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Rahmat Ali Khan
University of Malakand, Pakistan

Muhammad Usman
University of Dayton, musman1@udayton.edu

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A Study of the GAM Approach to Solve Laminar Boundary Layer Equations in the Presence of a Wedge

Rahmat Ali Khan

University of Malakand, Chakdara Dir(L)
Khyber Pukhtunkhwa, Pakistan
rahmat_alipk@yahoo.com

Muhammad Usman

Department of Mathematics, University of Dayton
300 College Park, Dayton, OH 45469-2316, USA
musman1@udayton.edu

Abstract. We apply an easy and simple technique, the generalized approximation method (GAM) to investigate the temperature field associated with the Falkner-Skan boundary-layer problem. The nonlinear partial differential equations are transformed to nonlinear ordinary differential equations using the similarity transformations. An iterative scheme for the non-linear ordinary differential equations associated with the velocity and temperature profiles are developed via GAM. Numerical results for the dimensionless velocity and temperature profiles of the wedge flow are presented graphically for different values of the wedge angle and Prandtl number.

Mathematics Subject Classification: 03B22, 45A03, 90C92

Keywords: Falkner-Skan equation; Heat transfer; approximation method; GAM

1. Introduction

One of the well-known problems of classical fluid mechanics is the laminar flow along a stationary plate. When a free stream is parallel to a plate and the velocity is constant, the situation is known as the Blasius problem. When the wall makes a positive angle with the free stream, then the problem is called a wedge flow problem. The temperature distribution associated with two-dimensional steady and incompressible wedge flow was first analyzed in 1930s by Falkner and Skan to illustrate the application of Prandtl's boundary layer theory. Falkner and Skan developed a similarity transformation method which reduced a partial differential boundary-layer equation to a nonlinear third order ordinary differential equation. For further historical context, we refer the readers to the recent paper by T. Fang and J. Zhanga [7]. Since then several researchers, including mathematicians, engineers and physicists have introduced new computational techniques to solve the wedge flow problem. For example, Lin and Lin [18] provided a very accurate numerical solution using Runge-Kutta method for forced convection heat transfer from isothermal or uniform-flux surfaces to fluids of any Prandtl number. Hsu et al. [8] studied the temperature and flow fields of the flow past a wedge using three methods: the series expansion method, Runge Kutta integration and the shooting method. Kuo [13] investigated the temperature field associated with the Falkner- Skan boundary-layer problem by the differential transformation method. More recently, Yao [21] studied series solution of the temperature distribution in the Falkner-Skan wedge flow by the homotopy analysis method. N. S. Elgazery [2] studies the Falkner-Skan equation using Adomian Decomposition Method and shooting method.

Motivated by [13, 21], in this paper, we revisited the problem of flow of an incompressible viscous fluid along a wedge placed in a flowing fluid and obtain estimates for the exact solution of the problem. Here we remark that mesh generation for the problem domain is the prerequisite for the numerical simulations of most of the numerical methods and in most cases it becomes more expensive than solving the problem itself. Moreover, series solution methods such as the series expansion method, homotopy analysis method, Adomian Decomposition Method etc yield wonderful results for a very short domain and for very small values of the parameters. But in most cases with large domain and big values of the parameters, the series may not converges and yield inaccurate results. Our contribution to this problem is that, we provide estimates for the exact solution and study the effect of the fluid parameters on the velocity field. These estimates determine the region of existence for the solution. Based on these estimates, we apply a simple analytical technique, the generalized approximation method (GAM), [9, 10, 11, 12], a kind of quasilinearization method [3, 4, 5, 14, 15, 16, 17, 19] which uses linear iterations to approximate the solution of the nonlinear problems. It is worthwhile to note that GAM is a theoretical iterative scheme which for each practical problem need to be developed and such iterative scheme for the problem under consideration have not been developed previously. For practical point of view, the importance of the method is that it uses linear problems to generates a bounded monotone sequence which converges uniformly and rapidly to the solution of the original problem. The boundedness and monotonicity of the sequence guaranteed convergence of the sequence to the exact solution of the problem which most of the stated methods fail to guarantee. For Mathematical point of view, the important feature of the method is that, at each iteration, the solution is bracketed between the iterates and a fixed upper solution. Moreover, our results are accurate and consistent with the theoretical results for any value of the parameters and any domain.

