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2.8: Single Node Adjustment Method

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2.8. Single Node Adjustment Method

From the previous section we saw that identifying the nodal heads that produce flows which subsequently satisfy continuity is no easy task. We seek a systematic way of finding correction factors that will – in a methodical way – drive heads to produce flows that will satisfy continuity.

We start with the general form of the continuity equation given below.

$$\sum_i Q_{in,j} - \sum_i Q_{out,j} - D_j = 0 \quad j = 1,2, \ldots J$$

Eq. (1)

We know from Eq. (71) and Eq. (73) that flows can be expressed in terms of nodal heads. Therefore Eq. (1) can be rewritten as:

$$F(H)_j = \sum_i Q(H)_{in,j} - \sum_i Q(H)_{out,j} - D_j = 0 \quad j = 1,2, \ldots J$$

Eq. (74)

Eq. (71) can be expressed in terms of the head difference between node k and node j regardless of the flow direction with the equation shown below. Note that we make use of the sign operator such that sign(H_k-H_j) is positive if $H_k > H_j$ and is negative if $H_k < H_j$.

In other words, if $H_k > H_j$ then flow from the pipe enters node j while flow from the pipe leaves node j if $H_k < H_j$. The sign convention used throughout this document assumes a positive value of the pipeline flow if flow enters a node and a negative value if flow leaves a node.

$$Q(H)_i = sign(H_k - H_j)\left(\frac{|H_k - H_j|}{K_{P,i}}\right)^{1/N}$$

Eq. (75)

Where H_j is the head at node j, H_k is the head at the connecting node, $K_{P,I}$ is the pipe resistance coefficient for pipe i, and N is the exponent used for Darcy-Weisbach (N=2) or Hazen-Williams (N=1.852).

Now Eq. (74) can now be written compactly as shown in Eq. (76) below. Note that the i subscript refers to all the pipes that are connected at node j.

$$F(H)_j = \sum_i Q(H)_i - D_j = \sum \left[sign\big(H_k - H_j\big)\left(\frac{|H_k - H_j|}{K_{P,i}}\right)^{\frac{1}{N}}\right]_i - D_j = 0$$

Eq. (76)

We can obtain a head correction factor, ΔH, for each node by using a first order Taylor Series expansion about some initial nodal head, H_o, we have:

$$F(H)_j = \mathrm{F}(H_0)_j + \frac{\partial F(H_0)}{\partial H}\Delta H_j \quad j = 1,2, \dots .J$$

Eq. (77)

We find the gradient or derivative term, $\partial F(H_0)/\partial H$, by taking the derivative of Eq. (75) with respect to head. Doing so yields:

$$\frac{\partial Q(H)_i}{\partial H} = \left(\frac{1}{NK_{P,i}}\right) sign(H_k - H_j)\left(\frac{|H_k - H_j|}{K_{P,i}}\right)^{\left(\frac{1}{N}\right)-1}$$

Eq. (78)

Eq. (78) can be rewritten as:

$$\frac{\partial Q(H)_i}{\partial H} = \left(\frac{1}{NK_{P,i}}\right) sign(H_k - H_j)\left(\frac{|H_k - H_j|}{K_{P,i}}\right)^{\left(\frac{1}{N}\right)}\left(\frac{|H_k - H_j|}{K_{P,i}}\right)^{-1}$$

Eq. (79)

The second to last term in Eq. (79) is simply the pipeline discharge, Q_i. Recognizing this and inverting the last term we have:

$$\frac{\partial Q(H)_i}{\partial H} = \left(\frac{1}{NK_{P,i}}\right) sign(H_k - H_j)\left(\frac{K_{P,i}}{|H_k - H_j|}\right) Q_i$$

Eq. (80)

Or

$$\frac{\partial Q(H)_i}{\partial H} = \left(\frac{1}{N}\right) sign(H_k - H_j)\left(\frac{1}{|H_k - H_j|}\right) Q_i$$

Eq. (81)

Using the earlier definition of the net head change, F(Q) we have:

$$\frac{\partial Q(H)_i}{\partial H} = \left(\frac{1}{N}\right)\left(\frac{1}{F(Q)_i}\right) Q_i$$

Eq. (82)

Or

$$\frac{\partial Q(H)_i}{\partial H} = \left(\frac{1}{N}\right)\left(\frac{1}{K_{P,i}Q_i|Q_i|^{0.852}}\right)Q_i$$

Eq. (83)

$$\frac{\partial Q(H)_i}{\partial H} = \left(\frac{1}{NK_{P,i}|Q_i|^{0.852}}\right) = \frac{1}{G(|Q_i|)}$$

Eq. (84)

Where $G(|Q_i|)$ is the gradient of $F(Q_i)$ with respect to the pipeline flow. The gradient term can be found from Eq. (43).

Substituting Eq. (84) into Eq. (77) gives:

$$F(H)_j = \mathrm{F}(H_0)_j + \left(\sum_i \frac{1}{G(|Q_i|)}\right)\Delta H_j$$

Eq. (85)

Recall that the way we have set up the continuity equation as shown in Eq. (74), F(H) = 0 at the correct heads. Therefore Eq. (85) can be solved for the head correction factor:

$$\Delta H_j = -\frac{\mathrm{F}(H_0)_j}{\left(\sum_i \frac{1}{G(|Q_i|)}\right)}$$

Eq. (86)

The update formula is used to update the head at Node j is:

$$H_{j,New} = H_{j,Old} - \Delta H_j$$

Eq. (87)

An example of the Single Node Adjustment method is presented below. In this approach a head correction factor is developed and applied to each continuity equation. The process is repeated until head correction factors are close to zero – that is – the approach has converged.

2.8.1. Single Node Adjustment Method – Example #1

Consider the network shown in Figure 106 with the corresponding pipe characteristics given in Table 257 and pump characteristics given in Table 258. Note that this is the same problem presented as Example #1 in the previous sections. From Example #1 in the previous section on the Simultaneous Path Adjustment method, the head-discharge relationship for the pump when flow is expressed in Cfs is:

$$E(Q) = 350 - 2.392Q^{1.760}$$

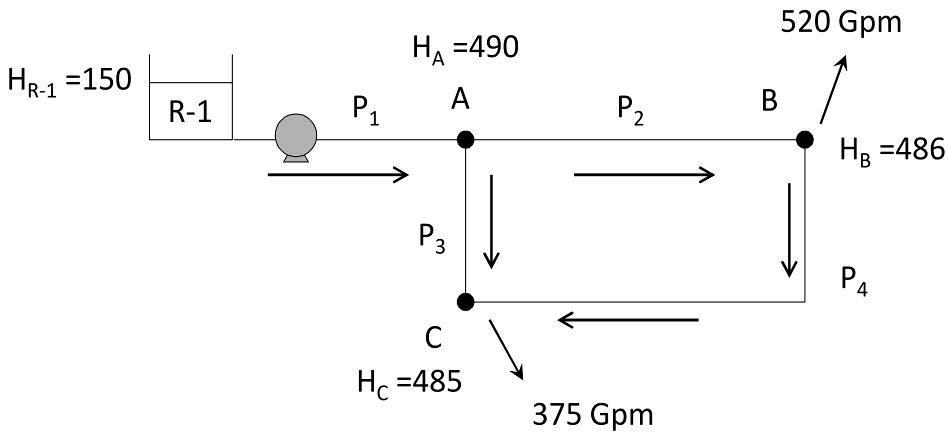

Figure 106 – Four Pipe System (Iteration #0)

Table 257 – Pipe Characteristics for Four Pipe System

Pipe Label	Length (Ft)	Diameter (In)	C-Factor	Minor Loss Coeff	Kp	Km
P_1	800	12	120	10	0.534	0.252
P_2	1,200	10	120	0	1.945	0.000
P_3	1,500	8	120	0	7.209	0.000
P_4	2,200	6	120	0	42.920	0.000

Table 258 – Pump Data for Four Pipe System

Flow (Gpm)	Pump Head (Ft)
0	350
1500	330
2800	290

Table 259 – Node Data for Four Pipe System

Node	Demand (Gpm)	Demand (Cfs)
Node A	0	0
Node B	520	1.1585
Node C	375	0.8355

Iteration #0: When using the node equations an initial head at each node must be provided, i.e. an initial guess. It is not necessary for the initial head to produce flows that satisfy continuity. In fact, what we aim to do is converge to a set of nodal heads that produce flows which satisfy continuity. The initial set of heads for this example are given in Table 260.

Table 260 – Heads for Iteration #0

Node Label	Nodal Head (Ft)
A	490.00
B	486.00
C	485.00

Notice that the flow directions shown in Figure 106 are consistent with the heads shown in Table 260. We know from basic fluid mechanics that flow is from a high head to a low head. Thus for pipe P_2 we know that flow is from Node A to Node B. For pipe P_3 we know that flow is from Node A to Node C. For pipe P_4 we know that flow is from Node B to Node C. There is a pump in pipe P_1 which adds head and causes the hydraulic grade to increase from 150 ft (at R-1) to an assumed value of 490 ft at Node A.

The first step in the solution process is to compute the flow rate in each pipe using Eq. (71) for those pipes with no minor losses or pumps and using Eq. (73) within a root-solving framework for those

pipes that do have minor losses and/or pumps. Referring to the section on the Manual Solution of the Node Equations for details we see that a flow in pipe P_1 of Q_1 = 1.894 Cfs results in a solution to the root-solving problem:

$$F(Q)_1 = 0.534Q_1^{1.852} + 0.252Q_1^2 - (350 - 2.392Q_1^{1.760})$$
$$= (150 - 490) = -340$$

Or

$$F(Q)_1 = 0.534Q_1^{1.852} + 0.252Q_1^2 - (350 - 2.392Q_1^{1.760}) + 340 = 0$$

$$F(Q = 1.894)_1$$
$$= 0.252(1.894)^2 + 0.534(1.894)^{1.852} + 2.392(1.894)^{1.760}$$
$$- 10 = 0.006 \approx 0$$

For pipe P_2:

$$Q_2 = \left(\frac{490 - 486}{1.945}\right)^{1/1.852} = 1.476\ Cfs$$

For pipe P_3:

$$Q_3 = \left(\frac{490 - 485}{7.209}\right)^{1/1.852} = 0.821\ Cfs$$

For pipe P_4:

$$Q_4 = \left(\frac{486 - 485}{42.920}\right)^{1/1.852} = 0.131\ Cfs$$

We have now computed the flow in each pipeline associated with the initial set of heads. The flows are shown in Table 261 below. Now that we have the flow in each pipes, we can see if continuity at each junction node is satisfied.

Table 261 – Flows for Iteration #0

Pipe Label	Flow (Cfs)
P_1	1.894
P_2	1.476
P_3	0.821
P_4	0.131

Node A:

$$Q_1 - (Q_2 + Q_3) - D_A = 0$$
$$1.894 - (1.476 + 0.821) - 0 = -0.403 \neq 0$$

Node B:

$$Q_2 - Q_4 - D_B = 0$$
$$1.476 - 0.131 - 1.1585 = 0.186 \neq 0$$

Node C:

$$Q_3 + Q_4 - D_C = 0$$
$$0.821 + 0.131 - 0.8355 = 0.117 \neq 0$$

From the expressions above we can see that continuity is not satisfied; therefore, the nodal heads presented in Table 260 are not the correct nodal heads. Consequently, the flows shown in Table 261 are not the correct flows. We must change the values of the nodal heads in such a way that we drive the flows so that they satisfy continuity.

<u>Iteration #1</u>: Let's develop head correction factors for Node A, Node B and Node C. Similar to the Hardy-Cross method for flow correction factors, we will compute a head correction factor for a node and apply the correction factor to compute a new set of flows. Note that only the pipes connected to the node where the updated head is computed will experience a change in flows.

Node A: We start by developing a head correction factor for Node A. The head correction factor is computed from Eq. (86).

$$\Delta H_A = -\frac{\mathrm{F}(H_0)_A}{\left(\sum_i \frac{1}{G(|Q_i|)}\right)}$$

Notice that $F(H_o)$ is the continuity equation for Node A evaluated at the initial set of heads. Recall that the flow in each pipe for the initial guess of nodal heads is presented in Table 261.

