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2.7: Manual Solution of the Node Equations

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2.7. Manual Solution of the Node Equations

2.7.1. Manual Solution of the Node Equations – Example #1

Consider the network shown in Figure 102 with the corresponding pipe characteristics given in
Table 237 and pump characteristics given in Table 238. Note that this is the same problem presented as Example #1 in the previous sections. From Example #1 in the previous section on the Simultaneous Path Adjustment method, the head-discharge relationship for the pump when flow is expressed in Cfs is:

$$E(Q) = 350 - 2.392Q^{1.760}$$

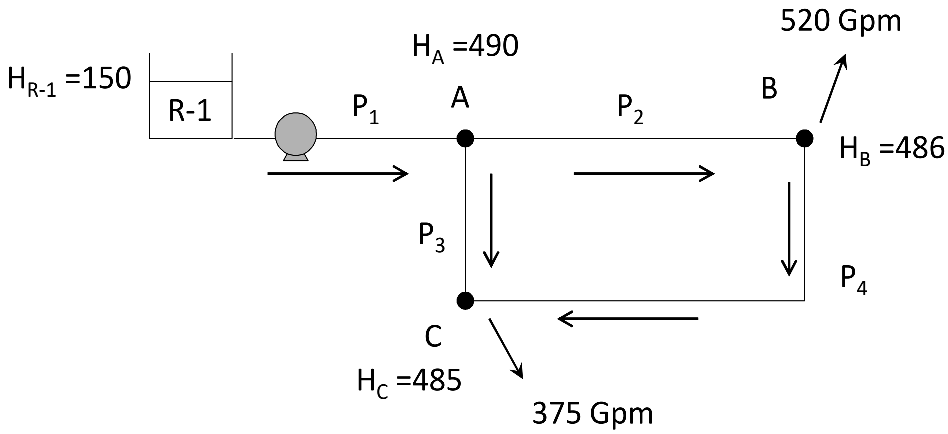

Figure 102 – Four Pipe System (Iteration #0)

Table 237

Pipe Label	Length (Ft)	Diameter (In)	C-Factor	Minor Loss Coeff	Kp	Km
P_1	800	12	120	10	0.534	0.252
P_2	1,200	10	120	0	1.945	0.000
P_3	1,500	8	120	0	7.209	0.000
P_4	2,200	6	120	0	42.920	0.000

Table 238 – Pump Data

Flow (Gpm)	Pump Head (Ft)
0	350
1500	330
2800	290

Table 239 – Node Data

Node	Demand (Gpm)	Demand (Cfs)
Node A	0	0
Node B	520	1.1585
Node C	375	0.8355

<u>Iteration #0</u>: When using the node equations an initial head at each node must be provided, i.e. an initial guess. It is not necessary for the initial head to produce flows that satisfy continuity. In fact, what we aim to do is converge to a set of nodal heads that produce flows which satisfy continuity. The initial set of heads for this example are given in Table 240.

Table 240 – Heads for Iteration #0

Node Label	Nodal Head (Ft)
A	490.00
B	486.00
C	485.00

Notice that the flow directions shown in Figure 102 are consistent with the heads shown in Table 240. We know from basic fluid mechanics that flow is from a high head to a low head. Thus for pipe P_2 we know that flow is from Node A to Node B. For pipe P_3 we know that flow is from Node A to Node C. For pipe P_4 we know that flow is from Node B to Node C. There is a pump in pipe P_1 which adds head and causes the hydraulic grade to increase from 150 ft (at R-1) to an assumed value of 490 ft at Node A.

The first step in the solution process is to compute the flow rate in each pipe using Eq. (71) for those pipes with no minor losses or pumps and using Eq. (73) within a root-solving framework for those pipes that do have minor losses and/or pumps. Using the pipeline and pump data for this system, along with the initial nodal heads, the net head change equation for pipe P_1 becomes:

$$F(Q)_1 = 0.534Q_1^{1.852} + 0.252Q_1^2 - (350 - 2.392Q_1^{1.760}) \\ = (150 - 490) = -340$$

Or

$$F(Q)_1 = 0.534Q_1^{1.852} + 0.252Q_1^2 - (350 - 2.392Q_1^{1.760}) + 340 = 0$$

Simplifying the net head change equation for pipe P_1 we obtain:

$$F(Q)_1 = 0.252Q_1^2 + 0.534Q_1^{1.852} + 2.392Q_1^{1.760} - 10 = 0$$

We will attempt to solve the net head change equation above for pipe P_1 using guess and check, although more elegant solution methods exist.

