Non-spinning (NSRS)	\$5.36	\$4.63	0%	0%	0%

Table 2: Ancillary Service Capacity Allocation by Model (15) Based on ERCOT March Data

under the "worst" scenario, when $\theta = 1$, resource reservation method starts to allocate resource to multiple services to spread the risk. So does the case when $\theta = 1.5$. As θ increases, the service with less MCP range is more likely to receive increasingly amount of capacity.

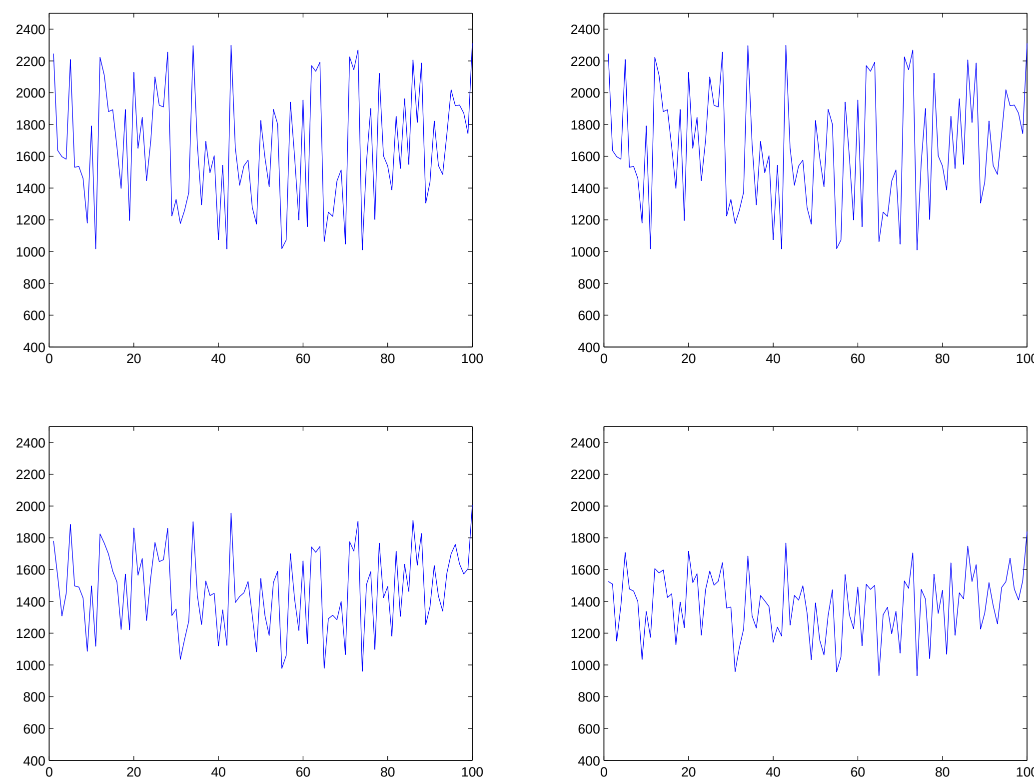

Figure 6: Summary of different allocation plans with $\theta = 0, 0.5, 1, 1.5$ without nesting. The horizontal axis is the replication index and the vertical axis is the revenue by simulation.

To compare and illustrate the performance of all the methods, we simulate the price of continuous uniform distribution at the range of $[p^* - \Delta p_i, p_i^* + \Delta p_i]$ with 100 replications with the same random numbers used. In Figure 6, the four charts are about the revenues in the order of current methods, resource reservation method at $\theta = 0.5$ without nesting, the resource reservation method at $\theta = 1$ without nesting, and resource reservation method at $\theta = 1.5$ without nesting by 100 replications. The resource reservation method will greatly reduce the standard deviation as expected when $\theta > 1$. However, the revenue is lower compared to the current method without nesting.