The remaining part of this paper is organized as follows: In Section 2, we will present the problem under consideration. In Section 3, we will discuss some theorems and definitions about upper and lower solutions of the problem. In Sections 4 and 5, we formulate the generalized approximation method and study error analysis, while numerical simulations are presented in Section 6. Finally we present a brief conclusion in Section 7. We have used MATHEMATICA for numerical simulations.

2. THE MATHEMATICAL MODEL

The temperature of the wall T_w is assumed to be uniform and constant and is greater than the free stream temperature T_∞. It is further assumed that the mainstream velocity U_∞ is uniform and constant and that the flow in the laminar boundary layer is two-dimensional, and that the changes in temperature due to viscous dissipation are small, the flow and heat transfer are governed by the following equations [13]:

- continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}$$

- momentum equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = U\frac{dU}{dx} + \nu\frac{\partial^2 u}{\partial y^2}, \tag{2.2}$$

- energy equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}, \tag{2.3}$$

where u, v are components of velocity in x and y directions of the fluid flow, ν is the viscosity, $U(x)$ is the reference velocity at the edge of the boundary layer, α is the thermal diffusivity of the fluid and T is the temperature. Consider a general case of a power law free stream velocity, that is, $U(x) = U_\infty(x/L)^m$, where L is the length of the wedge, x is measured from the tip of the wedge and m is the Falkner-Skan power-law parameter. For the sketch of the flow layout, we refer the readers to [13]. The appropriate boundary conditions are given by [13]

$$\begin{aligned} u(x,0) = v(x,0) = 0,\ T(x,0) = T_w, \\ u(x,y) \to U(x),\ T(x,y) \to T_\infty \text{ as } y \to \infty. \end{aligned} \tag{2.4}$$

The continuity equation (2.1) is automatically satisfied by the stream function $\psi(x,y)$ such that

$$u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x},$$

and the momentum equation (2.2) takes the form

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = U\frac{dU}{dx} + \nu \frac{\partial^3 \psi}{\partial y^3}. \tag{2.5}$$

By the transformation [13]

$$f(\eta) = \Big[\frac{1+m}{2}\frac{L^m}{\mu U_\infty}\frac{1}{x^{1+m}}\Big]^{\frac{1}{2}}\psi,\ \eta = \Big[\frac{1+m}{2}\frac{U_\infty}{\mu L^m}\frac{1}{x^{1-m}}\Big]^{\frac{1}{2}} y,\ \theta = \frac{T - T_w}{T_\infty - T_w},$$

where f is a dimensionless stream function and η is a similarity variable, the dimensionless momentum equation (2.5) and the dimensionless boundary-layer energy equation (2.3) become:

$$f'''(\eta) + f(\eta)f''(\eta) + \lambda[1 - (f'(\eta))^2] = 0,\ \eta \in (0,\infty), \tag{2.6}$$

$$\theta''(\eta) + Pr.f(\eta)\theta'(\eta) = 0,\ \eta \in (0,\infty), \tag{2.7}$$

where $\lambda\pi$ is the wedge angle and is related to the Falkner–Skan power-law parameter m through the expression $\lambda = \frac{2m}{1+m}$, and Pr is the Prandtl number, equal to the ratio of the momentum diffusivity to thermal diffusivity ($Pr = \nu/\alpha$) of the fluid and prime $'$ denotes differentiation with respect to η.

The boundary conditions (2.4) can be written as

$$f(0) = f'(0) = 0,\ \theta(0) = 0, \text{ and } f'(\eta) \to 1,\ \theta(\eta) \to 1 \text{ as } \eta \to \infty. \tag{2.8}$$

Here we remark that the system of boundary value problems (2.6), (2.7), (2.8) is nonlinear and the available applied mathematical techniques will not be sufficient to solve exactly the system. We need to construct computational algorithm via GAM. Our main contribution is to study the nonlinear system (2.6), (2.7) under the boundary conditions (2.8). Using the transformation $f(\eta) = \int_0^\eta w(s)ds$, the boundary value problem (2.6), (2.7), (2.8) can be written as a second order nonlinear boundary value problem

$$\begin{aligned} -w''(\eta) - w'(\eta) = g(w, w'),\ \eta \in (0,\infty), \\ w(0) = 0,\ w(\infty) = 1, \end{aligned} \tag{2.9}$$