Thus for the initial heads:

$$F(H_o)_A = Q_1 - (Q_2 + Q_3) - D_A = 1.894 - (1.476 + 0.821) = -0.403$$

Now the gradient terms for each pipe must be computed.

$$G(Q)_1 = (1.852)0.534Q_1^{0.852} + (2)0.252Q_1 + (1.760)2.392Q_1^{0.760}$$

For a flow of Q_1 = 1.894 Cfs, then the gradient term for P_1 is:

$$G(Q = 1.894)_1 = (1.852)0.534(1.894)^{0.852} + (2)0.252(1.894) + (1.760)2.392(1.894)^{0.760} = 9.497$$

For P_2 evaluated at its initial flow of Q_2 = 1.476 Cfs:

$$G(Q)_2 = (1.852)1.945Q_2^{0.852} = (1.852)1.945(1.476)^{0.852} = 5.020$$

For P_3 evaluated at its initial flow of Q_3 = 0.821 Cfs:

$$G(Q)_3 = (1.852)7.209Q_3^{0.852} = (1.852)7.209(0.821)^{0.852} = 11.286$$

Now the head correction factor for Node A is:

$$\Delta H_A = -\frac{-0.403}{\left(\frac{1}{9.497}+\frac{1}{5.020}+\frac{1}{11.286}\right)} = -\frac{-0.403}{0.393} = 1.025$$

Now we update the head at Node A and recompute the flows.

$$H_{A,New} = H_{A,Old} - \Delta H_A = 490 - 1.025 = 488.975$$

For pipe P_1 a root-solving procedure is necessary to find the pipeline flow due to the presence of the pump. Note that the flow in pipe P_1 changes because the head at Node A changes. The updated root-solving equation is:

$$F(Q)_1 = 0.534Q_1^{1.852} + 0.252Q_1^2 - (350 - 2.392Q_1^{1.760}) \\ = (150 - 488.975) = -338.975$$

$$F(Q = 1.998)_1 \\ = 0.252(1.998)^2 + 0.534(1.998)^{1.852} + 2.392(1.998)^{1.760} \\ - 11.025 = -0.007 \approx 0$$

For pipe P_2:

$$Q_2 = \left(\frac{488.975 - 486}{1.945}\right)^{1/1.852} = 1.258\, Cfs$$

For pipe P_3:

$$Q_3 = \left(\frac{488.975 - 485}{7.209}\right)^{1/1.852} = 0.725\, Cfs$$

Node B: Now we examine Node B and develop a head correction factor for this node. We start by computing continuity at this node to obtain $F(Q)_B$. Then we find the gradient terms for all pipes connected to this node. We end examination of this node by developing a node correction factor.

$$F(H_o)_B = Q_2 - Q_4 - D_B = 1.258 - 0.131 - 1.1585 = -0.0315$$

For P_2 evaluated at its current flow of Q_2 = 1.258 Cfs:

$$G(Q)_2 = (1.852)1.945Q_2^{0.852} = (1.852)1.945(1.258)^{0.852} = 4.381$$

For P_4 evaluated at its initial flow of Q_4 = 0.131 Cfs:

$$G(Q)_4 = (1.852)42.920Q_4^{0.852} = (1.852)42.920(0.131)^{0.852}$$
$$= 14.067$$

Now the head correction factor for Node B is:

$$\Delta H_B = -\frac{-0.0315}{\left(\frac{1}{4.381} + \frac{1}{14.067}\right)} = -\frac{-0.0315}{0.299} = 0.105$$

Now we update the head at Node B and recompute the flows.

$$H_{B,New} = H_{B,Old} - \Delta H_B = 486 - 0.105 = 485.895$$

For pipe P_2:

$$Q_2 = \left(\frac{488.975 - 485.895}{1.945}\right)^{1/1.852} = 1.282\ Cfs$$

For pipe P_4:

$$Q_4 = \left(\frac{485.895 - 485}{42.920}\right)^{1/1.852} = 0.124\,Cfs$$

Node C: Now we examine Node C and develop a head correction factor for this node. We start by computing continuity at this node to obtain $F(Q)_C$. Then we find the gradient terms for all pipes connected to this node. We end examination of this node by developing a node correction factor.

$$F(H_o)_C = Q_3 + Q_4 - D_C = 0.725 + 0.124 - 0.8355 = 0.0135$$

For P_3 evaluated at its current flow of Q_3 = 0.725 Cfs:

$$G(Q)_3 = (1.852)7.209Q_3^{0.852} = (1.852)7.209(0.725)^{0.852} = 10.151$$

For P_4 evaluated at its current flow of Q_4 = 0.124 Cfs:

$$G(Q)_4 = (1.852)42.920Q_4^{0.852} = (1.852)42.920(0.124)^{0.852} = 13.424$$

Now the head correction factor for Node C is:

$$\Delta H_C = -\frac{0.0135}{\left(\frac{1}{10.151} + \frac{1}{13.424}\right)} = -\frac{0.0135}{0.173} = -0.078$$

Now we update the head at Node C and recompute the flows.

$$H_{C,New} = H_{C,Old} - \Delta H_C = 485 - (-0.078) = 485.078$$

For pipe P_3:

$$Q_3 = \left(\frac{488.975 - 485.078}{7.209}\right)^{1/1.852} = 0.717\ Cfs$$

For pipe P_4:

$$Q_4 = \left(\frac{485.895 - 485.078}{42.920}\right)^{1/1.852} = 0.118\ Cfs$$

We will summarize the function evaluations, that is, continuity at each node to see if continuity is satisfied. If continuity is not satisfied, then we will need to perform one or more additional iterations.

Table 262 – Flows for Iteration #1

Pipe Label	Flow (Cfs)
P_1	1.998
P_2	1.282
P_3	0.717
P_4	0.118

Table 263 – Function Evaluations for Iteration #1

Node	F(Q)
A	-0.001
B	0.006
C	0.001

The function evaluations shown in Table 263 indicate that the solution has, for all practical purposes, converged. The reason that only one iteration was required to converge to a solution is because the initial guess for nodal heads was very close to the final solution. Let's retry evaluating this system but using an initial guess for nodal heads that is further away from the true solution.

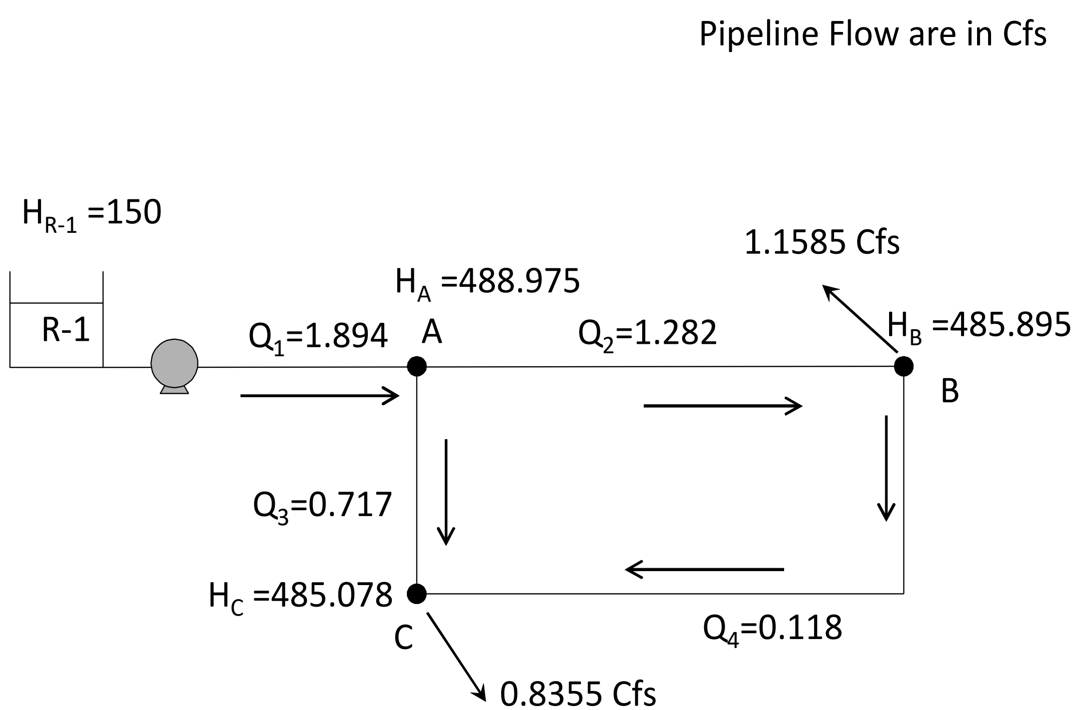

Figure 107 – Four Pipe System (Iteration #1)

2.8.2. Single Node Adjustment Method – Example #1A

In this example we use the initial guess of nodal head shown in Table 264. Note that the head at Node A is governed somewhat by the cutoff head of the pump. The cutoff head of the pump is 350 ft. Noting that the reservoir level is 150 ft, the maximum head at Node A is 500 ft.

Iteration #0:

Table 264 – Heads for Iteration #0

Node Label	Nodal Head (Ft)
A	498.00
B	475.00
C	480.00

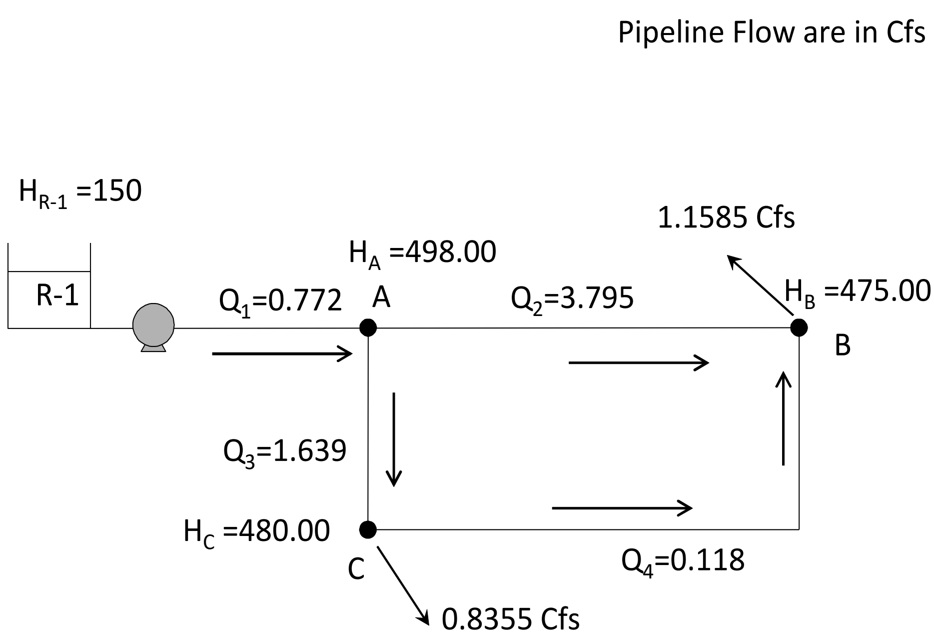

Figure 108 – Example #1A (Iteration #0)

Node A: We compute the pipeline flow for pipe P_1 using a root-solving technique. With a head of 498 ft for Node A, the energy equation for pipe P_1 is:

$$F(Q)_1 = 0.534Q_1^{1.852} + 0.252Q_1^2 - (350 - 2.392Q_1^{1.760}) \\ = (150 - 498) = -348$$

Or

$$F(Q)_1 = 0.534Q_1^{1.852} + 0.252Q_1^2 - (350 - 2.392Q_1^{1.760}) + 348 = 0$$

$$F(Q = 0.772)_1 \\ = 0.252(0.772)^2 + 0.534(0.772)^{1.852} + 2.392(0.772)^{1.760} \\ - 2 = -0.002 \approx 0$$

For pipe P_2:

$$Q_2 = \left(\frac{498 - 475}{1.945}\right)^{1/1.852} = 3.795\ Cfs$$

For pipe P_3:

$$Q_3 = \left(\frac{498 - 480}{7.209}\right)^{1/1.852} = 1.639\ Cfs$$

For pipe P_4:

$$Q_4 = \left(\frac{480 - 475}{42.920}\right)^{1/1.852} = 0.313\ Cfs$$

The function evaluation for Node A is now:

$F(H_o)_A = Q_1 - (Q_2 + Q_3) - D_A = 0.772 - (3.795 + 1.639) = -4.662$

Now the gradient terms for each pipe must be computed.