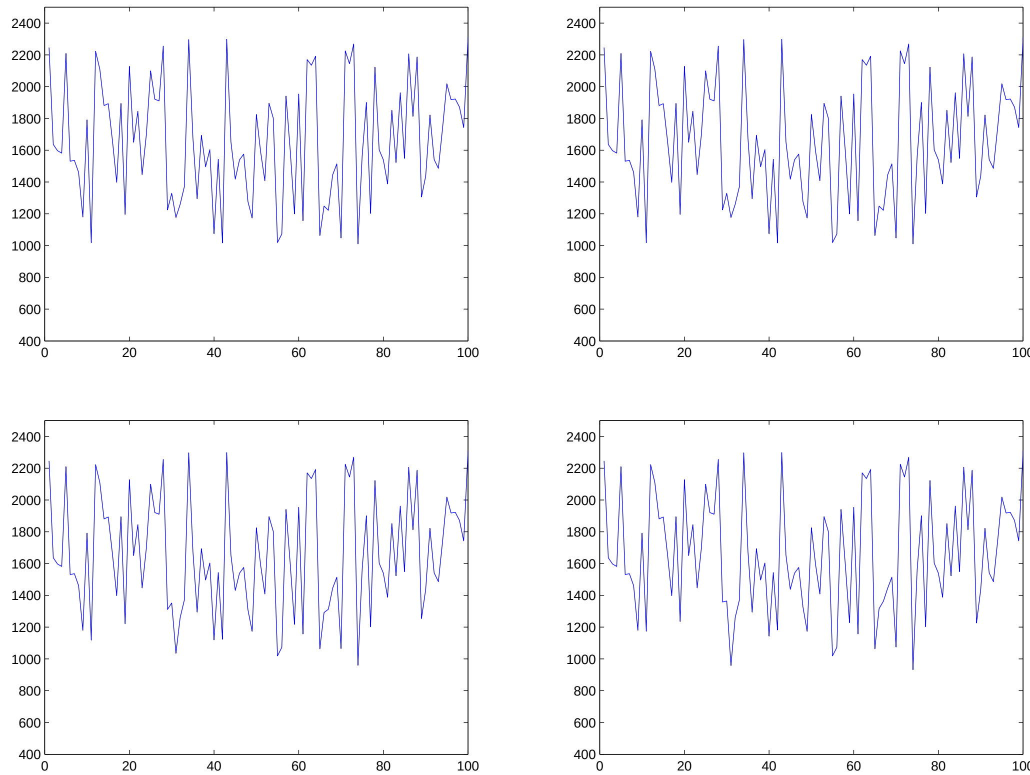

Figure 7: Summary of different allocation plans with $\theta = 0, 0.5, 1, 1.5$ with nesting. The horizontal axis is the replication index and the vertical axis is the revenue by simulation.

In Figure 7, the four charts are about the revenues in the order of current methods, resource reservation method at $\theta = 0.5$ *with* nesting, the resource reservation method at $\theta = 1$ *with* nesting, and resource reservation method at $\theta = 1.5$ *with* nesting by 100 replications. The nesting results dominate the current method in terms of revenue and its standard deviation. If it were applied in the real business, the capacity allocation plan would lead to a higher revenue with a lower variation, i.e. less risk.

In Table 3, we present the simulation data with $10,000$ replications to compare all the methods. The simulation is based on the same random number. Clearly the resource reservation method with/without nesting will reduce the standard deviation. The resource reservation method without

Setting	Mean	St. Deviation	Max	Min
Current method	\$1653.1	\$381.7	\$2311.5	\$997.6
SP based method	\$1653.1	\$381.7	\$2311.5	\$997.6
RR, $\theta = 0.5$ without nesting	\$1653.1	\$381.7	\$2311.5	\$997.6
RR, $\theta = 1$ without nesting	\$1467.3	\$257.2	\$2012.7	\$925.2
RR, $\theta = 1.5$ without nesting	\$1365.6	\$206.5	\$1850.7	\$882.7
RR, $\theta = 0.5$ with nesting	\$1653.1	\$381.7	\$2311.5	\$997.6
RR, $\theta = 1$ with nesting	\$1657.7	\$375.7	\$2311.5	\$925.2
RR, $\theta = 1.5$ with nesting	\$1660.2	\$373.4	\$2311.5	\$882.7

Table 3: Simulation Summary for March ERCOT Data, Replication $= 10,000$

nesting controls risk by almost halving the standard deviation. However, since there is no nesting operation, the revenue is lower as well. The resource reservation method with nesting consistently outperforms all the other methods in revenue and outperforms the current method in terms of standard variation as well. We notice the improvement on the revenue may not be quite significant. Our interpretation on this less significant revenue is that the regulation (URS) MCP is substantially greater than the other two services. The overlap between URS and RRS is quite small which means the URS will yield more revenue. Thus, the resource reservation method will tend to allocate less capacity for other services (see Table 2). Despite the dominating URS MCP, the resource reservation method with nesting still yields a greater revenue and smaller standard deviation. When URS MCP is not dominating others, the improvement of applying the resource reservation method with nesting becomes more substantial (see Subsection 4.1.2).