$$
\begin{aligned}
&\theta''(\eta) + Pr.\theta'(\eta)\int_0^\eta w(s)ds = 0,\ \eta \in (0,\infty), \\
&\theta(0) = 0,\ \theta(\infty) = 1,
\end{aligned} \tag{2.10}
$$

where $g(w,w') = (\int_0^\eta w(s)ds - 1)w'(\eta) + \lambda[1 - w^2(\eta)]$. To determine the temperature distribution using (2.10), we need to solve the nonlinear boundary value problem (2.9) for $w(\eta)$. Equivalently a solution of (2.9), is a solution of the corresponding integral equation

$$
w(\eta) = (1 - e^{-\eta}) + \int_0^\infty G(\eta,s)g(w(s),w'(s))ds,\ \eta \in (0,\infty), \tag{2.11}
$$

where

$$
G(\eta,s) = \begin{cases} 1 - e^{-\eta}, & 0 \le \eta < s \le \infty \\ (1 - e^{-\eta})e^{s-\eta}, & 0 \le s < \eta \le \infty \end{cases}
$$

is the Green's function of the corresponding homogeneous linear problem. Clearly, $G(\eta,s) > 0$ on $(0,\infty) \times (0,\infty)$.

3. Upper and Lower Solutions: Estimates for the exact solution

Recall the concept of lower and upper solutions corresponding to the BVP (2.9).

Definition 3.1. A function $\alpha \in C^1(I)$ is called a lower solution of the BVP (2.9) if it satisfies the following inequalities,

$$
\begin{aligned}
-\alpha'' \quad &(\eta) - \alpha'(\eta) \le g(\alpha(\eta), \alpha'(\eta)), \quad \eta \in (0,\infty) \\
&\alpha(0) \le 0,\ \alpha(\infty) \le 1.
\end{aligned}
$$

An upper solution $\beta \in C^1(I)$ of the BVP (2.9) is defined similarly by reversing the inequalities.

Here, $\alpha = 0$ and $\beta = 1$ are lower and upper solutions of the BVP (2.9).

Definition 3.2. For $T > 0$, a continuous function $\omega : (0,\infty) \to (0,\infty)$ is called a Nagumo function if

$$
\int_\gamma^\infty \frac{sds}{\omega(s)} = \infty,
$$

where $\gamma = \max\{|\alpha(0) - \beta(T)|, |\alpha(T) - \beta(0)|\}$. We say that $g \in C[\mathbb{R} \times \mathbb{R}]$ satisfies a Nagumo condition on $[0,T]$ relative to α, β if for $x \in [\min\alpha, \max\beta]$, there exists a Nagumo function ω such that $|g(w,w')| \le \omega(|w'|)$.

The following result is known [1] (Theorem 1.7.1, Page 31).

Theorem 3.3. *Assume that for each $T > 0$, $g(w, w')$ satisfies Nagumo's condition on $[0, T]$ relative to the pair $\alpha, \beta \in C^1[[0,\infty), \mathbb{R}]$ with $\alpha \le \beta$ on $[0,\infty)$. Suppose also that α, β are lower and upper solutions of (2.9) on $[0,\infty)$, respectively. Then the BVP (2.9) has a solution $w \in C^2[[0,\infty), \mathbb{R}]$ such that $\alpha \le w \le \beta$ on $[0,\infty)$.*

Since $\alpha = 0$, $\beta = 1$, therefore for each $T > 0$, $\eta \in [0, T]$ and $w(\eta) \in [0, 1]$, we have

$$|g(w, w')| = |(\int_0^\eta w(s)ds - 1)w'(\eta) + \lambda[1 - w^2(\eta)]| \le (T+1)|w'(\eta)| + |\lambda| = \omega(|w'|).$$

Hence, for each $T > 0$, g satisfies a Nagomo condition with $\omega(s) = (T+1)s + |\lambda|$ as a Nagumo function and $\gamma = 1$. By Theorem 3.3, the the BVP (2.9) has a solution u such that $\alpha \le w \le \beta$ on $[0,\infty)$. Hence

$$0 \le w(\eta) \le 1,\ 0 \le f(\eta) \le \eta,\ \eta \in [0,\infty).$$

These are the estimates for w and f.