$G(Q)_1 = (1.852)0.534Q_1^{0.852} + (2)0.252Q_1 + (1.760)2.392Q_1^{0.760}$

For a flow of Q_1 = 0.772 Cfs, then the gradient term for P_1 is:

$$G(Q = 0.772)_1 = (1.852)0.534(0.772)^{0.852} + (2)0.252(0.772) + (1.760)2.392(0.772)^{0.760} = 4.645$$

For P_2 evaluated at its initial flow of Q_2 = 3.795 Cfs:

$G(Q)_2 = (1.852)1.945Q_2^{0.852} = (1.852)1.945(3.795)^{0.852} = 11.224$

For P_3 evaluated at its initial flow of Q_3 = 1.639 Cfs:

$G(Q)_3 = (1.852)7.209Q_3^{0.852} = (1.852)7.209(1.639)^{0.852} = 20.339$

Now the head correction factor for Node A is:

$$\Delta H_A = -\frac{-4.662}{\left(\frac{1}{4.645} + \frac{1}{11.224} + \frac{1}{20.339}\right)} = -\frac{-4.662}{0.353} = 13.204$$

Now we update the head at Node A and recompute the flows using the updated heads.

$$H_{A,New} = H_{A,Old} - \Delta H_A = 498 - 13.204 = 484.796$$

The new flows are:

$$F(Q)_1 = 0.534Q_1^{1.852} + 0.252Q_1^2 - (350 - 2.392Q_1^{1.760})$$
$$= (150 - 484.796) = -334.796$$

$$F(Q = 2.390)_1$$
$$= 0.252(2.390)^2 + 0.534(2.390)^{1.852} + 2.392(2.390)^{1.760}$$
$$- 15.204 = 0.002 \approx 0$$

For pipe P_2:

$$Q_2 = \left(\frac{484.796 - 475}{1.945}\right)^{1/1.852} = 2.394\ Cfs$$

For pipe P_3:

$$Q_3 = \left(\frac{484.796 - 480}{7.209}\right)^{1/1.852} = 0.802\ Cfs$$

Figure 109 shows the nodal heads and the pipeline flows associated with the updated head at Node A.

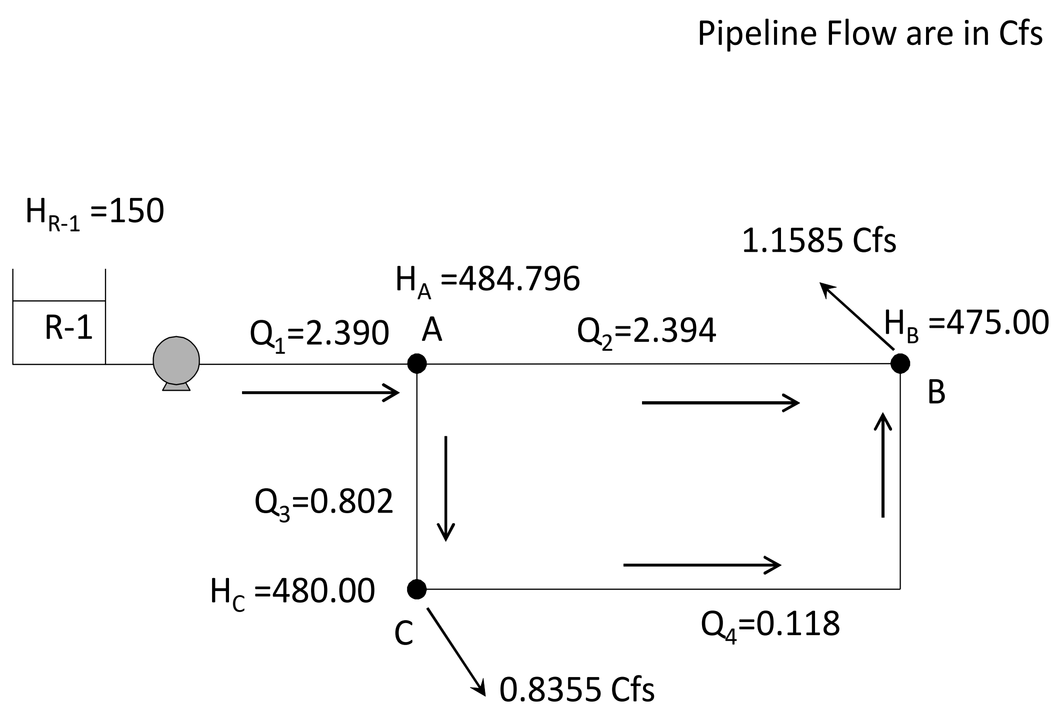

Figure 109 – Example #1A (Iteration #0)

Node B: Now we examine Node B and develop a head correction factor for this node. We start by computing continuity at this node to obtain $F(Q)_B$. Then we find the gradient terms for all pipes connected to this node. We end examination of this node by developing a node correction factor.

$$F(H_o)_B = \ Q_2 + Q_4 - D_B = 2.394 + 0.313 - 1.1585 = 1.549$$

For P_2 evaluated at its current flow of Q_2 = 2.394 Cfs:

$$G(Q)_2 = (1.852)1.945Q_2^{0.852} = (1.852)1.945(2.394)^{0.852} = 7.580$$

For P_4 evaluated at its initial flow of Q_4 = 0.313 Cfs:

$$G(Q)_4 = (1.852)42.920Q_4^{0.852} = (1.852)42.920(0.313)^{0.852}$$
$$= 29.546$$

Now the head correction factor for Node B is:

$$\Delta H_B = -\frac{1.549}{\left(\frac{1}{7.580} + \frac{1}{29.546}\right)} = -\frac{1.549}{0.166} = -9.328$$

Now we update the head at Node B and recompute the flows.

$$H_{B,New} = H_{B,Old} - \Delta H_B = 475 - (-9.328) = 484.328$$

For pipe P_2:

$$Q_2 = \left(\frac{484.796 - 484.328}{1.945}\right)^{1/1.852} = 0.463\ Cfs$$

For pipe P_4:

$$Q_4 = \left(\frac{484.328 - 480}{42.920}\right)^{1/1.852} = 0.290\ Cfs$$

Figure 110 shows the updated heads and flows after the head correction factor for Node B is applied. Notice that there is a change in flow direction for pipe P_4 as the head at Node B is greater than the head at Node C.

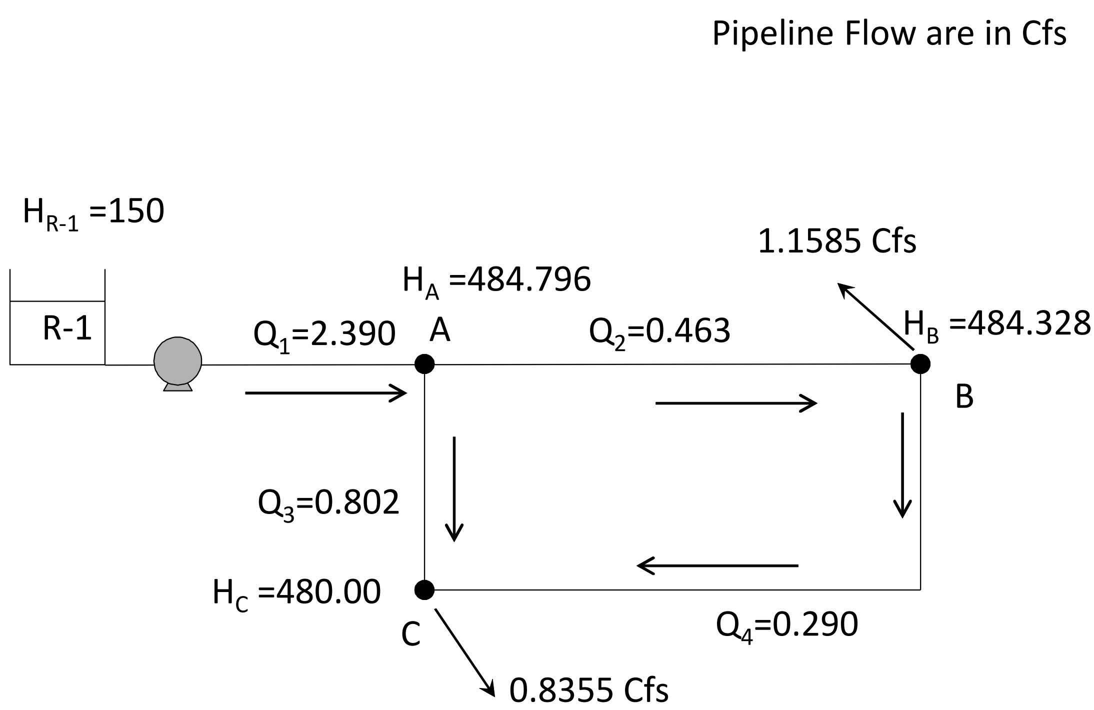

Figure 110 – Example #1A (Iteration #0)

Node C: Now we examine Node C and develop a head correction factor for this node. We start by computing continuity at this node to obtain $F(Q)_C$. Then we find the gradient terms for all pipes connected to this node. We end examination of this node by developing a node correction factor.

$$F(H_o)_C = Q_3 + Q_4 - D_C = 0.802 + 0.290 - 0.8355 = 0.257$$

For P_3 evaluated at its current flow of Q_3 = 0.802 Cfs:

$$G(Q)_3 = (1.852)7.209Q_3^{0.852} = (1.852)7.209(0.802)^{0.852} = 11.063$$

For P_4 evaluated at its current flow of Q_4 = 0.290 Cfs:

$$G(Q)_4 = (1.852)42.920Q_4^{0.852} = (1.852)42.920(0.290)^{0.852}$$
$$= 27.686$$

Now the head correction factor for Node C is:

$$\Delta H_C = -\frac{0.257}{\left(\frac{1}{11.063} + \frac{1}{27.686}\right)} = -\frac{0.257}{0.126} = -2.036$$

Now we update the head at Node C and recompute the flows using the most recent head values.

$$H_{C,New} = H_{C,Old} - \Delta H_C = 480 - (-2.036) = 482.036$$

For pipe P_3:

$$Q_3 = \left(\frac{484.796 - 482.036}{7.209}\right)^{1/1.852} = 0.595\ Cfs$$

For pipe P_4:

$$Q_4 = \left(\frac{484.328 - 482.036}{42.920}\right)^{1/1.852} = 0.206\ Cfs$$

Figure 111 shows the updated heads and flows after the head correction factor for Node C is applied.

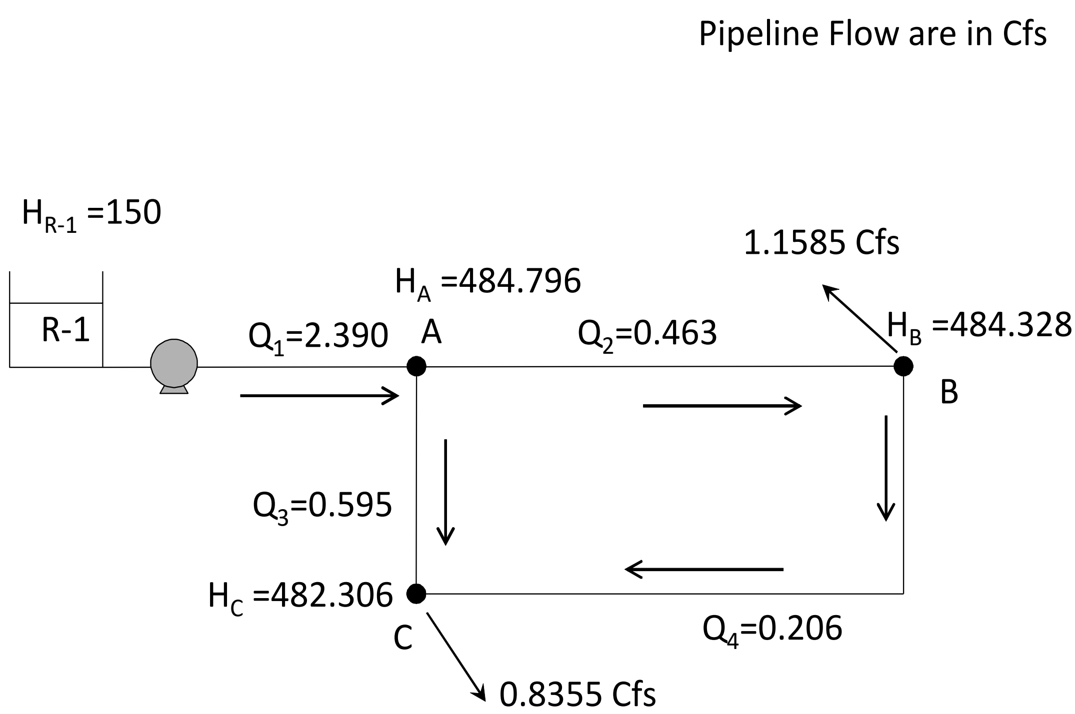

Figure 111 – Example #1A (Iteration #0)

Iteration #1:

The new set of heads for Iteration #1 are given in Table 265. Table 266 shows the more recent set of pipeline flows. These flows are based upon the most current nodal heads. Figure 111 shows the current set of flows and their directions.