4.1.2 ERCOT AS market, April data

The same market participant tries to allocate a pre-determined capacity in April. The experiment settings remain the same as the previous subsection. In Table 4, we present the capacity allocation plan by the current method and the RR methods. Since the MCP of URS is not dominating in April, the capacity is well spread out that 44% of capacity is allocated to NSRS service when $\theta = 1.5$.

In Figure 8, we illustrate the simulation result with 100 replications by presenting charts in the order of current methods, resource reservation method at $\theta = 0.5$ *without* nesting, resource reservation method at $\theta = 1$ *without* nesting, and resource reservation method at $\theta = 1.5$ *without* nesting. All the illustrations are based on the identical flow of random numbers and we claim the differences in revenue and standard deviation are meaningful. We draw the same conclusion as the

Service Name	p_i^*	Δp_i	$\theta = 0.5$, Capacity %	$\theta = 1$, Capacity %	$\theta = 1.5$, Capacity %
Regulation (URS)	\$7.71	\$7.04	30.9%	34.9%	20.2%
Spinning (RRS)	\$8.44	\$5.62	69.1%	65.1%	35.6%
Non-spinning (NSRS)	\$3.52	\$2.38	0%	0%	44.2%

Table 4: Ancillary Service Capacity Allocation by Model (15) Based on ERCOT April Data

previous experiment that the resource reservation method will control the risk but lead to a lower revenue without nesting.

In Figure 9, we illustrate the simulation result with 100 replications by presenting charts in the order of current methods, resource reservation method at $\theta = 0.5$ *with* nesting, resource reservation method at $\theta = 1$ *with* nesting, and resource reservation method at $\theta = 1.5$ *with* nesting. Clearly, the resource reservation method with nesting method leads to the best performance in revenue.

Setting	Mean	St. Deviation	Max	Min
Current method	\$1154.2	\$613.6	\$2212.5	\$100.6
SP based method	\$1154.2	\$613.6	\$2212.5	\$100.6
RR, $\theta = 0.5$ without nesting	\$1237.7	\$388.9	\$2129.5	\$336.4
RR, $\theta = 1$ without nesting	\$1232.8	\$385.1	\$2134.1	\$324.5
RR, $\theta = 1.5$ without nesting	\$921.4	\$232.8	\$1562.1	\$292.5
RR, $\theta = 0.5$ with nesting	\$1419.4	\$446.4	\$2212.5	\$336.4
RR, $\theta = 1$ with nesting	\$1403.9	\$450.9	\$2212.5	\$324.5
RR, $\theta = 1.5$ with nesting	\$1463.2	\$434.3	\$2212.5	\$372.9

Table 5: Simulation Summary for April ERCOT Data, Replication $= 10,000$

In Table 5, we simulate the bidding process with $10,000$ replications. In this experiment, there is no dominating MCP, the resource reservation method outperforms the current method without the help of nesting by up to 7% and reduces the standard deviation by 37%. The resource reservation method with nesting outperforms the current method in terms of revenue by up to 26% and reduce the standard deviation by up to 29%.

4.2 Numerical Results for ISO's total AS amount problem

We performed computational tests on a probabilistic version of the ISO's total ancillary services amount problem. We have a set of market participants $I, |I| = 40$ and a set of ancillary services demands D with $|D| = 20$. The ancillary services demands vector was generated by the multivariate normal distribution with the mean and covariance matrix which is suggested by the ERCOT's