Let Q_1 = 2 Cfs

$$F(Q)_1 = 0.252(2)^2 + 0.534(2)^{1.852} + 2.392(2)^{1.760} - 10 = 1.037 \neq 0$$

Since a flow of 2 Cfs does not cause the net head change across pipe P_1 to equal zero, we must try a different flow.

Let Q_1 = 1.9 Cfs

$$F(Q)_1 = 0.252(1.9)^2 + 0.534(1.9)^{1.852} + 2.392(1.9)^{1.760} - 10 = 0.063 \neq 0$$

Let Q_1 = 1.894 Cfs

$$F(Q)_1 = 0.252(1.894)^2 + 0.534(1.894)^{1.852} + 2.392(1.894)^{1.760} - 10 = 0.006 \approx 0$$

At a flow of Q_1 = 1.894 $F(Q_1)$ = 0.006 which is sufficiently close to zero. Now we find the discharge in pipes P_2, P_3 and P_4 using Eq. (71).

For pipe P_2:

$$Q_2 = \left(\frac{490 - 486}{1.945}\right)^{1/1.852} = 1.476\,Cfs$$

For pipe P_3:

$$Q_3 = \left(\frac{490 - 485}{7.209}\right)^{1/1.852} = 0.821\,Cfs$$

For pipe P_4:

$$Q_4 = \left(\frac{486 - 485}{42.920}\right)^{1/1.852} = 0.131\,Cfs$$

We have now computed the flow in each pipeline associated with the initial set of heads. The flows are shown in Table 241 below. Now that we have the flow in each pipes, we can see if continuity at each junction node is satisfied.

Table 241 – Flows for Iteration #0

Pipe Label	Flow (Cfs)
P_1	1.894
P_2	1.476
P_3	0.821
P_4	0.131

Node A:

$$Q_1 - (Q_2 + Q_3) - D_A = 0$$
$$1.894 - (1.476 + 0.821) - 0 = -0.403 \neq 0$$

Node B:

$$Q_2 - Q_4 - D_B = 0$$
$$1.476 - 0.131 - 1.1585 = 0.186 \neq 0$$

Node C:

$$Q_3 + Q_4 - D_C = 0$$
$$0.821 + 0.131 - 0.8355 = 0.117 \neq 0$$

From the expressions above we can see that continuity is not satisfied; therefore, the nodal heads presented in Table 240 are not the correct nodal heads. Consequently, the flows shown in Table 241 are not the correct flows. We must change the values of the nodal heads in such a way that we drive the flows so that they satisfy continuity.

From the continuity equation for Node B we can see that the flow in pipes P_2 and P_4 is perhaps a bit too low. Therefore, in order to increase the flow in these pipes we must increase the head difference across pipes. Simultaneous to this, we must remain aware of the effect that any head changes have on the flow in pipe P_4 as well. Let's update the nodal heads and use the values presented in Table 242

Table 242 – Heads for Iteration #1

Node Label	Nodal Head (Ft)
A	489.00
B	485.50
C	485.60

Iteration #1: We use a root-solving technique to find the discharge in pipe P_1 given that the net head difference across pipe P_1 is 489.00 – 150.00 = 339.00 ft.

$$F(Q)_1 = 0.534Q_1^{1.852} + 0.252Q_1^2 - (350 - 2.392Q_1^{1.760}) \\ = (150 - 480) = -339$$

Or

$$F(Q)_1 = 0.534Q_1^{1.852} + 0.252Q_1^2 - (350 - 2.392Q_1^{1.760}) + 339 = 0$$

Simplifying the net head change equation for pipe P_1 we obtain:

$$F(Q)_1 = 0.252Q_1^2 + 0.534Q_1^{1.852} + 2.392Q_1^{1.760} - 11 = 0$$

We will attempt to solve the net head change equation above for pipe P_1 using guess and check, although more elegant solution methods exist.

Let Q_1 = 2 Cfs

$$F(Q)_1 = 0.252(2)^2 + 0.534(2)^{1.852} + 2.392(2)^{1.760} - 11 = 0.035 \neq 0$$

Since a flow of 2 Cfs does not cause the net head change across pipe P_1 to equal zero, we must try a different flow.