Table 265 – Heads for Iteration #1

Node Label	Nodal Head (Ft)
A	484.796
B	484.328
C	482.036

Table 266 – Flows for Iteration #1

Pipe Label	Flow (Cfs)
P_1	2.390
P_2	0.463
P_3	0.595
P_4	0.206

Node A: We compute the function evaluation for Node A using the most current pipeline flows. The function evaluation for Node A is now:

$$F(H_o)_A = Q_1 - (Q_2 + Q_3) - D_A = 2.390 - (0.463 + 0.595) = 1.332$$

Now the gradient terms for each pipe must be computed.

$$G(Q)_1 = (1.852)0.534Q_1^{0.852} + (2)0.252Q_1 + (1.760)2.392Q_1^{0.760}$$

For a flow of Q_1 = 2.390 Cfs, then the gradient term for P_1 is:

$$\begin{aligned} G(Q = 2.390)_1 \\ &= (1.852)0.534(2.390)^{0.852} + (2)0.252(2.390) \\ &+ (1.760)2.392(2.390)^{0.760} = 11.443 \end{aligned}$$

For P_2 evaluated at its initial flow of Q_2 = 0.463 Cfs:

$$G(Q)_2 = (1.852)1.945Q_2^{0.852} = (1.852)1.945(0.463)^{0.852} = 1.870$$

For P_3 evaluated at its initial flow of Q_3 = 0.595 Cfs:

$$G(Q)_3 = (1.852)7.209Q_3^{0.852} = (1.852)7.209(0.595)^{0.852} = 8.578$$

Now the head correction factor for Node A is:

$$\Delta H_A = -\frac{1.332}{\left(\frac{1}{11.443} + \frac{1}{1.870} + \frac{1}{8.578}\right)} = -\frac{1.332}{0.739} = -1.802$$

Now we update the head at Node A and recompute the flows using the updated heads.

$$H_{A,New} = H_{A,Old} - \Delta H_A = 484.796 - (-1.802) = 486.598$$

The new flows are:

$$\begin{aligned} F(Q)_1 &= 0.534Q_1^{1.852} + 0.252Q_1^2 - (350 - 2.392Q_1^{1.760}) \\ &= (150 - 486.598) = -336.598 \end{aligned}$$

$$\begin{aligned} &F(Q = 2.228)_1 \\ &\quad = 0.252(2.228)^2 + 0.534(2.228)^{1.852} + 2.392(2.228)^{1.760} \\ &\quad - 13.402 = 0.0002 \approx 0 \end{aligned}$$

For pipe P_2:

$$Q_2 = \left(\frac{486.598 - 484.328}{1.945}\right)^{1/1.852} = 1.087\ Cfs$$

For pipe P_3:

$$Q_3 = \left(\frac{486.598 - 482.036}{7.209}\right)^{1/1.852} = 0.781\ Cfs$$

Figure 112 shows the nodal heads and the pipeline flows associated with the updated head at Node A.

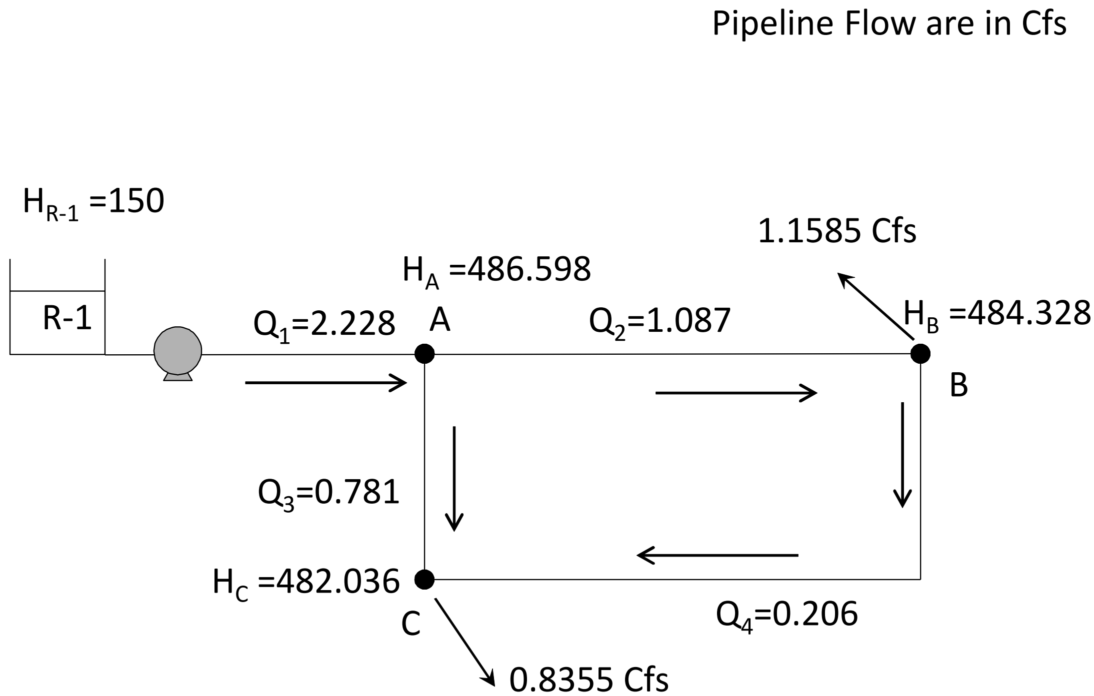

Figure 112 – Example #1A (Iteration #1)

Node B: Now we examine Node B and develop a head correction factor for this node. We start by computing continuity at this node to obtain $F(Q)_B$. Then we find the gradient terms for all pipes connected to this node. We end examination of this node by developing a node correction factor.

$$F(H_o)_B = Q_2 - Q_4 - D_B = 1.087 - 0.206 - 1.1585 = -0.278$$

For P_2 evaluated at its current flow of Q_2 = 1.087 Cfs:

$$G(Q)_2 = (1.852)1.945Q_2^{0.852} = (1.852)1.945(1.087)^{0.852} = 3.868$$

For P_4 evaluated at its initial flow of Q_4 = 0.206 Cfs:

$$G(Q)_4 = (1.852)42.920Q_4^{0.852} = (1.852)42.920(0.206)^{0.852}$$
$$= 20.688$$

Now the head correction factor for Node B is:

$$\Delta H_B = -\frac{-0.278}{\left(\frac{1}{3.868} + \frac{1}{20.688}\right)} = -\frac{-0.278}{0.307} = 0.904$$

Now we update the head at Node B and recompute the flows.

$$H_{B,New} = H_{B,Old} - \Delta H_B = 484.328 - 0.904 = 483.424$$

For pipe P_2:

$$Q_2 = \left(\frac{486.598 - 483.424}{1.945}\right)^{1/1.852} = 1.303\ Cfs$$

For pipe P_4:

$$Q_4 = \left(\frac{483.424 - 482.036}{42.920}\right)^{1/1.852} = 0.157\ Cfs$$

Figure 113 shows the updated heads and flows after the head correction factor for Node B is applied.

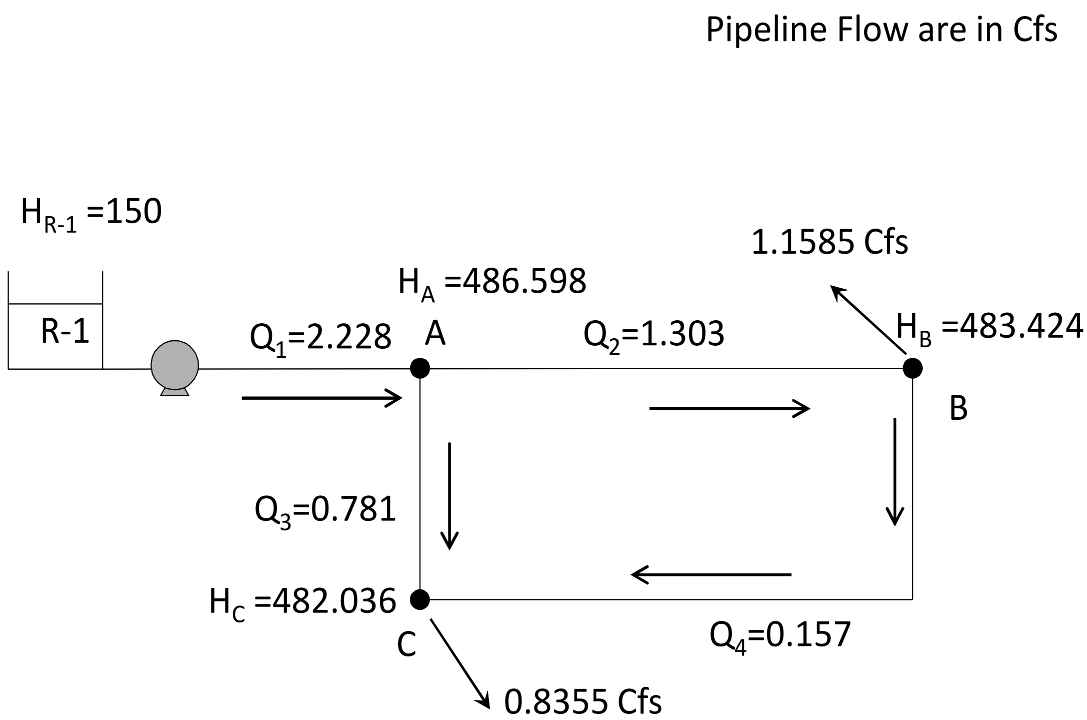

Figure 113 – Example #1A (Iteration #1)

Node C: Now we examine Node C and develop a head correction factor for this node. We start by computing continuity at this node to obtain $F(Q)_C$. Then we find the gradient terms for all pipes connected to this node. We end examination of this node by developing a node correction factor.

$$F(H_o)_C = Q_3 + Q_4 - D_C = 0.781 + 0.157 - 0.8355 = 0.103$$

For P_3 evaluated at its current flow of Q_3 = 0.781 Cfs:

$$G(Q)_3 = (1.852)7.209Q_3^{0.852} = (1.852)7.209(0.781)^{0.852} = 10.816$$

For P_4 evaluated at its current flow of Q_4 = 0.157 Cfs:

$$G(Q)_4 = (1.852)42.920Q_4^{0.852} = (1.852)42.920(0.157)^{0.852}$$
$$= 16.414$$

Now the head correction factor for Node C is:

$$\Delta H_C = -\frac{0.103}{\left(\frac{1}{10.816} + \frac{1}{16.414}\right)} = -\frac{0.103}{0.153} = -0.670$$

Now we update the head at Node C and recompute the flows using the most recent head values.

$$H_{C,New} = H_{C,Old} - \Delta H_C = 482.036 - (-0.67) = 482.706$$

For pipe P_3:

$$Q_3 = \left(\frac{486.598 - 482.706}{7.209}\right)^{1/1.852} = 0.717\ Cfs$$

For pipe P_4:

$$Q_4 = \left(\frac{483.424 - 482.706}{42.920}\right)^{1/1.852} = 0.110\ Cfs$$

Figure 114 shows the updated heads and flows after the head correction factor for Node C is applied.