4. GENERALIZED APPROXIMATION METHOD (GAM)

Differentiating g with respect to w, w', we obtain

$$g_w = -2\lambda w,\ g_{w'} = f(\eta) - 1,\ g_{ww} = -2\lambda,\ g_{ww'} = 0,\ g_{w'w'} = 0.$$

The quadratic form

$$(w - z)^2 g_{ww}(z, z') + 2(w - z)(w' - z')g_{ww'}(z, z') + (w' - z')^2 g_{w'w'}(z, z') \le 0,$$

which implies that

$$\begin{aligned} g(w, w') &\le g(z, z') + g_w(z, z')(w - z) + g_{w'}(z, z')(w' - z') \\ &= B(z, z') + (\int_0^\eta z(s)ds - 1)w' - 2\lambda z w, \end{aligned} \tag{4.1}$$

where z, $z' \in \mathbb{R}$, $B(z, z') = \lambda(1 - z^2(\eta) + 2z(\eta))$.

Define $h : \mathbb{R}^4 \to \mathbb{R}$ by

$$h(w, w'; z, z') = B(z, z') + (\int_0^\eta z(s)ds - 1)w' - 2\lambda z w.$$

Clearly, h is continuous and satisfies the following relations

$$\begin{cases} g(w, w') \le h(w, w'; z, z'), \\ g(w, w') = h(w, w'; w, w'). \end{cases} \tag{4.2}$$

As an initial approximation, choose $w_0 = \beta = 1$ and consider the following linear BVP

$$\begin{aligned} w''(\eta) - w'(\eta) &= h(w(\eta), w'(\eta); w_0'(\eta), w_0'(\eta)), \\ w(0) = 0, &\quad w(\infty) = 1. \end{aligned} \tag{4.3}$$

Using (4.2) and the definition of lower and upper solutions, we obtain

$$h(w_0(\eta), w_0'(\eta);\, w_0(\eta), w_0'(\eta)) = g(w_0(\eta), w_0'(\eta)) \leq -w_0''(\eta) - w_0'(\eta),\ \eta \in (0,\infty)$$
$$h(\alpha(\eta), \alpha'(\eta);\, w_0(\eta), w_0'(\eta)) \geq g(\alpha(\eta), \alpha'(\eta)) \geq -\alpha''(\eta) - \alpha\beta'(\eta),\ \eta \in (0,\infty),$$

which implies that α and w_0 are lower and upper solutions of (4.3). Hence, by Theorem 3.3, solution w_1 of (4.3) satisfies $\alpha \leq w_1 \leq w_0$ on $[0,\infty)$. Moreover, in view of (4.2) and the fact that w_1 is a solution of (4.3), we obtain

$$-w_1''(\eta) - w_1'(\eta) = h(w_1(\eta), w_1'(\eta);\, w_0(\eta), w_0'(\eta)) \geq g(w_1(\eta), w_1'(\eta)),\ \eta \in (0,\infty), \tag{4.4}$$

which implies that w_1 is an upper solution of (2.9).

Similarly, we can show that α and w_1 are lower and upper solutions of the linear BVP

$$\begin{gathered} -w''(\eta) - w'(\eta) = h(w(\eta), w'(\eta); w_1'(\eta), w_1'(\eta)), \\ w(0) = 0, \quad w(\infty) = 1. \end{gathered} \tag{4.5}$$

By Theorem 3.3, there exists a solution w_2 of (4.5) such that $\alpha \leq w_2 \leq w_1$ on $(0,\infty)$.

Continuing this process we obtain a monotone sequence $\{w_n\}$ of solutions of linear problems satisfying

$$\beta = w_0 \geq w_1 \geq w_2 \geq w_3 \geq ... \geq w_{n-1} \geq w_n \geq \alpha \text{ on } (0,\infty),$$

where the element w_n is a solution of the following linear problem

$$\begin{gathered} -w''(\eta) - w'(\eta) = g(w(\eta), w'(\eta); w_{n-1}(\eta), w_{n-1}'(\eta)), \\ w(0) = 0, \quad w(\infty) = 1, \end{gathered}$$

and is given by

$$w_n(y) = (1 - e^{-\eta}) + \int_0^\infty G(\eta, s) h(w_n(s), w_n'(s); w_{n-1}(s), w_{n-1}'(s)) ds,\ \eta \in (0,\infty). \tag{4.6}$$