Let Q_1 = 1.95 Cfs

$$F(Q)_1 = 0.252(1.95)^2 + 0.534(1.95)^{1.852} + 2.392(1.95)^{1.760} - 11 = -0.465 \neq 0$$

Let Q_1 = 1.99 Cfs

$$F(Q)_1 = 0.252(1.99)^2 + 0.534(1.99)^{1.852} + 2.392(1.99)^{1.760} - 11 = -0.064 \neq 0$$

Let Q_1 = 1.996 Cfs

$$F(Q)_1 = 0.252(1.996)^2 + 0.534(1.996)^{1.852} + 2.392(1.996)^{1.760} - 11 = -0.004 \approx 0$$

At a flow of Q_1 = 1.996 $F(Q_1)$ = -0.004 which is sufficiently close to zero. Now we find the discharge in pipes P_2, P_3 and P_4 using Eq. (71).

For pipe P_2:

$$Q_2 = \left(\frac{489 - 485.5}{1.945}\right)^{1/1.852} = 1.373\ Cfs$$

For pipe P_3:

$$Q_3 = \left(\frac{489 - 485.6}{7.209}\right)^{1/1.852} = 0.666\ Cfs$$

For pipe P_4:

$$Q_4 = \left(\frac{485.6 - 485.5}{42.920}\right)^{1/1.852} = 0.038\ Cfs$$

Notice that pipe P_4 has experienced a change in flow direction. Since the head at Node C is greater than the head at Node B, the flow in pipe P_4 is from Node C to Node B. This is shown in Figure 103 below.

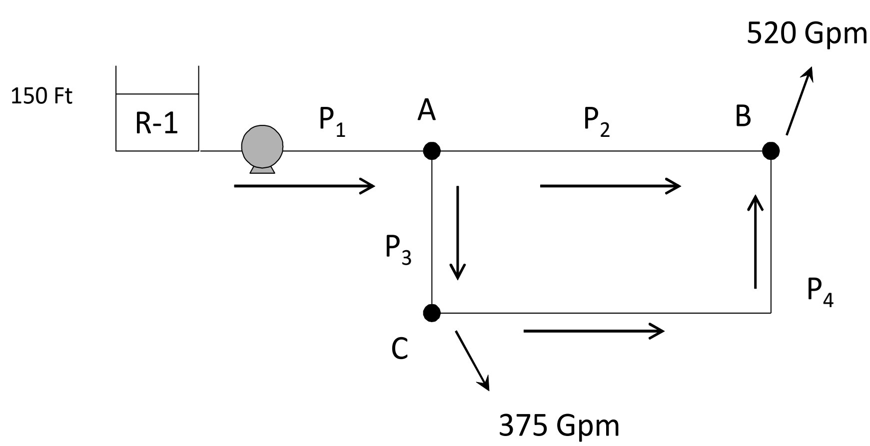

Figure 103 – Four Pipe System (Iteration #1)

We have now computed the flow in each pipeline associated with the initial set of heads. The flows are shown in Table 243 below. Now that we have the flow in each pipes, we can see if continuity at each junction node is satisfied.

Table 243 – Flows for Iteration #1

Pipe Label	Flow (Cfs)
P_1	1.996
P_2	1.373
P_3	0.666
P_4	0.038

Node A:

$$Q_1 - (Q_2 + Q_3) - D_A = 0$$
$$1.996 - (1.373 + 0.666) - 0 = -0.044 \neq 0$$

Node B:

$$Q_2 + Q_4 - D_B = 0$$
$$1.373 + 0.038 - 1.1585 = 0.252 \neq 0$$

Node C:

$$Q_3 - Q_4 - D_C = 0$$
$$0.666 - 0.038 - 0.8355 = -0.207 \neq 0$$

While overall the error associated with continuity is decreasing, we still have not yet converged to a solution. Therefore, we must perform another iteration. We will use the nodal heads provided in Table 244 below.

Table 244 – Heads for Iteration #2

Node Label	Nodal Head (Ft)
A	489.00
B	486.00
C	485.10

Iteration #2: We use a root-solving technique to find the discharge in pipe P_1 given that the net head difference across pipe P_1 is 489.00 – 150.00 = 339.00 ft.

$$F(Q)_1 = 0.534Q_1^{1.852} + 0.252Q_1^2 - (350 - 2.392Q_1^{1.760}) = (150 - 489) = -339$$

Or

$$F(Q)_1 = 0.534Q_1^{1.852} + 0.252Q_1^2 - (350 - 2.392Q_1^{1.760}) + 339 = 0$$

Simplifying the net head change equation for pipe P_1 we obtain:

$$F(Q)_1 = 0.252Q_1^2 + 0.534Q_1^{1.852} + 2.392Q_1^{1.760} - 11 = 0$$

We will attempt to solve the net head change equation above for pipe P_1 using guess and check, although more elegant solution methods exist.