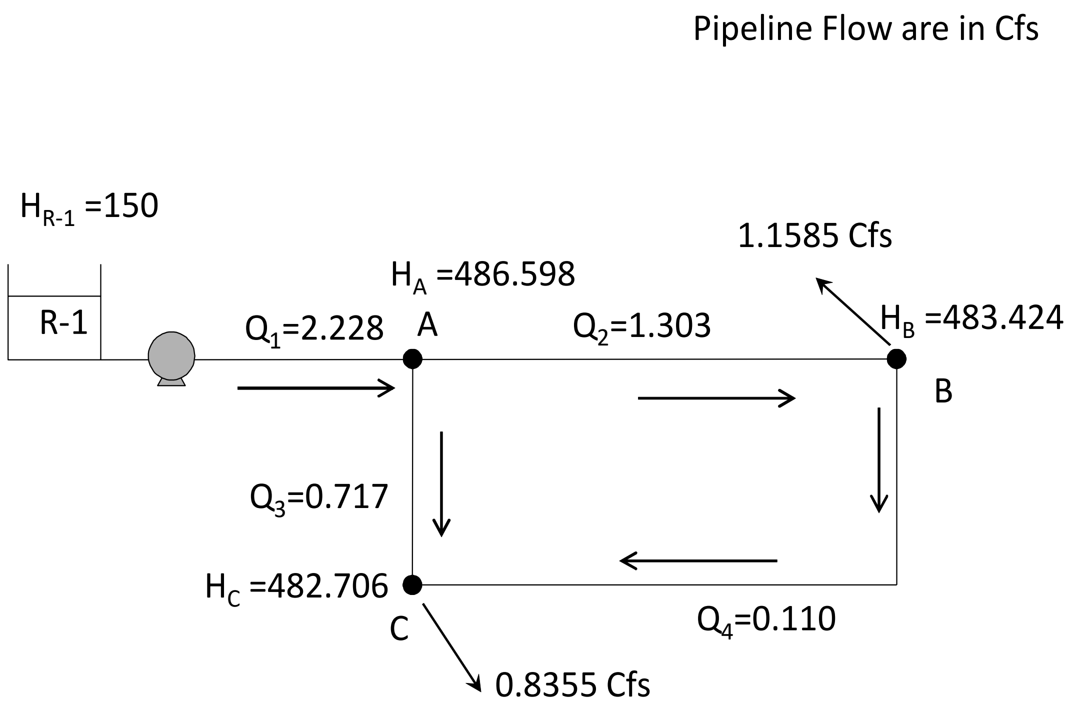

Figure 114 – Example #1A (Iteration #1)

Iteration #2:

The new set of heads for Iteration #2 are given in Table 267. Table 268 shows the more recent set of pipeline flows. These flows are based upon the most current nodal heads. Figure 114 shows the current set of flows and their directions.

Table 267 – Heads for Iteration #2

Node Label	Nodal Head (Ft)
A	486.598
B	483.424
C	482.706

Table 268 – Flows for Iteration #2

Pipe Label	Flow (Cfs)
P_1	2.228
P_2	1.303
P_3	0.717
P_4	0.110

Node A: We compute the function evaluation for Node A using the most current pipeline flows. The function evaluation for Node A is now:

$$F(H_o)_A = Q_1 - (Q_2 + Q_3) - D_A = 2.228 - (1.303 + 0.717) = 0.208$$

For a flow of Q_1 = 2.228 Cfs, then the gradient term for P_1 is:

$$\begin{aligned} G(Q = 2.228)_1 \\ &= (1.852)0.534(2.228)^{0.852} + (2)0.252(2.228) \\ &+ (1.760)2.392(2.228)^{0.760} = 10.817 \end{aligned}$$

For P_2 evaluated at its initial flow of Q_2 = 1.303 Cfs:

$$G(Q)_2 = (1.852)1.945Q_2^{0.852} = (1.852)1.945(1.303^{0.852} = 4.514$$

For P_3 evaluated at its initial flow of Q_3 = 0.717 Cfs:

$$G(Q)_3 = (1.852)7.209Q_3^{0.852} = (1.852)7.209(0.717)^{0.852} = 10.056$$

Now the head correction factor for Node A is:

$$\Delta H_A = -\frac{0.208}{\left(\frac{1}{10.817} + \frac{1}{4.514} + \frac{1}{10.056}\right)} = -\frac{0.208}{0.413} = -0.504$$

Now we update the head at Node A and recompute the flows using the updated heads.

$$H_{A,New} = H_{A,Old} - \Delta H_A = 486.598 - (-0.504) = 487.102$$

The new flows are:

$$F(Q)_1 = 0.534Q_1^{1.852} + 0.252Q_1^2 - (350 - 2.392Q_1^{1.760}) \\ = (150 - 487.102) = -337.102$$

$$F(Q = 2.181)_1 \\ = 0.252(2.181)^2 + 0.534(2.181)^{1.852} + 2.392(2.181)^{1.760} \\ - 12.898 = 0.0001 \approx 0$$

For pipe P_2:

$$Q_2 = \left(\frac{487.102 - 483.424}{1.945}\right)^{1/1.852} = 1.410\ Cfs$$

For pipe P_3:

$$Q_3 = \left(\frac{487.102 - 482.706}{7.209}\right)^{1/1.852} = 0.766\ Cfs$$

Figure 115 shows the nodal heads and the pipeline flows associated with the updated head at Node A.

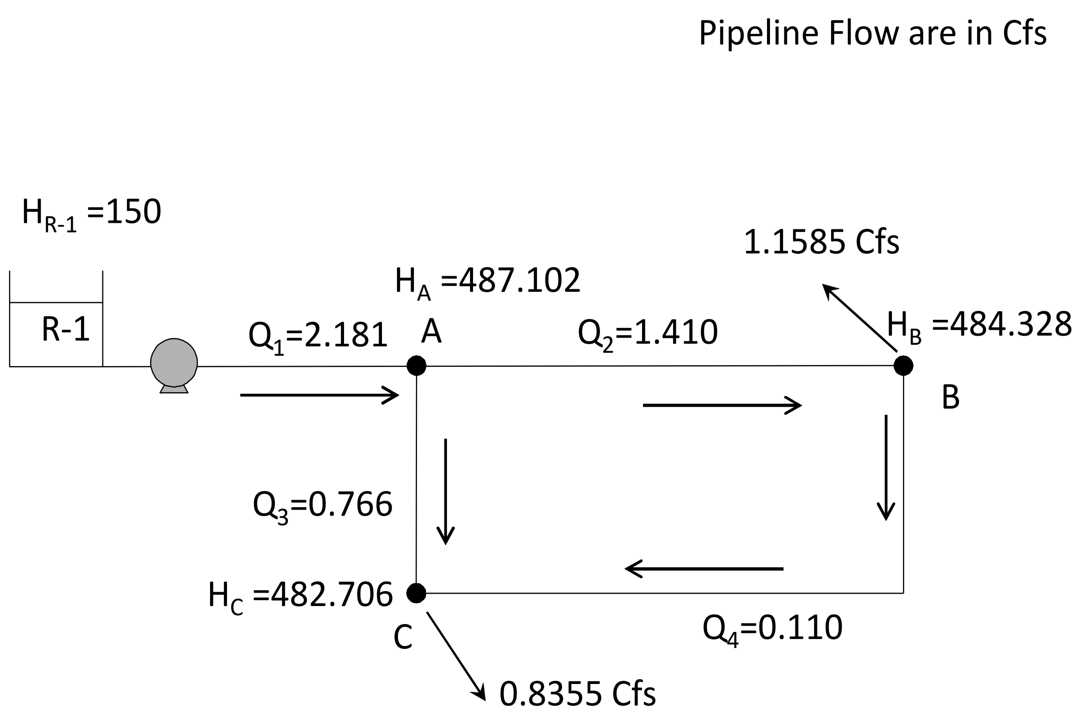

Figure 115 – Example #1A (Iteration #2)

Node B: Now we examine Node B and develop a head correction factor for this node. We start by computing continuity at this node to obtain $F(Q)_B$. Then we find the gradient terms for all pipes connected to this node. We end examination of this node by developing a node correction factor.

$$F(H_o)_B = Q_2 - Q_4 - D_B = 1.410 - 0.110 - 1.1585 = 0.142$$

For P_2 evaluated at its current flow of Q_2 = 1.410 Cfs:

$$G(Q)_2 = (1.852)1.945Q_2^{0.852} = (1.852)1.945(1.410)^{0.852} = 4.828$$

For P_4 evaluated at its initial flow of Q_4 = 0.110 Cfs:

$$G(Q)_4 = (1.852)42.920Q_4^{0.852} = (1.852)42.920(0.110)^{0.852} = 12.122$$

Now the head correction factor for Node B is:

$$\Delta H_B = -\frac{0.142}{\left(\frac{1}{4.828} + \frac{1}{12.122}\right)} = -\frac{0.142}{0.289} = -0.490$$

Now we update the head at Node B and recompute the flows.

$$H_{B,New} = H_{B,Old} - \Delta H_B = 483.424 - (-0.490) = 483.914$$

For pipe P_2:

$$Q_2 = \left(\frac{487.102 - 483.914}{1.945}\right)^{1/1.852} = 1.306\ Cfs$$

For pipe P_4:

$$Q_4 = \left(\frac{483.914 - 482.706}{42.920}\right)^{1/1.852} = 0.145\ Cfs$$

Figure 116 shows the updated heads and flows after the head correction factor for Node B is applied.

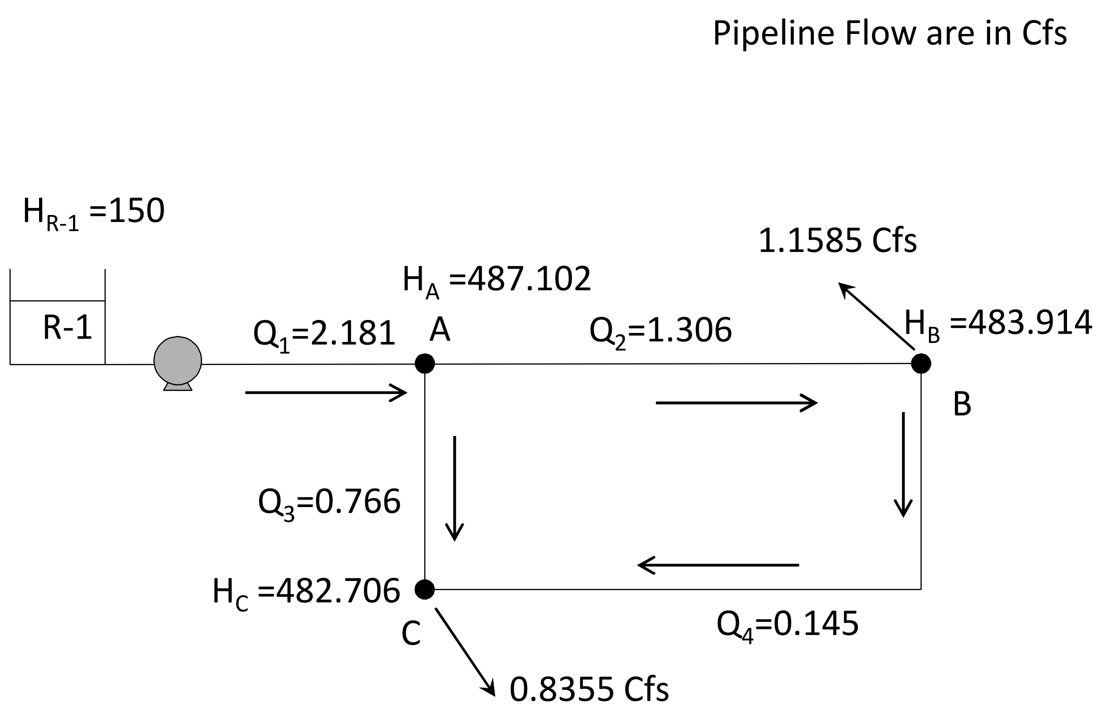

Figure 116 – Example #1A (Iteration #2)

Node C: Now we examine Node C and develop a head correction factor for this node. We start by computing continuity at this node to obtain $F(Q)_C$. Then we find the gradient terms for all pipes connected to this node. We end examination of this node by developing a node correction factor.

$$F(H_o)_C = Q_3 + Q_4 - D_C = 0.766 + 0.145 - 0.8355 = 0.076$$

For P_3 evaluated at its current flow of Q_3 = 0.766 Cfs:

$$G(Q)_3 = (1.852)7.209Q_3^{0.852} = (1.852)7.209(0.766)^{0.852} = 10.638$$

For P_4 evaluated at its current flow of Q_4 = 0.145 Cfs:

$$G(Q)_4 = (1.852)42.920Q_4^{0.852} = (1.852)42.920(0.145)^{0.852}$$
$$= 15.338$$

Now the head correction factor for Node C is:

$$\Delta H_C = -\frac{0.076}{\left(\frac{1}{10.638} + \frac{1}{15.338}\right)} = -\frac{0.076}{0.159} = -0.475$$

Now we update the head at Node C and recompute the flows using the most recent head values.

$$H_{C,New} = H_{C,Old} - \Delta H_C = 482.706 - (-0.475) = 483.181$$

For pipe P_3:

$$Q_3 = \left(\frac{487.102 - 483.181}{7.209}\right)^{1/1.852} = 0.720\ Cfs$$

For pipe P_4:

$$Q_4 = \left(\frac{483.914 - 483.181}{42.920}\right)^{1/1.852} = 0.111\ Cfs$$

Figure 117 shows the updated heads and flows after the head correction factor for Node C is applied.

