Fall 2008

2008 (Fall)

University of Dayton. Department of Mathematics

Abstracts of the Colloquium Talks: Fall 2008
Department of Mathematics

Date	Speaker and Title	Time/Location
Thursday, Sep 4	Paul Eloe, University of Dayton Uniqueness Implies Existence and Uniqueness Conditions for a Class of (k +j)-Point Boundary Value Problems for nth Order Differential Equations	3:00 PM, SC 323
Thursday, Sep 11	Paul Eloe, University of Dayton Uniqueness Implies Existence and Uniqueness Conditions for a Class of (k +j)-Point Boundary Value Problems for nth Order Differential Equations	3:00 PM, SC 323
Thursday, Sep 18	Youssef Raffoul, University of Dayton Boundedness And Exponential Stability In Highly Nonlinear Stochastic Differential Equations	3:00 PM, SC 323
Thursday, Sep 25	Hannah Samuel, University of Dayton “Money illusion or resale option?” - A Model-Based Mispricing Approach to Understanding the Housing Market Crisis	3:00 PM, SC 323
Thursday, Oct 2	Jeremiah Kermes, University of Dayton Sheaves and Schemes: The Gadgets of Algebraic Geometry	3:00 PM, SC 323
Thursday, Oct 16	Jeremiah Kermes, University of Dayton Sheaves and Schemes: The Gadgets of Algebraic Geometry	3:00 PM, SC 323
Thursday, Oct 23	Rahmat Khan, National University of Sciences and Technology, Pakistan Analytical techniques to nonlinear problems arising in heat transfer	3:00 PM, SC 323
Thursday, Oct 30	Rahmat Khan, National University of Sciences and Technology, Pakistan Existence of positive solutions of some four-point boundary value problems	3:00 PM, SC 323
Thursday, Nov 13	Muhammad Usman, University of Dayton Forced Oscillations of the Korteweg-de Vries Equation on a Bounded Domain and Their Stability	3:00 PM, SC 323
Thursday, Nov 20	Locksley Todd, University of Dayton	3:00 PM, SC 323
Thursday, Dec 4	Philip Myers, University of Dayton Transfer Function Model Application for Forecasting Automotive Warranty Claims	3:00 PM, SC 323

Uniqueness Implies Existence and Uniqueness Conditions for a Class of *(k + j)*– Point Boundary Value Problems for *n*th Order Differential Equations

Paul Eloe

Abstract For the nth order nonlinear differential equation

$$y\,(n)\ =\ f(x, y, y\,0\,, \ldots, y(n-1)),$$

we consider uniqueness implies existence results for solutions satisfying certain $(k+j)$ −point boundary conditions, $1\ \leq\ j\ \leq\ n-1$, and $1\ \leq\ k\ \leq\ n-j$. We define $(k;\ j)$ −point unique solvability in analogy to k −point disconjugacy and we show that $(n\ -\ j0;\ j0)$ −point unique solvability implies $(k;\ j)$ −point unique solvability for $1\ \leq\ j\ \leq\ j0$, and $1\ \leq\ k\ \leq\ n-j$. This result is in analogy to n −point disconjugacy implies k −point disconjugacy, $2\ \leq\ k\ \leq\ n\ -\ 1$.

Boundedness And Exponential Stability In Highly Nonlinear Stochastic Differential Equations
Youssef Raffoul

Abstract: Let $B(t)\ =\ \big(B1(t), B2(t), \ldots, Bm(t)\big)^T$ be a m −dimensional standard Brownian motion defined on a complete probability space $(\Omega, \Im, P)$. Consider *n*-dimensional stochastic systems

$$dx(t)\ =\ f(x(t), t)dt\ +\ g(x(t), t)dB(t), t\ \geq\ 0, \qquad (1)$$

with initial condition $x(t_0)\ =\ x_0\ \in\ \mathbb{R}\text{^}n$, where $t0\ \geq\ 0, x(t)\ =\ \big(x1(t), x2(t), \ldots, xn(t)\big)T\ \in\ \mathbb{R}^n$, and $f\ :\ \mathbb{R}^n\ \times\ \mathbb{R}^+\ \rightarrow\ \mathbb{R}^n and\ g\ :\ \mathbb{R}^n\ \times\ \mathbb{R}^+\ \rightarrow\ \mathbb{R}^{n\times m}$ are given nonlinear continuous functions. It is known that if the functions *f* and *g* satisfy a general Lipschitz condition and linear growth condition, then all solutions of system (1) exist stochastically.

In this research, we use the method of Lyapunov functions to obtain sufficient conditions for stochastic boundedness and exponential asymptotic stability of system (1) without the above requirement on the functions *f* and *g*.

Our theorems will make significant contribution to the theory of stochastic differential equations differential equations when dealing with equations that might contain unbounded terms. The theory is illustrated with several examples.

“Money illusion or resale option?” - A Model-Based Mispricing Approach to Understanding the Housing Market Crisis
Hannah Samuel; Research Advisors: Dr. Peter Lung Dr. Paul Eloe
Abstract The objective of this research is to study factors apart from inflation or the money illusion (“Money Illusion and Housing Frenzies”, Brunnermeier and Julliard, 2007) that drive the housing market crisis.