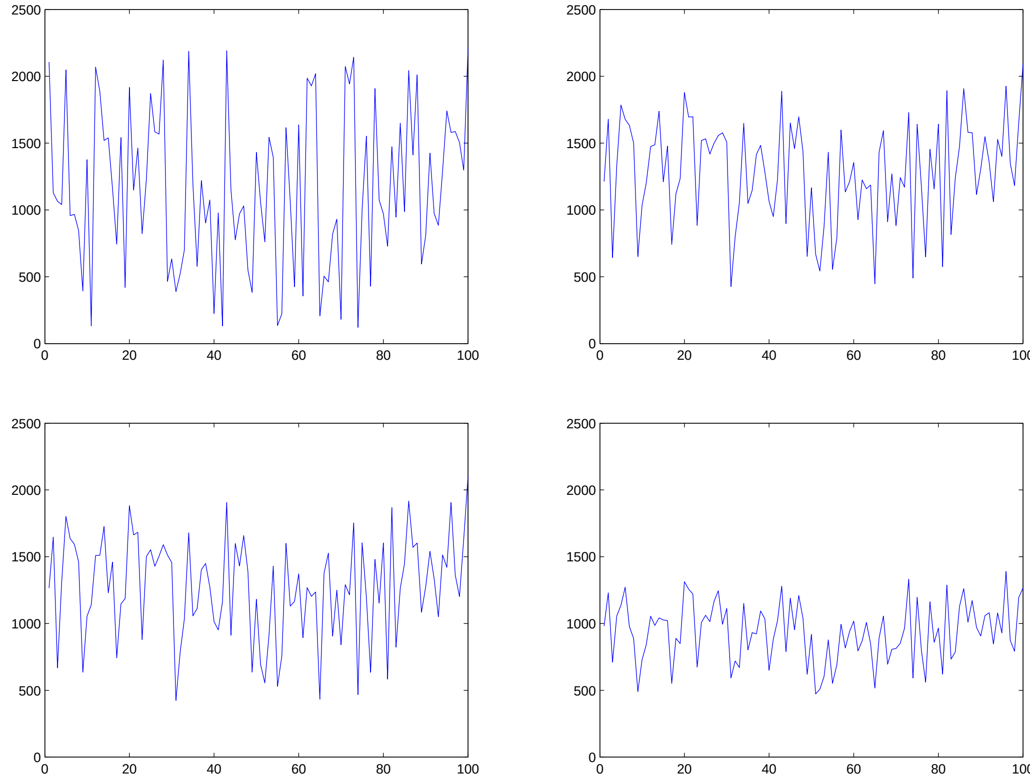

Figure 8: Summary of different allocation plans with $\theta = 0, 0.5, 1, 1.5$ without nesting. The horizontal axis is the replication index and the vertical axis is the revenue by simulation.

historical data. Essentially, any logarithmic concave and continuous distribution, e.g. Normal, Exponential, etc would be valid too if it fits the historical data. The cost coefficients are pre-generated constants. We use the Monte Carlo sampling to evaluate $g(x)$ and $\nabla g(x)$ and apply the feasible region methods to obtain the optimal solution. During the Monte Carlo sampling, all scenarios occur evenly likely with probability $\frac{1}{N}$ where $N = 10,000$ is the sample size. We obtain the market participants' service capacity, $M_i, i = 1, \ldots, 40$ from the numerical experiments of market participant's ancillary services capacity allocation problem. A matlab based CVX package is used and all experiments were done on a computer with four 2.4 Ghz processors and 12.0 Gb of memory. The imposed chance constraint with $\alpha = 0.05$ which means the procured ancillary services capacity will ensure a reliability at 95%.

We compared the proposed method with the market clearing process in which the capacity allocation x_{ij} is determined by the resource reservation method with nesting operation. We summarize the result in Table 6. The results suggest that imposing the chance constraints will greatly improve