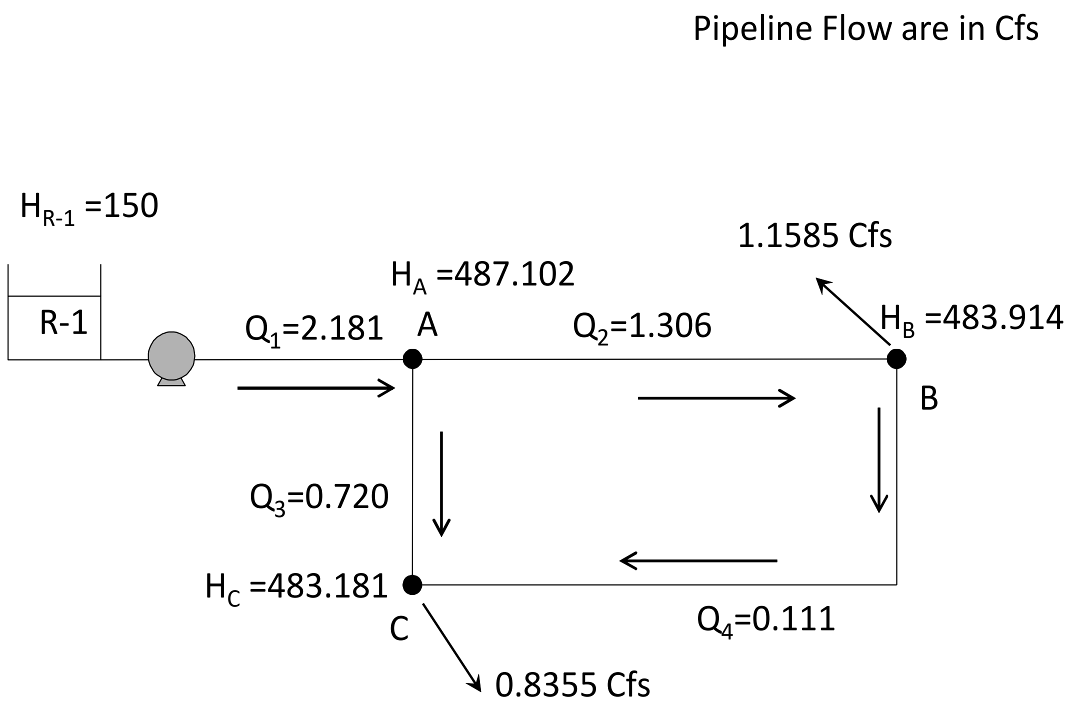

Figure 117 – Example #1A (Iteration #2)

Iteration #3:

The new set of heads for Iteration #3 are given in Table 269.

Table 270 shows the more recent set of pipeline flows. These flows are based upon the most current nodal heads. Figure 117 shows the current set of flows and their directions.

Table 269 – Heads for Iteration #3

Node Label	Nodal Head (Ft)
A	487.102
B	483.914
C	483.181

Table 270 – Flows for Iteration #3

Pipe Label	Flow (Cfs)
P_1	2.181
P_2	1.306
P_3	0.720
P_4	0.111

Node A: We compute the function evaluation for Node A using the most current pipeline flows. We also compute the gradient term using the most recent set of pipeline flows for the pipes connected to Node A. These values are summarized in the table below.

Note that H(j) represents the current head at Node A while H(i) represents the head at the connecting node. A positive flow rate indicates that flow enters the node while a negative flow indicates that discharge is leaving the node.

Table 271 – Summary of Pipeline Flows and Gradients for Node A

Pipe	H(i)	H(j)	Flow (Cfs)	G(Q)	1/G(Q)
P1	150	487.102	2.181	10.634	0.094
P2	483.914	487.102	-1.306	4.523	0.221
P3	483.181	487.102	-0.720	10.092	0.099
			$F(H)_A$=0.155		Σ = 0.414

$$\Delta H_A = -\frac{0.155}{\left(\frac{1}{10.634}+\frac{1}{4.523}+\frac{1}{10.092}\right)} = -\frac{0.155}{0.414} = -0.374$$

Now we update the head at Node A and recompute the flows using the updated heads.

$$H_{A,New} = H_{A,Old} - \Delta H_A = 487.102 - (-0.374) = 487.476$$

The new flows are:

$$F(Q)_1 = 0.534Q_1^{1.852} + 0.252Q_1^2 - (350 - 2.392Q_1^{1.760}) = (150 - 487.476) = -337.476$$

$$F(Q = 2.146)_1 = 0.252(2.146)^2 + 0.534(2.146)^{1.852} + 2.392(2.146)^{1.760} - 12.524 \approx 0$$

For pipe P_2:

$$Q_2 = \left(\frac{487.476 - 483.914}{1.945}\right)^{1/1.852} = 1.306\,Cfs$$

For pipe P_3:

$$Q_3 = \left(\frac{487.476 - 483.181}{7.209}\right)^{1/1.852} = 0.720\,Cfs$$

Figure 118 shows the nodal heads and the pipeline flows associated with the updated head at Node A. Note that there is a flow imbalance at Node A as shown below when we apply continuity to this node. This indicates that the flows in these pipes are not the correct flows.

$$F(H_o)_A = Q_1 - Q_2 - Q_3 = 2.146 - 1.306 - 0.720 = 0.120 \neq 0$$

Even though there is a flow imbalance at Node A, the magnitude of the flow imbalance, $F(H_o)_A = 0.120\,Cfs$, represents an improvement over the value of the function $F(H_o)_A = 0.155\,Cfs$ before the head correction was made.

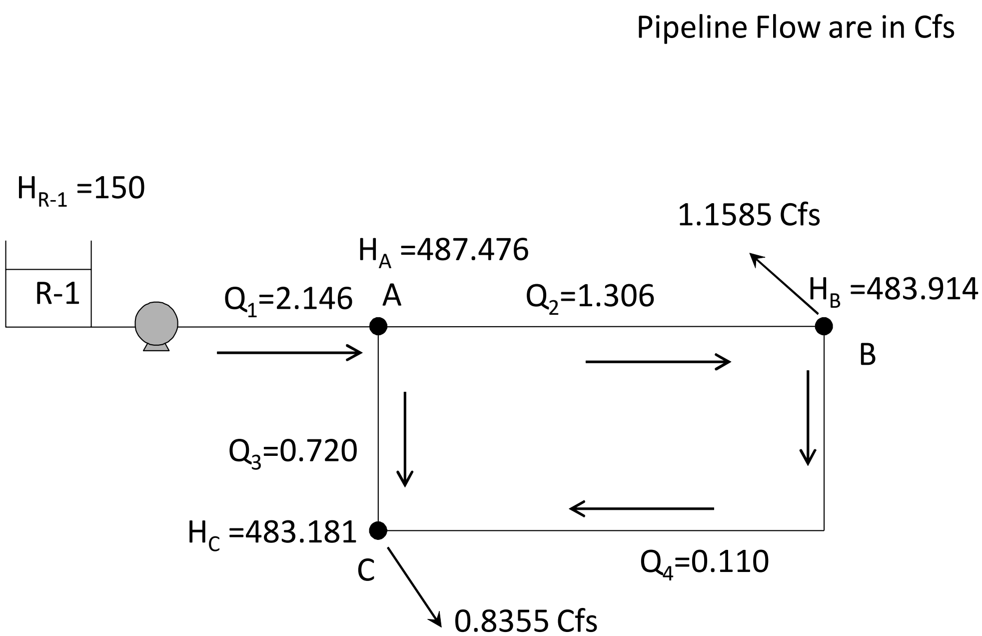

Figure 118 – Example #1A (Iteration #3)

Node B: We compute the function evaluation for Node B using the most current pipeline flows. We also compute the gradient term using the most recent set of pipeline flows for the pipes connected to Node B. These values are summarized in the table below.

Note that H(j) represents the current head at Node B while H(i) represents the head at the connecting node. A positive flow rate indicates that flow enters the node while a negative flow indicates that discharge is leaving the node.

Table 272 – Summary of Pipeline Flows and Gradients for Node B

Pipe	H(i)	H(j)	Flow (Cfs)	G(Q)	1/G(Q)
P2	487.476	483.914	1.386	4.758	0.210
P4	483.181	483.914	-0.111	12.216	0.082
			$F(H)_B$=0.117		Σ = 0.292

Now the head correction factor for Node B is:

$$\Delta H_B = -\frac{0.117}{\left(\frac{1}{4.758} + \frac{1}{12.216}\right)} = -\frac{0.117}{0.292} = -0.399$$

Now we update the head at Node B and recompute the flows.

$$H_{B,New} = H_{B,Old} - \Delta H_B = 483.914 - (-0.399) = 484.313$$

For pipe P_2:

$$Q_2 = \left(\frac{487.476 - 484.313}{1.945}\right)^{1/1.852} = 1.300\ Cfs$$

For pipe P_4:

$$Q_4 = \left(\frac{484.313 - 483.313}{42.920}\right)^{1/1.852} = 0.140\ Cfs$$

Figure 119 shows the updated heads and flows after the head correction factor for Node B is applied.

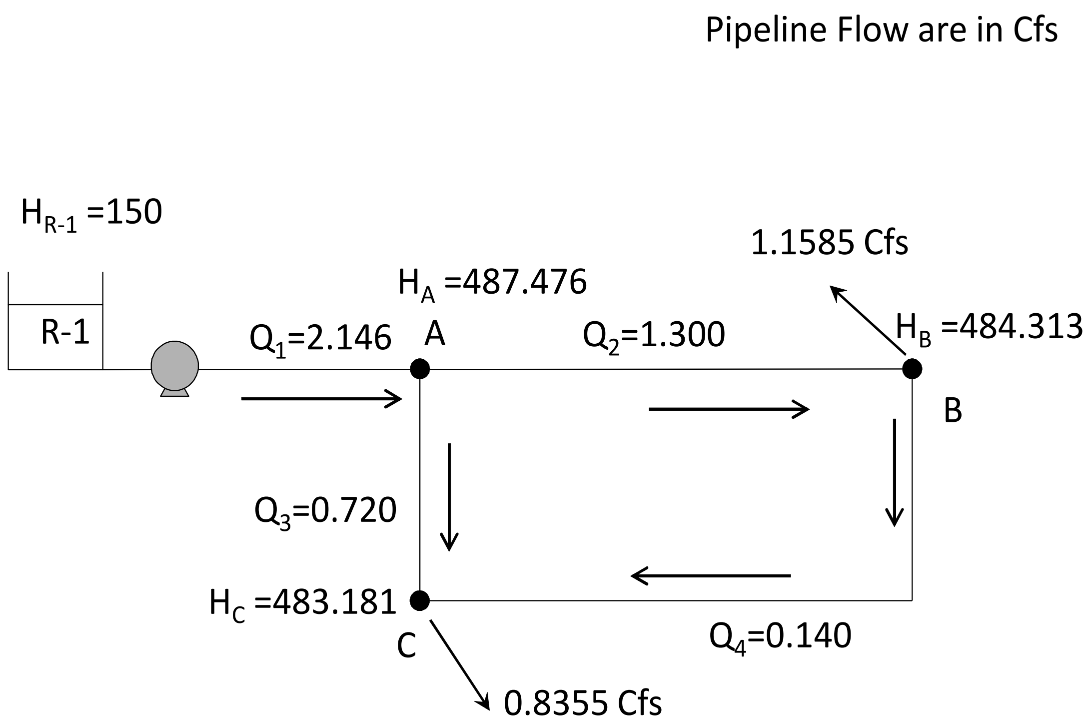

Figure 119 – Example #1A (Iteration #2)

Node C: We compute the function evaluation for Node C using the most current pipeline flows. We also compute the gradient term using the most recent set of pipeline flows for the pipes connected to Node C. These values are summarized in the table below.

Note that H(j) represents the current head at Node C while H(i) represents the head at the connecting node. A positive flow rate indicates that flow enters the node while a negative flow indicates that discharge is leaving the node.