The sequence of functions w_n is uniformly bounded and equicontinuous. The monotonicity and uniform boundedness of the sequence $\{w_n\}$ implies the existence of a pointwise limit ω on $(0,\infty)$. From the boundary conditions, we have

$$0 = w_n(0) \to \omega(0) \text{ and } 1 = w_n(\infty) \to \omega(\infty).$$

Hence ω satisfy the boundary conditions. Moreover, by the dominated convergence theorem, for any $\eta \in (0,\infty)$, we have

$$\int_0^\infty G(\eta, s) h(w_n(s), w_n'(s); w_{n-1}(s), w_{n-1}'(s)) ds \to \int_0^\infty G(\eta, s) g(\omega(s), \omega'(s)) ds.$$

Passing to the limit as $n \to \infty$, (4.6) yields

$$\omega(\eta) = (1 - e^{-\eta}) + \int_0^\infty G(\eta, s) g(\omega(s), \omega'(s)) ds,\ \eta \in (0, \infty), \tag{4.7}$$

which implies that ω is a solution of (2.9). Hence, the BVP (2.10) takes the form

$$\begin{aligned} &\theta''(\eta) + Pr.\theta'(\eta) \int_0^\eta \omega(s) ds = 0,\ \eta \in (0, \infty), \\ &\theta(0) = 0,\ \theta(\infty) = 1. \end{aligned} \tag{4.8}$$

5. Error Analysis: Rapid convergence

In order to justify that the convergence is faster, we study error analysis and show that the convergence is quadratic. Define $e_n = w_n - w$ on I, then we have $e_n(0) = 0$, $e_n(\infty) = 0$ and

$$-e_n''(\eta) - e_n'(\eta) = h(\eta, w_n, w_n'; w_{n-1}, w_{n-1}') - g(\eta, w, w'),\ t \in [0, \infty),$$

which in view of the mean value theorem, the definition of h and the relation $g_u \leq 0$, yields

$$-e_n''(\eta) - e_n'(\eta) \leq (f(\eta) - 1) e_n'(\eta) + \lambda e'^2_{n-1}.$$

By comparison result, we have $e_n \leq r$ on $[0, \infty)$, where r is a solution of the linear boundary value problem,

$$-r_n''(\eta) - r_n'(\eta) = (f(\eta) - 1) r_n'(\eta) + \lambda e'^2_{n-1},\ r(0) = r(\infty) = 0$$

and is given by

$$r(\eta) = \int_0^\infty G(\eta, s)[(f(s) - 1) r_n'(s) + \lambda e'^2_{n-1}] ds. \tag{5.1}$$

In view of the boundary conditions, there exists $t_1 \in (0, \infty)$ such that $r'(t_1) = 0$ and in view of the differential equation in (5.1), it follows that the function $r' e^{\int f}$ is decreasing. Hence $r'(\eta) \geq 0$, $\eta \in [0, t_1]$ and $r'(\eta) \leq 0$, $\eta \in [t_1, \infty)]$. Moreover, $f(\eta)$ being an increasing function of η ensure the existence of $t_2 \in (0, \infty)$ such that $f(\eta) \leq 1$, $\eta \in [0, t_2]$ and $f(\eta) \geq 1$, $\eta \in [t_2, \infty)]$. Choose $t_3 = \min\{t_1, t_2\}$ and $t_4 = \max\{t_1, t_2\}$, then

$$(f(\eta)-1) r_n'(\eta) \leq 0,\ \eta \in [0, t_3],\ (f(\eta)-1) r_n'(\eta) \geq 0,\ \eta \in [t_3, t_4],\ (f(\eta)-1) r_n'(\eta) \leq 0,\ t \in [t_4, \infty).$$

The negative part of the function $(f(\eta) - 1) r_n'(\eta)$ is larger than its positive part. Hence,

$$\int_0^\infty G(\eta, s)[(f(s) - 1) r_n'(s)] ds \leq 0$$

and consequently, (5.1) reduces to

$$r(\eta) = \int_0^\infty G(\eta, s) \lambda e'^2_{n-1} ds \leq \int_0^\infty (1 - e^{-s}) \lambda e'^2_{n-1} ds \lambda \|e_{n-1}'\|^2, \tag{5.2}$$

where $\|x\| = \int_0^\infty (1-e^{-s})^{\frac{1}{2}} x(s)ds$ is the L norm.