Let Q_1 = 2 Cfs

$$F(Q)_1 = 0.252(2)^2 + 0.534(2)^{1.852} + 2.392(2)^{1.760} - 11 = 0.035 \neq 0$$

Since a flow of 2 Cfs does not cause the net head change across pipe P_1 to equal zero, we must try a different flow.

Let Q_1 = 1.95 Cfs

$$F(Q)_1 = 0.252(1.95)^2 + 0.534(1.95)^{1.852} + 2.392(1.95)^{1.760} - 11 = -0.465 \neq 0$$

Let Q_1 = 1.99 Cfs

$$F(Q)_1 = 0.252(1.99)^2 + 0.534(1.99)^{1.852} + 2.392(1.99)^{1.760} - 11 = -0.064 \neq 0$$

Let Q_1 = 1.996 Cfs

$$F(Q)_1 = 0.252(1.996)^2 + 0.534(1.996)^{1.852} + 2.392(1.996)^{1.760} - 11 = -0.004 \sim 0$$

At a flow of Q_1 = 1.996 F(Q_1) = -0.004 which is sufficiently close to zero. Now we find the discharge in pipes P_2, P_3 and P_4 using Eq. (71).

For pipe P_2:

$$Q_2 = \left(\frac{489 - 486}{1.945}\right)^{1/1.852} = 1.263\ Cfs$$

For pipe P_3:

$$Q_3 = \left(\frac{489 - 485.1}{7.209}\right)^{1/1.852} = 0.718\ Cfs$$

For pipe P_4:

$$Q_4 = \left(\frac{486 - 485.1}{42.920}\right)^{1/1.852} = 0.124\ Cfs$$

Notice that pipe P_4 has once again experienced a change in flow direction. Since the head at Node B is greater than the head at Node C, the flow in pipe P_4 is from Node B to Node C. This is shown in Figure 104 below.

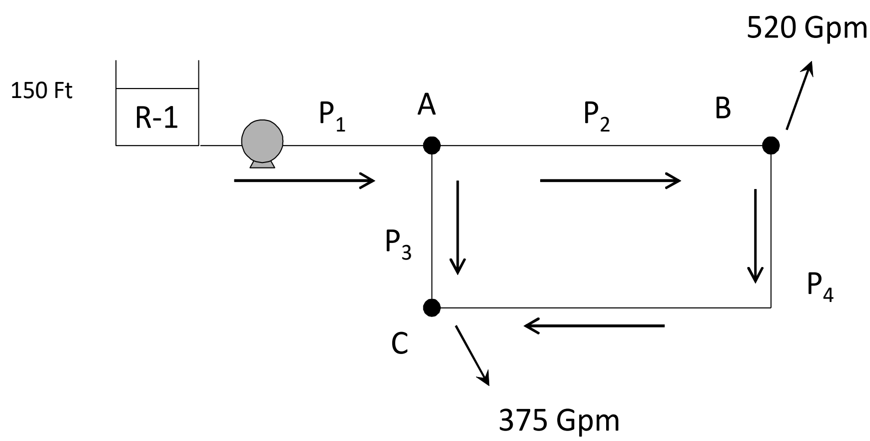

Figure 104 – Four Pipe System (Iteration #2)

We have now computed the flow in each pipeline associated with the initial set of heads. The flows are shown in Table 245 below. Now that we have the flow in each pipes, we can see if continuity at each junction node is satisfied.

Table 245 – Flows for Iteration #2

Pipe Label	Flow (Cfs)
P_1	1.996
P_2	1.263
P_3	0.718
P_4	0.124

Node A:

$$Q_1 - (Q_2 + Q_3) - D_A = 0$$
$$1.996 - (1.263 + 0.718) - 0 = 0.015 \approx 0$$

Node B:

$$Q_2 - Q_4 - D_B = 0$$
$$1.263 - 0.124 - 1.1585 = -0.020 \approx 0$$

Node C:

$$Q_3 + Q_4 - D_C = 0$$
$$0.718 + 0.124 - 0.8355 = 0.007 \approx 0$$

The error associated with continuity is sufficiently small for each of the individual junction nodes. Consequently, we have converged to a solution. The pipeline flows are thus given in Table 245 and the nodal heads are presented in Table 246.