Table 273 – Summary of Pipeline Flows and Gradients for Node C

Pipe	H(i)	H(j)	Flow (Cfs)	G(Q)	1/G(Q)
P3	487.476	483.181	0.756	10.520	0.095
P4	484.313	483.181	0.140	14.887	0.067
			$F(H)_B$=0.061		Σ = 0.162

Now the head correction factor for Node C is:

$$\Delta H_C = -\frac{0.061}{\left(\frac{1}{10.520} + \frac{1}{14.887}\right)} = -\frac{0.061}{0.162} = -0.373$$

Now we update the head at Node C and recompute the flows using the most recent head values.

$$H_{C,New} = H_{C,Old} - \Delta H_C = 483.181 - (-0.373) = 483.554$$

For pipe P_3:

$$Q_3 = \left(\frac{487.476 - 483.554}{7.209}\right)^{1/1.852} = 0.720\ Cfs$$

For pipe P_4:

$$Q_4 = \left(\frac{484.313 - 483.554}{42.920}\right)^{1/1.852} = 0.113\ Cfs$$

Figure 117 shows the updated heads and flows after the head correction factor for Node C is applied.

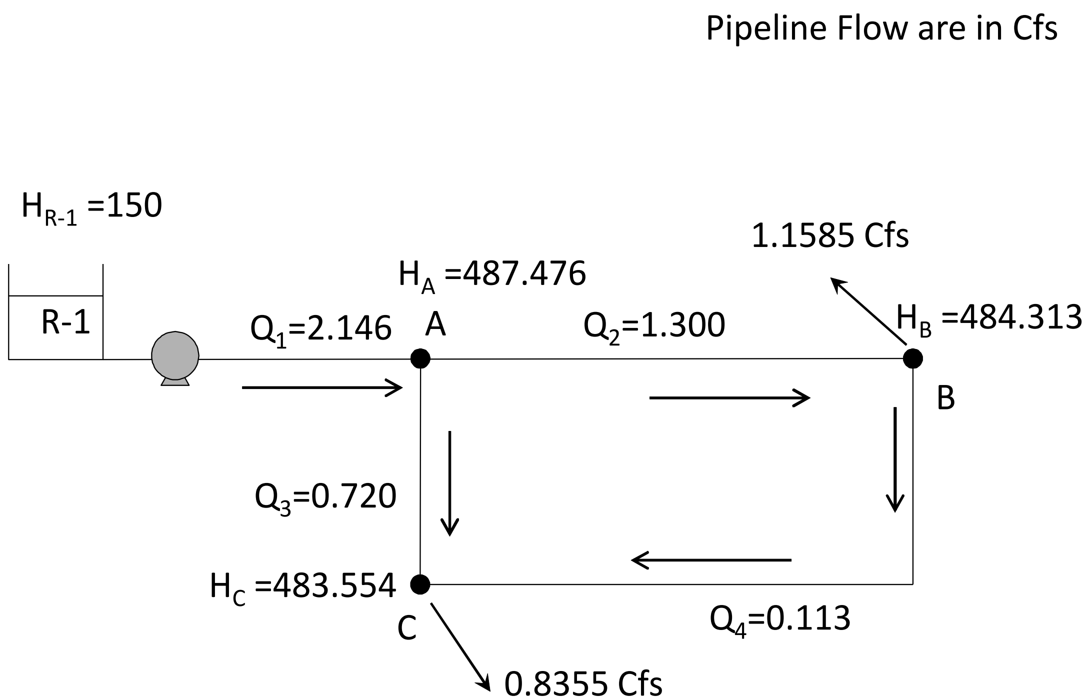

Figure 120 – Example #1A (Iteration #2)

The Single Node Adjustment method can, like the Single Path Adjustment method, exhibit slow convergence characteristics.

Table 274 – Single Node Adjustment Method, Example #1A

	Iteration #4		
	Node A	Node B	Node C
Old Head	487.476	484.313	483.554
Q1	2.146		
Q2	-1.300	1.366	
Q3	-0.720		0.749
Q4		-0.113	0.137
Demand	0.000	1.1585	0.8355
F(H)	0.126	0.095	0.051
∆H	-0.303	-0.321	-0.308
New Head	487.779	484.634	483.862
	Iteration #5		
Old Head	487.779	484.634	483.862
Q1	2.117		
Q2	-1.296	1.349	
Q3	-0.719		0.743
Q4		-0.114	0.134
Demand	0.000	1.1585	0.8355
F(H)	0.102	0.076	0.042
∆H	-0.224	-0.259	-0.250
New Head	488.023	484.893	484.112
	Iteration #6		
Old Head	488.023	484.893	484.112
Q1	2.093		
Q2	-1.293	1.335	
Q3	-0.719		0.738
Q4		-0.115	0.131
Demand	0.000	1.1585	0.8355
F(H)	0.081	0.061	0.034
∆H	-0.193	-0.208	-0.199
New Head	488.216	485.101	484.311
	Iteration #7		
Old Head	488.216	485.101	484.311
Q1	2.074		
Q2	-1.289	1.325	
Q3	-0.718		0.734

Q4		-0.116	0.128
Demand	0.000	1.1585	0.8355
F(H)	0.067	0.050	0.027
ΔH	-0.159	-0.170	-0.156
New Head	488.375	485.271	484.467
	Iteration #8		
Old Head	488.375	485.271	484.467
Q1	2.059		
Q2	-1.287	1.315	
Q3	-0.718		0.731
Q4		-0.117	0.127
Demand	0.000	1.1585	0.8355
F(H)	0.054	0.039	0.023
ΔH	-0.128	-0.133	-0.132
New Head	488.503	485.404	484.599
	Iteration #9		
Old Head	488.503	485.404	484.599
Q1	2.046		
Q2	-1.286	1.308	
Q3	-0.718		0.728
Q4		-0.117	0.125
Demand	0.000	1.1585	0.8355
F(H)	0.042	0.033	0.018
ΔH	-0.100	-0.109	-0.102
New Head	488.603	485.513	484.701
	Iteration #10		
Old Head	488.603	485.513	484.701
Q1	2.036		
Q2	-1.284	1.302	
Q3	-0.718		0.726
Q4		-0.117	0.124
Demand	0.000	1.1585	0.8355
F(H)	0.034	0.027	0.015
ΔH	-0.081	-0.088	-0.084
New Head	488.684	485.601	484.785

2.8.3. Single Node Adjustment Method – Example #2

We now apply the Single Node Adjustment method to the eight-pipe system shown in the figure below. Recall that an initial set of heads must be provided with the Single Node Adjustment method. The flow arrows in the figure below indicate the direction of flow recognizing that flow is from a high head to a low head. Pipeline and node data are presented in Table 275 and Table 276 respectively. Also presented is a table of head-discharge values for the pump in Table 277.

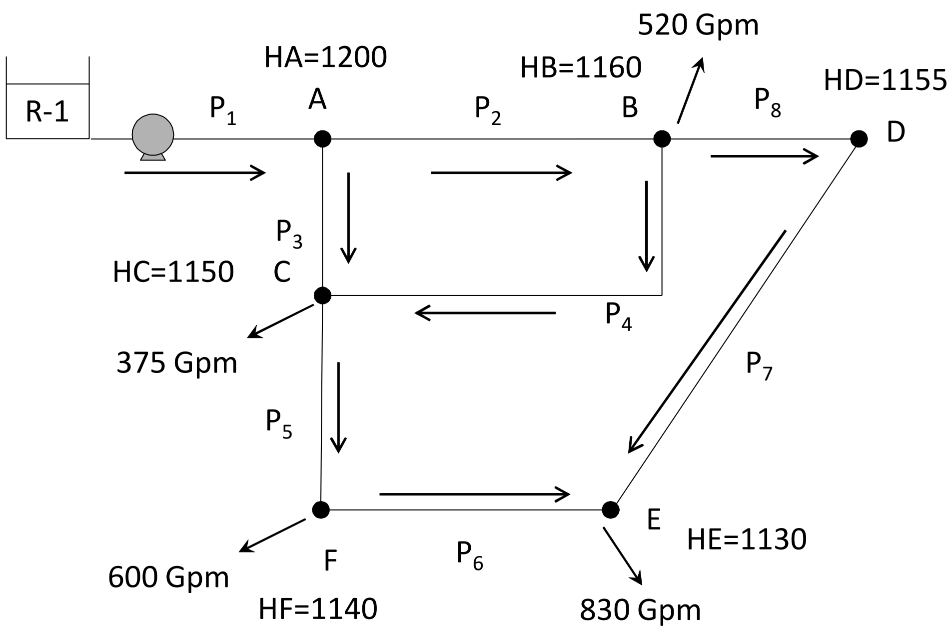

Figure 121 – Initial Heads and Flow Directions for Example #2

Table 275 – Pipeline Data for Example #2

Pipe Label	Length (Ft)	Diameter (In)	C-Factor	Minor Loss Coeff
P_1	800	12	120	10
P_2	1200	10	120	0
P_3	1500	8	120	0
P_4	2200	6	120	0
P_5	1500	6	110	0
P_6	2000	8	120	0
P_7	3100	6	110	0
P_8	1200	8	120	0

Table 276 – Node Data for Example #2

Node Label	Demand (Gpm)	Elevation	Initial Head (Ft)
R-1	N/A	990	990
A	0	1020	1200
B	520	1030	1160
C	375	1030	1150
D	0	1100	1155
E	830	1080	1130
F	600	1055	1140

Table 277 – Pump Data for Example #2

Flow (Gpm)	Pump Head (Ft)
0	350
1500	330
2800	290

Using the three pump head-discharge points provided above, a pump curve having the form shown below can be developed.

$$E(Q) = 350 - 2.392Q^{1.760}$$

We begin the process by computing pipeline flows for all pipes connected to Node A. The current set of nodal heads is used to compute these flows. Then a head correction factor at Node A is found using Eq. (86). Once a head correction factor is found, then an updated head at Node A is computed. The process is repeated for the other nodes in the system.

Iteration #0:

Node A: We compute the function evaluation for Node A using Eq. (76) and the most current pipeline flows. We also compute the gradient term using the most recent set of pipeline flows for the pipes connected to Node A. These values are summarized in the table below.

Note that H(j) represents the head at the node being examined - Node A in this case while H(k) represents the head at the other end of the pipe that is connected to Node j. For example, for pipe P_1 H(k) is the head at Reservoir R-1 while for pipe P_2 H(k) is the head at Node B. A positive flow rate indicates that flow enters Node A while a negative flow indicates that discharge is leaving the node.

Table 278 – Summary of Pipeline Flows and Gradients for Node A

Pipe	H(j)	H(k)	Flow (Cfs)	G(Q)	1/G(Q)
P1	1200	990	8.191	30.879	0.032
P2	1200	1160	-5.117	14.476	0.069
P3	1200	1150	-2.845	32.543	0.031
		Demand	0.000		
			$F(H)_A$=0.229		Σ = 0.132

We show how the flows are computed using the initial set of nodal heads. Because pipe P_1 contains a pump and because this pipe

has minor losses, then we must use a root-solving technique to find the pipeline flow. The head loss equation for pipe P_1 can be written as:

$$F(Q)_1 = 0.534Q_1^{1.852} + 0.252Q_1^2 - (350 - 2.392Q_1^{1.760})$$
$$= (1200 - 990) = 0$$

Using any of a wide variety of root-solving or optimization techniques we can determine that the discharge in pipe P_1 is Q_1 = 8.191 Cfs.

$$F(Q = 8.191)_1$$
$$= 0.534(8.191)^{1.852} + 0.252(8.191)^2$$
$$- (350 - 2.392(8.191)^{1.760}) + 210 \approx 0$$

We can use Eq. (75) to find the discharge in pipes P_2 and P_3 as illustrated below.

For pipe P_2:

$$Q_2 = sign(1160 - 1200)\left(\frac{1200 - 1160}{1.945}\right)^{1/1.852} = -5.117\,Cfs$$

For pipe P_3:

$$Q_3 = sign(1150 - 1200)\left(\frac{1200 - 1150}{7.209}\right)^{\frac{1}{1.852}} = -2.845\,Cfs$$

The continuity equation at Node A evaluated at the current flows, which result from the current heads, is:

$$F(H_o)_A = Q_1 + Q_2 + Q_3 = 8.191 + (-5.117) + (-2.845) = 0.229$$
$$\neq 0$$

The gradient term for the three pipes connected at Node A can be found from:

$$G(Q)_1 = (1.852)0.534Q_1^{0.852} + (2)0.252Q_1 - (-(1.760)2.392Q_1^{0.760})$$

$$G(Q)_1 = (1.852)0.534(8.191)^{0.852} + (2)0.252(8.191) \\ - (-(1.760)2.392(8.191)^{0.760}) = 30.879$$

$$G(Q)_2 = (1.852)1.945(5.117)^{0.852} = 14.476$$

$$G(Q)_3 = (1.852)7.209(2.845)^{0.852} = 32.543$$

Now a head correction factor for Node A can be computed:

$$\Delta H_A = -\frac{0.228}{\left(\frac{1}{30.879} + \frac{1}{14.476} + \frac{1}{32.543}\right)} = -\frac{0.228}{0.132} = -1.727$$

Now we update the head at Node A and proceed to examine Node B using the most up to date heads.

$$H_{A,New} = H_{A,Old} - \Delta H_A = 1200 - (-1.727) = 1201.727$$

Node B: We find pipeline flows and then compute the function evaluation for the current node (Node B) using Eq. (76). We also compute the gradient term using the most recent set of pipeline flows for the pipes connected to this node. These values are summarized in the table below.