6. NUMERICAL RESULTS AND DISCUSSION

Results for f, f', f'' and θ via GAM for different values of the parameter λ and the Prandtl number Pr are obtained. Numerical simulations show that only few iterations generated by the GAM is enough to approximate the exact solution of the problem independent of the choices of the parameters and the convergence is very fast. For example, see Figures 6.1 for $f'(\eta)$ [or $u(\eta)$] corresponding to $\lambda = 0.5$ and $\lambda = 1$. The corresponding results for $\lambda = 0.5$ is also shown in Table 1. Recall that $\lambda = 0$ corresponds to the flat plane and $\lambda = 1$ corresponds to the plane stagnation point. The behavior of f, f', f'' are shown in Fig.6.2 corresponding to $\lambda = 0.5$ and $\lambda = 1$. The velocity profile corresponding to $\lambda = 0$, 0.5, 1 is shown in Table 2. From Table 2, it follows that the asymptotic behavior of $f'(\eta)$ is observed quicker for larger values of λ which show the effect of the parameter λ on $f'(\eta)$. Thus f' achieve its asymptotic behavior faster as the value of λ increases. In Fig. 6.3, we study the effect of the Prandtl number Pr on the temperature distribution θ. We observe that both Prandtl number Pr and λ produce some effect on the temperature field θ. For $\lambda = 0.5$, 1, the temperature field θ is shown for different Prandtl number $Pr = 1, 2, 3, 4, 5$ in Fig 6.3. We observe that θ increases and achieve its asymptotic behavior faster as the value of Pr and λ increases.

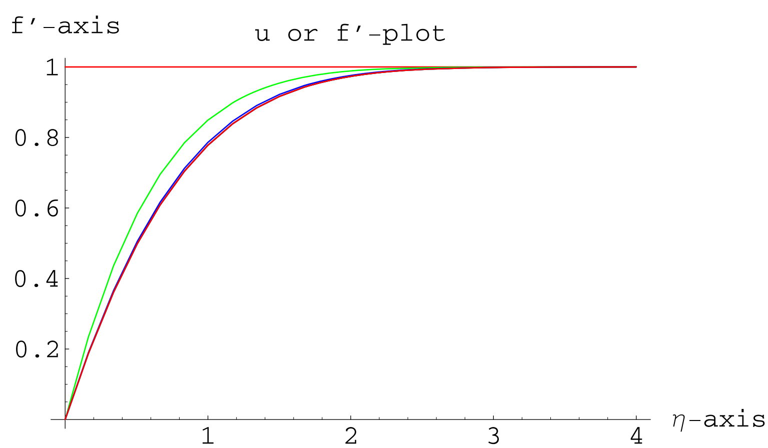

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Fig.1, Results for $f'(\eta)$ via GAM for $\lambda = 0.5$(left) and $\lambda = 1$ (right) indicating that the iterations converges rapidly to the exact solution

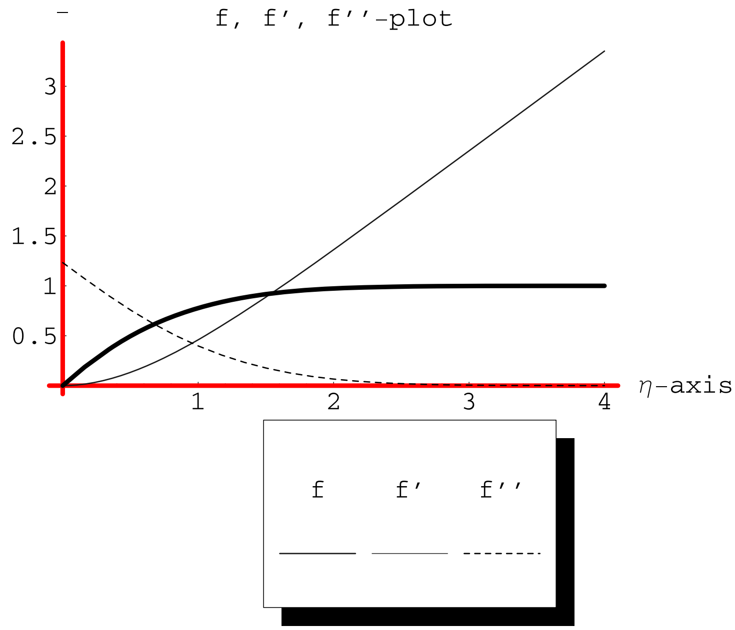

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Fig.2, Results for $f(\eta)$, $f'(\eta)$ and $f''(\eta)$ via GAM for $\lambda = 0.5$ (left) and $\lambda = 1$(right)