For comparison purposes, this same system was evaluated in EPANET. EPANET uses the Gradient-Based method which is discussed later in this document. Essentially the Gradient-Based method solves the loop and node equations simultaneously to compute pipeline flows and nodal head within a coupled system. The results from EPANET are provided in the tables below.

Table 246 – Heads for Example #1

Node Label	EPANET Nodal Head (Ft)	Nodal Head (Ft)
A	489.03	489.00
B	485.97	486.00
C	485.14	485.10

Table 247 – Flows for Example #1

Pipe Label	EPANET Flow (Cfs)	Flow (Cfs)
P_1	1.99	1.996
P_2	1.28	1.263
P_3	0.72	0.718
P_4	0.12	0.124

2.7.2. Manual Solution of the Node Equations – Example #2

Now let's apply the manual solution of the node equations to a more complex system containing two loops and six nodes as shown in Figure 105. This example will illustrate how the complexity of manually solving the node equations increases significantly when we increase the size of the system.

Consider the eight-pipe system shown below with the flow arrows indicating the flow directions associated with the initial guess of nodal heads. Recall that flow is from a high head to a low head. During the course of the calculations we may, and most likely will, experience flow direction changes in one or more pipelines. Pipeline and node data are presented in Table 248 and

Table 249 respectively. Also presented is a table of head-discharge values for the pump in Table 250.

Table 248 – Pipeline Data

Pipe Label	Length (Ft)	Diameter (In)	C-Factor	ΣKL	Kp	Km
P_1	800	12	120	10	0.534	0.252
P_2	1200	10	120	0	1.945	0.000
P_3	1500	8	120	0	7.209	0.000
P_4	2200	6	120	0	42.920	0.000
P_5	1500	6	110	0	34.380	0.000
P_6	2000	8	120	0	9.612	0.000
P_7	3100	6	110	0	71.053	0.000
P_8	1200	8	120	0	5.767	0.000

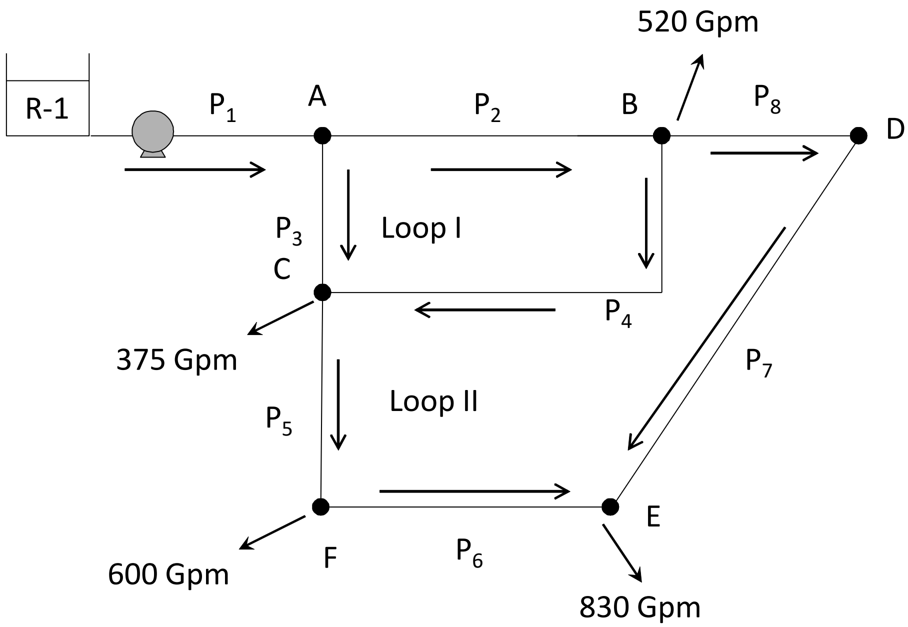

Figure 105 – Eight Pipe System (Iteration #0)

Table 249 – Node Data

Node Label	Demand (Gpm)	Demand (Cfs)	Elevation
R-1	N/A	N/A	990
A	0	0	1020
B	520	1.1585	1030
C	375	0.8355	1030
D	0	0	1100
E	830	1.8492	1080
F	600	1.3368	1055

Table 250 – Pump Data

Flow (Gpm)	Flow (Cfs)	Pump Head (Ft)
0	0	350
1500	3.3419	330
2800	6.2383	290

Iteration #0: As with the first example, the first step is to develop a set of initial nodal heads. Let's use the nodal heads shown in Table 251. We now use these initial heads to compute pipeline flows and then check continuity at each junction node.