This research adopts the Campbell-Schiller (1988) Vector Autoregressive (VAR) model to derive a mispricing measure to estimate the asset’s fundamental value and the corresponding mispricing component. In this framework, we follow the approach of Brunnermeier and Julliard (2007) and Chen, Lung and Wang (2008) to measure mispricing as the difference between the market value and the fundamental value. To carry out our study, we obtain historical data of house prices and rental prices in the United States. The Campbell and Schiller decomposition (1988) takes into account risk factors that are specific to the housing industry to decompose the price-rent ratio into the fundamental value and the mispricing component. The mispricing component tells us if the asset is overvalued or undervalued. We propose to explain this mispricing component based on the investor’s subjective beliefs about resale

options for the asset in question and to study the relationship between the mispricing component and future returns.

Sheaves and Schemes: The Gadgets of Algebraic Geometry

Jeremiah Kermes

Abstract: The purpose of this talk is to introduce the main objects of study in modern algebraic geometry such as schemes. I will start from Hilbert's Nullstellenstatz and see how it motivated the use of affine schemes as generalizations of varieties. From there I introduce Serre's notion of a sheaf. This is then used to glue together affine schemes much in the way affine discs are patched together to form a manifold. This is Grothendieck's notion of a scheme. From here I will introduce the Picard group of divisors as the algebro-geometric version of a homology group. If time allows I'll cover the C˘ech cohomology of sheaves and explain its connection with divisors and line bundles.

With the wide variety of abstract concepts introduced in this talk, I will try to make it as accessible as possible by keeping the emphasis on the motivating examples like varieties. Projective and weighted projective spaces will be the primary examples of non-affine schemes. I will make an effort to strike a balance between examples that show the diversity of the theory, and those more patholgical examples.

ANALYTICAL TECHNIQUES TO NONLINEAR PROBLEMS ARISING IN HEAT TRANSFER

RAHMAT ALI KHAN

Abstract. Generalized approximation technique for the solution of steady state one-dimensional heat transfer problem in a fin made of materials with temperature dependent thermal conductivity, is developed. Results obtained by the generalized approximation method (GAM) are compared with those studied via homotopy perturbation method (HPM) and homotopy analysis method (HAM). Results obtained by the GAM are more accurate as compared to the HPM and HAM. Moreover, the (GAM) generates a bounded monotone sequence of solutions of linear problems that converges monotonically and rapidly to a solution of the original nonlinear problem, while the HPM series and HAM series diverges in some cases

EXISTENCE OF POSITIVE SOLUTIONS OF SOME FOUR-POINT BOUNDARY VALUE PROBLEMS

RAHMAT ALI KHAN

Abstract. Existence of positive solutions for a class of second order nonlinear equations

$$-x\text{“}(t) + \lambda x'(t) = f(t, x(t)), t \in [0,1]$$

subject to four-point boundary conditions of the type

$$x(0) = ax(\eta), \quad x(1) = bx(\delta), 0 < \eta \leq \delta < 1,$$

is established. For certain ranges of the parameters *a, b* loss of positivity of solution is investigated. Existence and uniqueness of $C\ 1$ -positive solution is established with the upper and lower solutions method. The results are extended to the case when the nonlinearity f is singular at $t = 0, t = 1$ and /or $x = 0$. An example is included to show the applicability of our results.

Forced Oscillations of the Korteweg-de Vries Equation on a Bounded Domain and Their Stability

Muhammad Usman; Joint work with Bing-Yu Zhang

Abstract In this work consider the pure KdV equation (without the damping terms) posed on a bounded domain. Consideration is given to the initial-boundary-value problem

$$\begin{cases} u_t + u_x + uu_x + u_{xxx} = 0, \quad u(x,0) = \varphi(x), \quad 0 < x < 1, t > 0, \\ u(0,t) = h(t), \quad u(1,t) = 0, \quad u_x(1,t) = 0, \quad t > 0. \end{cases} \qquad (*)$$

It is shown that if the boundary forcing h is periodic with small amplitude, then the small amplitude solution u of $(*)$ becomes eventually time-periodic. Viewing $(*)$ (without the initial condition) as an infinite-dimensional dynamical system in the Hilbert space $L^2(0,1)$, we also demonstrate that for a given periodic boundary forcing with small amplitude, the system $(*)$ admits a (locally) unique *limit cycle,* or *forced oscillation,* which is locally exponentially stable. A list of open problems are included for the interested readers to conduct further investigations.

Transfer Function Model Application for Forecasting Automotive Warranty Claims

Philip D. Myers

Abstract Honda and other automotive manufacturers are interested in ways to predict the quantity of warranty claims they may receive. Warranty claims are recorded by dealerships when a customer brings in a vehicle for any problem experienced during the vehicle's warranty period. This presentation shows an application of a Transfer Function Model, which attempts to predict the number of random warranty claims per month based on the number of sales per month for one particular Honda model.