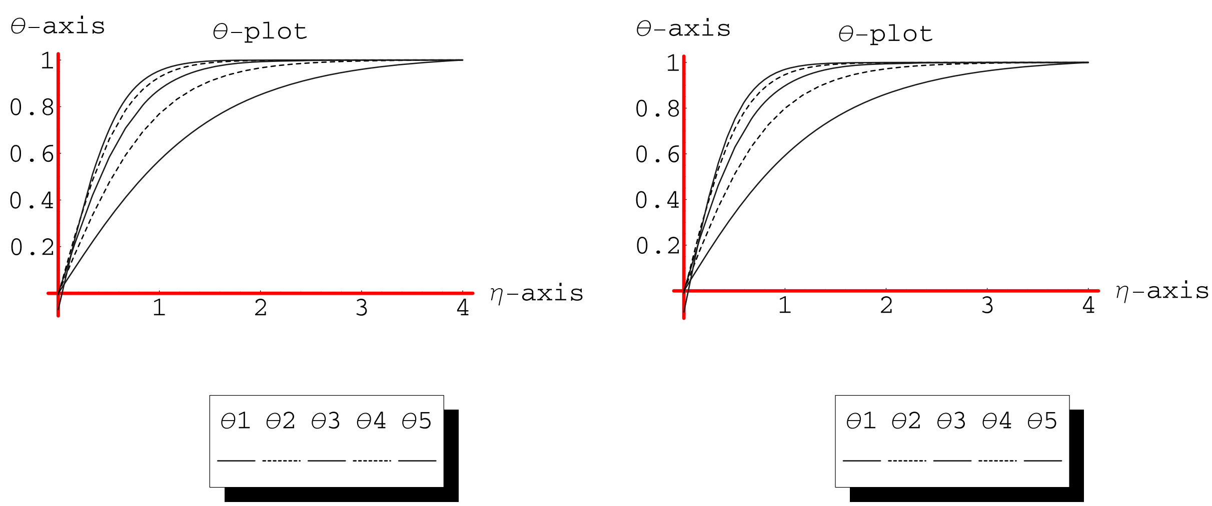

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Fig.3, Results for $Pr = 1, 2, 3, 4, 5$, results via GAM for $\theta(\eta)$ with $\lambda = 1$(left) and $\lambda = 2$(right)

η	0.5	1	1.5	2	2.5	3	3.5	4
w_1	0.504121	0.79113	0.926534	0.978723	0.994987	0.999055	0.999869	1
w_2	0.412332	0.697532	0.866992	0.950988	0.985119	0.996366	0.999354	1
w_3	0.402634	0.683055	0.854516	0.943547	0.981855	0.995298	0.999115	1
w_4	0.401596	0.681362	0.852888	0.942472	0.981341	0.995118	0.999073	1
w_5	0.401482	0.681173	0.852702	0.942346	0.98128	0.995096	0.999067	1
w_6	0.401469	0.681152	0.852681	0.942332	0.981273	0.995093	0.999067	1

Table 1; Results for f' obtained via GAM corresponding to $\lambda = 0.5$

η	0.5	1	1.5	2	2.5	3	3.5	4
$f'[\lambda = 0.5]$	0.401467	0.68115	0.852678	0.94233	0.981272	0.995093	0.999067	1
$f'[\lambda = 1]$	0.494612	0.777848	0.916165	0.973223	0.992865	0.998446	0.999747	1
$f'[\lambda = 1.5]$	0.609368	0.871622	0.96405	0.991449	0.998288	0.999715	0.999963	1

Table 2; Results for f' obtained via GAM corresponding to $\lambda = 0.5$, 1, 1.5

7. CONCLUSIONS

In this paper, we have used the generalized approximation method (GAM) to solve the third-order boundary value problem characterized by the well-known Falkner-Skan equation. The effectiveness of the method is illustrated by applying it successfully to various instances of the Falkner-Skan equation. Effect of parameters on the flow is simulated using MATHEMATICA. Numerical simulations show that the sequence of iterates (approximate solutions) converges rapidly to the exact solution of the problem.

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