Table 251 – Heads for Iteration #0

Node Label	Nodal Head (Ft)
R-1	990
A	1275
B	1260
C	1250
D	1255
E	1135
F	1140

As with Example #1, pipe P_1 contains a pump and a minor loss. Therefore, we must use a root-solving technique to find the flow through the pipeline since we cannot explicitly compute the flow. Recall that the varying exponents on the flow term prevent explicit calculation of the pipeline flow.

We are able to compute the coefficients for a continuous pump head-discharge equation using the pump head-discharge information presented in Table 250. The pump head curve for this pump is:

$$E(Q) = 350 - 2.392Q_1^{1.760}$$

The net head change equation for pipe P_1 is:

$$F(Q)_1 = 0.534Q_1^{1.852} + 0.252Q_1^2 - (350 - 2.392Q_1^{1.760})$$
$$= (990 - 1275) = -285$$

Or

$$F(Q)_1 = 0.534Q_1^{1.852} + 0.252Q_1^2 - (350 - 2.392Q_1^{1.760}) + 285 = 0$$

Simplifying the net head change equation for pipe P_1 we obtain:

$$F(Q)_1 = 0.252Q_1^2 + 0.534Q_1^{1.852} + 2.392Q_1^{1.760} - 65 = 0$$

We will attempt to solve the net head change equation above for pipe P_1 using guess and check, although more elegant solution methods exist.

Let Q_1 = 5 Cfs

$$F(Q)_1 = 0.252(5)^2 + 0.534(5)^{1.852} + 2.392(5)^{1.760} - 65 = -7.552 \neq 0$$

Since a flow of 5 Cfs does not cause the net head change across pipe P_1 to equal zero, we must try a different flow.

Let Q_1 = 5.5 Cfs

$$F(Q)_1 = 0.252(5.5)^2 + 0.534(5.5)^{1.852} + 2.392(5.5)^{1.760} - 65 = 3.221 \neq 0$$

Let Q_1 = 5.3 Cfs

$$F(Q)_1 = 0.252(5.3)^2 + 0.534(5.3)^{1.852} + 2.392(5.3)^{1.760} - 65 = -1.118 \neq 0$$

Let Q_1 = 5.355 Cfs

$$F(Q)_1 = 0.252(5.355)^2 + 0.534(5.355)^{1.852} + 2.392(5.355)^{1.760} - 65 = 0.012 \cong 0$$

At a flow of Q_1 = 5.355 F(Q_1) = 0.012 which is sufficiently close to zero. Now we find the discharge in pipes P_2, P_3, P_4, P_5, P_6, P_7 and P_8 using Eq. (71).

For pipe P_2:

$$Q_2 = \left(\frac{1275 - 1260}{1.945}\right)^{1/1.852} = 3.013\ Cfs$$

For pipe P_3:

$$Q_3 = \left(\frac{1275 - 1250}{7.209}\right)^{1/1.852} = 1.957\ Cfs$$

For pipe P_4:

$$Q_4 = \left(\frac{1260 - 1250}{42.920}\right)^{1/1.852} = 0.455\ Cfs$$

For pipe P_5:

$$Q_5 = \left(\frac{1250 - 1140}{34.380}\right)^{1/1.852} = 1.874\ Cfs$$

For pipe P_6:

$$Q_6 = \left(\frac{1140 - 1135}{9.612}\right)^{1/1.852} = 0.703\ Cfs$$

For pipe P_7:

$$Q_7 = \left(\frac{1255 - 1135}{71.053}\right)^{1/1.852} = 1.327\ Cfs$$

For pipe P_8:

$$Q_8 = \left(\frac{1260 - 1255}{5.767}\right)^{1/1.852} = 0.926\ Cfs$$

We have now computed the flow in each pipeline associated with the initial set of heads. The flows are shown in Table 252 below. Now that we have the flow in each pipes, we can see if continuity at each junction node is satisfied. This is also shown below.