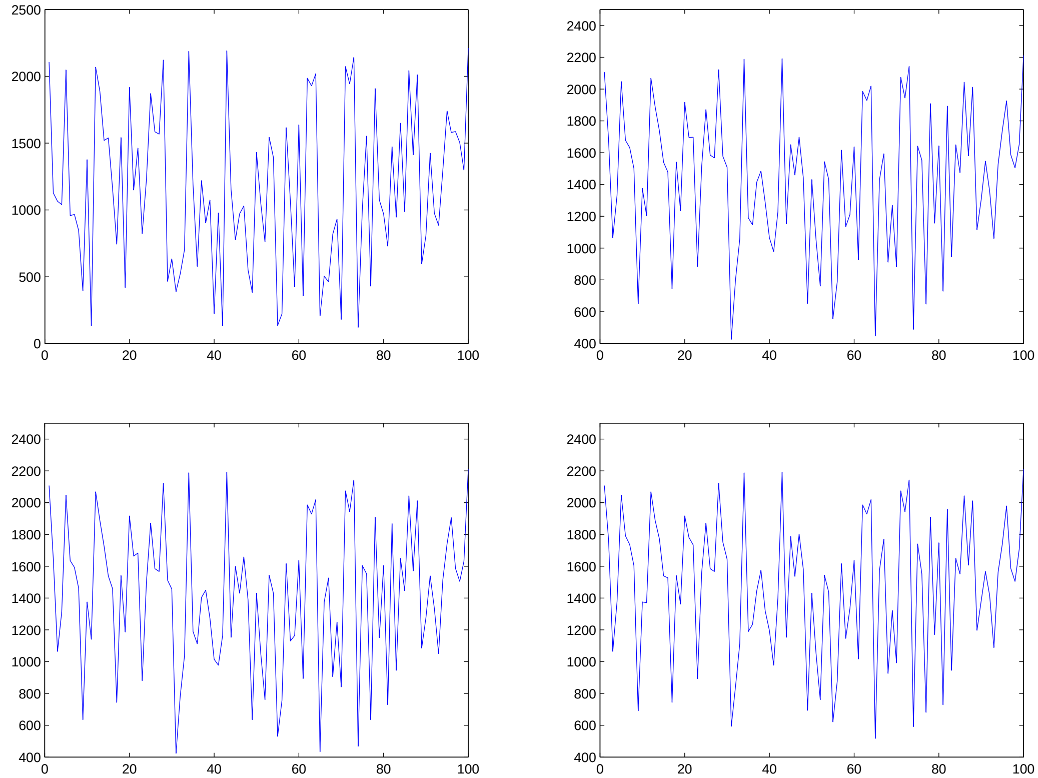

Figure 9: Summary of different allocation plans with $\theta = 0, 0.5, 1, 1.5$ with nesting. The horizontal axis is the replication index and the vertical axis is the revenue by simulation.

the system reliability by up to 35%. Both chance constrained optimizations take more than 5 hours. Considering their remarkable social benefit in terms of lifting power system reliability, the occurred computational cost becomes trivial. Our results will surely guarantee at least 95% system reliability.

Data	Chance constrained optimization	Current market clearing	gap
ERCOT March AS market	95.2%	71.4%	23.8%
ERCOT April AS market	96.1%	60.9%	35.2%

Table 6: System reliability under different methods for ISO's total AS demand amount

5 Conclusion

In this paper, we use stochastic models to solve the market participant's capacity allocation problem and the ISO's total ancillary service amount problem. We propose the resource reservation method for a market participant to place both the regular bids and the must-win bids with different purposes. The regular bids are calculated by stochastic programming model to pursue higher revenue under the uncertain demands. The must-win bids are to ensure a satisfactory performance under the worst scenario. Since the estimation of random factors could be incomplete, we will constantly revisit the previously made plans until the trustworthy distributional information is obtained. Meanwhile, we need to be prepared under the worst case scenario. Then we model the MCP as unknown-but-bounded random variables and adopt robust optimization methods. Thus, our resource reservation is by now a heuristic which combines the stochastic programming and robust optimization. In order to evaluate the performance, we conduct our numerical experiments on the real business data from ERCOT. We use the identical stream of random numbers to compare the performance of all the available methods, including the current methods, the stochastic programming only method, and the resource reservation method with and without nesting methods. The numerical experiment shows that the proposed method obtains a better revenue with a smaller standard deviation. Particularly when no ancillary service has a dominating MCP range, the gain by implementing our method is to reduce up to 29% of standard deviation and increase revenue by more than 26%.

We propose a chance constrained optimization model to determine the ISO's total ancillary service amount for a market participant. The proposed model yields two major impacts to the current ancillary services market. First, the proposed method suggests a proactive ISO action to pursue better system reliability. Our numerical results suggest that the proposed method will outperform the current market clearing process in terms of the system reliability up to 35%. Second, the proposed method becomes an alternative to the current market clearing process when determining the proper amount of capacity for individual market participant. Since the coefficients in the objective is the market participant's opportunity cost, this market participant needs to continuously reduce the operational cost to maintain or pursue a lasting advantage in the competition. Therefore, this central planning feature will help market participants solely concentrate on technical innovations to reduce the unit ancillary service cost rather than polishing the ancillary service bidding strategy. All involved parties, market participants, ISO, and public will benefit from this proposed research by taking advantage of significantly improved system reliability.

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