Table 252 – Flows for Iteration #0

Pipe Label	Flow (Cfs)
P_1	5.355
P_2	3.013
P_3	1.957
P_4	0.455
P_5	1.874
P_6	0.703
P_7	1.327
P_8	0.926

Node A:

$$Q_1 - (Q_2 + Q_3) - D_A = 0$$
$$5.355 - (3.013 + 1.957) - 0 = 0.385 \neq 0$$

Node B:

$$Q_2 - Q_4 - Q_8 - D_B = 0$$
$$3.013 - 0.455 - 0.926 - 1.1585 = 0.473 \neq 0$$

Node C:

$$Q_3 + Q_4 - Q_5 - D_C = 0$$
$$1.957 + 0.455 - 1.874 - 0.8355 = -0.297 \neq 0$$

Node D:

$$-Q_7 + Q_8 - D_D = 0$$
$$-1.327 + 0.926 - 0 = -0.401 \neq 0$$

Node E:

$$Q_6 + Q_7 - D_E = 0$$
$$0.703 + 1.327 - 1.8492 = 0.181 \neq 0$$

Node F:

$$Q_5 - Q_6 - D_F = 0$$
$$1.874 - 0.703 - 1.3368 = -0.166 \neq 0$$

Clearly we see that continuity is not satisfied at any of the junction nodes. Thus we must develop a new set of nodal heads that drive the flows to satisfy continuity. Let's use the set of heads presented in Table 253 below.

Table 253 – Heads for Iteration #1

Node Label	Nodal Head (Ft)
R-1	990
A	1278
B	1263
C	1249
D	1254
E	1136
F	1139

Iteration #1: We first find the flow in pipe P_1 using guess and check, although more elegant solution methods exist.

Let Q_1 = 5.2 Cfs

$$F(Q)_1 = 0.252(5.2)^2 + 0.534(5.2)^{1.852} + 2.392(5.2)^{1.760} - 62$$
$$= -0.342 \neq 0$$

Since a flow of 5.2 Cfs does not cause the net head change across pipe P_1 to equal zero, we must try a different flow.

Let Q_1 = 5.22 Cfs

$$F(Q)_1 = 0.252(5.22)^2 + 0.534(5.22)^{1.852} + 2.392(5.22)^{1.760} - 62 = 0.056 \neq 0$$

Let Q_1 = 5.215 Cfs

$$F(Q)_1 = 0.252(5.215)^2 + 0.534(5.215)^{1.852} + 2.392(5.215)^{1.760} - 62 = -0.021 \cong 0$$

At a flow of Q_1 = 5.215 $F(Q_1)$ = -0.021 which is sufficiently close to zero. Now we find the discharge in pipes P_2, P_3, P_4, P_5, P_6, P_7 and P_8 using Eq. (71).

For pipe P_2:

$$Q_2 = \left(\frac{1278 - 1263}{1.945}\right)^{1/1.852} = 3.013\ Cfs$$

For pipe P_3:

$$Q_3 = \left(\frac{1278 - 1249}{7.209}\right)^{1/1.852} = 2.120\ Cfs$$

For pipe P_4:

$$Q_4 = \left(\frac{1263 - 1249}{42.920}\right)^{1/1.852} = 0.546\ Cfs$$

For pipe P_5:

$$Q_5 = \left(\frac{1249 - 1139}{34.380}\right)^{1/1.852} = 1.874\ Cfs$$

For pipe P_6:

$$Q_6 = \left(\frac{1139 - 1136}{9.612}\right)^{1/1.852} = 0.533\ Cfs$$

For pipe P_7:

$$Q_7 = \left(\frac{1254 - 1136}{71.053}\right)^{1/1.852} = 1.315\ Cfs$$

For pipe P_8:

$$Q_8 = \left(\frac{1263 - 1254}{5.767}\right)^{1/1.852} = 1.272\ Cfs$$

We have now computed the flow in each pipeline associated with the initial set of heads. The flows are shown in Table 254 below. Now that we have the flow in each pipes, we can see if continuity at each junction node is satisfied. This is also shown below.

Table 254 – Flows for Iteration #1

Pipe Label	Flow (Cfs)
P_1	5.215
P_2	3.013
P_3	2.120
P_4	0.546
P_5	1.874
P_6	0.533
P_7	1.315
P_8	1.272

Node A:

$$Q_1 - (Q_2 + Q_3) - D_A = 0$$
$$5.215 - (3.013 + 2.120) - 0 = 0.082 \neq 0$$

Node B:

$$Q_2 - Q_4 - Q_8 - D_B = 0$$
$$3.013 - 0.546 - 1.272 - 1.1585 = 0.037 \neq 0$$

Node C:

$$Q_3 + Q_4 - Q_5 - D_C = 0$$
$$2.120 + 0.546 - 1.874 - 0.8355 = -0.043 \neq 0$$

Node D:

$$-Q_7 + Q_8 - D_D = 0$$
$$-1.315 + 1.272 - 0 = -0.043 \neq 0$$

Node E:

$$Q_6 + Q_7 - D_E = 0$$
$$0.533 + 1.315 - 1.8492 = -0.001 \neq 0$$

Node F:

$$Q_5 - Q_6 - D_F = 0$$
$$1.874 - 0.533 - 1.3368 = 0.004 \neq 0$$

Continuity is very nearly satisfied at each junction node. Yet we will try one more iteration. We will use the nodal heads presented in Table 255

Table 255 – Heads for Iteration #2

Node Label	Nodal Head (Ft)
R-1	990
A	1278.8
B	1263.6
C	1249.0
D	1254.0
E	1136.2
F	1139.2

Iteration #2: We first find the flow in pipe P_1 using guess and check, although more elegant solution methods exist.

Let Q_1 = 5.22 Cfs

$$F(Q)_1 = 0.252(5.22)^2 + 0.534(5.22)^{1.852} + 2.392(5.22)^{1.760} - 61.2 = 0.886 \neq 0$$

Since a flow of 5.22 Cfs does not cause the net head change across pipe P_1 to equal zero, we must try a different flow.

Let Q_1 = 5.17 Cfs

$$F(Q)_1 = 0.252(5.17)^2 + 0.534(5.17)^{1.852} + 2.392(5.17)^{1.760} - 61.2 = -0.182 \neq 0$$

Let Q_1 = 5.179 Cfs

$$F(Q)_1 = 0.252(5.179)^2 + 0.534(5.179)^{1.852} + 2.392(5.179)^{1.760} - 61.2 = 0.009 \cong 0$$

At a flow of Q_1 = 5.179 $F(Q_1)$ = 0.009 which is sufficiently close to zero. Now we find the discharge in pipes P_2, P_3, P_4, P_5, P_6, P_7 and P_8 using Eq. (71).

For pipe P_2:

$$Q_2 = \left(\frac{1278.8 - 1263.6}{1.945}\right)^{1/1.852} = 3.035\ Cfs$$

For pipe P_3:

$$Q_3 = \left(\frac{1278.8 - 1249.0}{7.209}\right)^{1/1.852} = 2.152\ Cfs$$

For pipe P_4:

$$Q_4 = \left(\frac{1263.6 - 1249.0}{42.920}\right)^{1/1.852} = 0.559\ Cfs$$

For pipe P_5:

$$Q_5 = \left(\frac{1249.0 - 1139.2}{34.380}\right)^{1/1.852} = 1.872\ Cfs$$

For pipe P_6:

$$Q_6 = \left(\frac{1139.2 - 1136.2}{9.612}\right)^{1/1.852} = 0.533\ Cfs$$

For pipe P_7:

$$Q_7 = \left(\frac{1254.0 - 1136.2}{71.053}\right)^{1/1.852} = 1.314\ Cfs$$

For pipe P_8:

$$Q_8 = \left(\frac{1263.2 - 1254.0}{5.767}\right)^{1/1.852} = 1.317\ Cfs$$

We have now computed the flow in each pipeline associated with the initial set of heads. The flows are shown in Table 256 below. Now that we have the flow in each pipes, we can see if continuity at each junction node is satisfied. This is also shown below.

Table 256 – Flows for Iteration #2

Pipe Label	Flow (Cfs)
P_1	5.179
P_2	3.035
P_3	2.152
P_4	0.559
P_5	1.872
P_6	0.533
P_7	1.314
P_8	1.317

Node A:

$$Q_1 - (Q_2 + Q_3) - D_A = 0$$
$$5.179 - (3.035 + 2.152) - 0 = -0.007 \cong 0$$

Node B:

$$Q_2 - Q_4 - Q_8 - D_B = 0$$
$$3.035 - 0.559 - 1.317 - 1.1585 = 0.001 \cong 0$$

Node C:

$$Q_3 + Q_4 - Q_5 - D_C = 0$$

$$2.152 + 0.559 - 1.872 - 0.8355 = 0.003 \cong 0$$

Node D:

$$-Q_7 + Q_8 - D_D = 0$$
$$-1.314 + 1.317 - 0 = 0.003 \cong 0$$

Node E:

$$Q_6 + Q_7 - D_E = 0$$
$$0.533 + 1.317 - 1.8492 = -0.002 \cong 0$$

Node F:

$$Q_5 - Q_6 - D_F = 0$$
$$1.872 - 0.533 - 1.3368 = 0.002 \cong 0$$

We can clearly see that continuity is, for all practical purposes, satisfied at each node. As a result, we have arrived at a solution. The pipeline flows are presented in Table 256 while the nodal heads are given in Table 255.