Table 279 – Summary of Pipeline Flows and Gradients for Node B

Pipe	H(j)	H(k)	Flow (Cfs)	G(Q)	1/G(Q)
P2	1160	1201.727	5.235	14.761	0.068
P4	1160	1150	-0.455	40.668	0.025
P8	1160	1155	-0.926	10.002	0.100
		Demand	-1.159		
			$F(H)_B$=2.695		Σ = 0.193

Now a head correction factor for Node B can be computed:

$$\Delta H_B = -\frac{2.695}{\left(\frac{1}{14.761}+\frac{1}{40.668}+\frac{1}{10.002}\right)} = -\frac{2.695}{0.193} = -13.964$$

Now we update the head at Node B and proceed to examine Node C using the most up to date heads.

$$H_{B,New} = H_{B,Old} - \Delta H_B = 1160 - (-13.964) = 1173.964$$

Node C: We find pipeline flows and then compute the function evaluation for the current node (Node C) using Eq. (76). We also compute the gradient term using the most recent set of pipeline flows for the pipes connected to this node. These values are summarized in the table below.

Table 280 – Summary of Pipeline Flows and Gradients for Node C

Pipe	H(j)	H(k)	Flow (Cfs)	G(Q)	1/G(Q)
P3	1150	1201.727	2.898	33.055	0.03
P4	1150	1173.964	0.730	60.794	0.016
P5	1150	1140	-0.513	36.076	0.028
		Demand	-0.835		
			$F(H)_C$=2.280		Σ = 0.074

Now a head correction factor for Node C can be computed:

$$\Delta H_C = -\frac{2.280}{\left(\frac{1}{33.055} + \frac{1}{60.794} + \frac{1}{36.076}\right)} = -\frac{2.280}{0.074} = -30.811$$

Now we update the head at Node C and proceed to examine Node D using the most up to date heads.

$$H_{C,New} = H_{C,Old} - \Delta H_C = 1150 - (-30.811) = 1180.811$$

Node D: We find pipeline flows and then compute the function evaluation for the current node (Node D) using Eq. (76). We also compute the gradient term using the most recent set of pipeline flows for the pipes connected to this node. These values are summarized in the table below.

Table 281 – Summary of Pipeline Flows and Gradients for Node D

Pipe	H(j)	H(k)	Flow (Cfs)	G(Q)	1/G(Q)
P7	1155	1130	-0.569	81.382	0.012
P8	1155	1173.964	1.902	18.468	0.054
		Demand	0.000		
			$F(H)_D$=1.333		Σ = 0.066

Now a head correction factor for Node D can be computed:

$$\Delta H_D = -\frac{1.333}{\left(\frac{1}{81.382} + \frac{1}{18.468}\right)} = -\frac{1.333}{0.066} = -20.197$$

Now we update the head at Node D and proceed to examine Node E using the most up to date heads.

$$H_{D,New} = H_{D,Old} - \Delta H_D = 1155 - (-20.197) = 1175.197$$

Node E: We find pipeline flows and then compute the function evaluation for the current node (Node E) using Eq. (76). We also compute the gradient term using the most recent set of pipeline flows for the pipes connected to this node. These values are summarized in the table below.

Table 282 – Summary of Pipeline Flows and Gradients for Node E

Pipe	H(j)	H(k)	Flow (Cfs)	G(Q)	1/G(Q)
P6	1130	1140	1.022	18.128	0.055
P7	1130	1175.197	0.783	106.865	0.009
		Demand	-1.849		
			$F(H)_E$=-0.044		Σ = 0.064

Now a head correction factor for Node E can be computed:

$$\Delta H_E = -\frac{-0.044}{\left(\frac{1}{18.128} + \frac{1}{106.865}\right)} = -\frac{-0.044}{0.064} = 0.688$$

Now we update the head at Node E and proceed to examine Node F using the most up to date heads.

$$H_{E,New} = H_{E,Old} - \Delta H_E = 1130 - (0.688) = 1129.312$$

Node F: We find pipeline flows and then compute the function evaluation for the current node (Node F) using Eq. (76). We also compute the gradient term using the most recent set of pipeline flows for the pipes connected to this node. These values are summarized in the table below.

Table 283 – Summary of Pipeline Flows and Gradients for Node F

Pipe	H(j)	H(k)	Flow (Cfs)	G(Q)	1/G(Q)
P5	1140	1180.811	1.097	68.898	0.015
P6	1140	1129.312	-1.059	18.692	0.053
		Demand	-1.337		
			$F(H)_F$=-1.299		Σ = 0.068

Now a head correction factor for Node F can be computed:

$$\Delta H_F = -\frac{-1.299}{\left(\frac{1}{68.898}+\frac{1}{18.692}\right)} = -\frac{-1.299}{0.068} = 19.103$$

Now we update the head at Node F. We stop the calculations if all head correction factors are below some specified tolerance. Otherwise we continue with another iteration.

$$H_{F,New} = H_{F,Old} - \Delta H_F = 1140 - (19.103) = 1120.897$$

A summary of the head correction factors for each node are presented in the table below. Note that the magnitude of all head correction values are not small – in fact some are quite large. Thus the need for more iterations.

Table 284 – Head Correction Factors Iteration #0

Node	Old Head (Ft)	Head Correction Factor, ΔH (Ft)	New Head (Ft)
A	1200.000	-1.720	1201.720
B	1160.000	-13.964	1173.964
C	1150.000	-30.811	1180.811
D	1155.000	-20.197	1175.197
E	1130.000	0.688	1129.312
F	1140.000	19.103	1120.897

Iteration #1:

Node A: We compute the function evaluation for Node A using Eq. (76) and the most current pipeline flows. We also compute the gradient term using the most recent set of pipeline flows for the pipes connected to Node A. These values are summarized in the table below. Note that H(j) represents the head at the node being examined - Node A in this case while H(k) represents the head at the other end of the pipe that is connected to Node j. For example, for pipe P_1 H(k) is the head at Reservoir R-1 while for pipe P_2 H(k) is the head at Node B. A positive flow rate indicates that flow enters Node A while a negative flow indicates that discharge is leaving the node.

Table 285 – Summary of Pipeline Flows and Gradients for Node A

Pipe	H(j)	H(k)	Flow (Cfs)	G(Q)	1/G(Q)
P1	1201.72	990	8.134	30.705	P1
P2	1201.72	1173.964	-4.201	12.236	P2
P3	1201.72	1180.811	-1.777	21.790	P3
		Demand	0.000		
			$F(H)_A$=0.229		Σ = 0.132

We show how the flows are computed using the initial set of nodal heads. Because pipe P_1 contains a pump and because this pipe has minor losses, then we must use a root-solving technique to find the pipeline flow. The head loss equation for pipe P_1 can be written as:

$$F(Q)_1 = 0.534Q_1^{1.852} + 0.252Q_1^2 - (350 - 2.392Q_1^{1.760}) \\ = (1201.720 - 990) = 0$$

Using any of a wide variety of root-solving or optimization techniques we can determine that the discharge in pipe P_1 is Q_1 = 8.191 Cfs.

$$F(Q = 8.134)_1 \\ = 0.534(8.134)^{1.852} + 0.252(8.134)^2 \\ - (350 - 2.392(8.134)^{1.760}) + 211.720 \approx 0$$

We can use Eq. (75) to find the discharge in pipes P_2 and P_3 as illustrated below.

For pipe P_2:

$$Q_2 = sign(1173.964 - 1201.720)\left(\frac{1201.720 - 1173.964}{1.945}\right)^{1/1.852} \\ = -4.201\ Cfs$$

For pipe P_3:

$$Q_3 = sign(1180.811 - 1201.720)\left(\frac{1201.720 - 1180.811}{7.209}\right)^{\frac{1}{1.852}} \\ = -1.777\ Cfs$$

The continuity equation at Node A evaluated at the current flows, which result from the current heads, is:

$$F(H_o)_A = Q_1 + Q_2 + Q_3 = 8.134 + (-4.201) + (-1.777) = 2.156 \neq 0$$

The gradient term for the three pipes connected at Node A can be found from:

$$G(Q)_1 = (1.852)0.534Q_1^{0.852} + (2)0.252Q_1 - (-(1.760)2.392Q_1^{0.760})$$

$$G(Q)_1 = (1.852)0.534(8.134)^{0.852} + (2)0.252(8.134) - (-(1.760)2.392(8.134)^{0.760}) = 30.705$$

$$G(Q)_2 = (1.852)1.945(4.201)^{0.852} = 12.236$$

$$G(Q)_3 = (1.852)7.209(1.777)^{0.852} = 21.790$$

Now a head correction factor for Node A can be computed:

$$\Delta H_A = -\frac{2.156}{\left(\frac{1}{30.705} + \frac{1}{12.236} + \frac{1}{21.790}\right)} = -\frac{2.156}{0.161} = -13.391$$

Now we update the head at Node A and proceed to examine Node B using the most up to date heads.

$$H_{A,New} = H_{A,Old} - \Delta H_A = 1201.720 - (-13.391) = 1215.111$$

Node B: We find pipeline flows and then compute the function evaluation for the current node (Node B) using Eq. (76). We also compute the gradient term using the most recent set of pipeline flows for the pipes connected to this node. These values are summarized in the table below.

Table 286 – Summary of Pipeline Flows and Gradients for Node B

Pipe	H(j)	H(k)	Flow (Cfs)	G(Q)	1/G(Q)
P2	1173.964	1215.111	5.196	14.666	0.068
P4	1173.964	1180.811	0.371	34.164	0.029
P8	1173.964	1175.197	0.435	5.253	0.19
		Demand	-1.159		
			$F(H)_B$=4.843		Σ = 0.287

Now a head correction factor for Node B can be computed:

$$\Delta H_B = -\frac{4.843}{\left(\frac{1}{14.666}+\frac{1}{34.164}+\frac{1}{5.253}\right)} = -\frac{4.843}{0.287} = -16.875$$

Now we update the head at Node B and proceed to examine Node C using the most up to date heads.

$$H_{B,New} = H_{B,Old} - \Delta H_B = 1173.964 - (-16.875) = 1190.839$$

Node C: We find pipeline flows and then compute the function evaluation for the current node (Node C) using Eq. (76). We also compute the gradient term using the most recent set of pipeline flows for the pipes connected to this node. These values are summarized in the table below.

Table 287 – Summary of Pipeline Flows and Gradients for Node C

Pipe	H(j)	H(k)	Flow (Cfs)	G(Q)	1/G(Q)
P3	1180.811	1215.111	2.322	27.363	0.037
P4	1180.811	1190.839	0.456	40.720	0.025
P5	1180.811	1120.897	-1.350	82.209	0.012
		Demand	-0.835		
			$F(H)_C$=0.593		Σ = 0.074

Now a head correction factor for Node C can be computed:

$$\Delta H_C = -\frac{0.593}{\left(\frac{1}{27.363} + \frac{1}{40.720} + \frac{1}{82.209}\right)} = -\frac{0.593}{0.074} = -8.014$$

Now we update the head at Node C and proceed to examine Node D using the most up to date heads.

$$H_{C,New} = H_{C,Old} - \Delta H_C = 1180.811 - (-8.014) = 1188.825$$

Node D: We find pipeline flows and then compute the function evaluation for the current node (Node D) using Eq. (76). We also compute the gradient term using the most recent set of pipeline flows for the pipes connected to this node. These values are summarized in the table below.

Table 288 – Summary of Pipeline Flows and Gradients for Node D

Pipe	H(j)	H(k)	Flow (Cfs)	G(Q)	1/G(Q)
P7	1175.197	1129.312	-0.790	107.611	0.009
P8	1175.197	1190.839	1.714	16.902	0.059
		Demand	0.000		
			$F(H)_D$=0.924		Σ = 0.068

Now a head correction factor for Node D can be computed:

$$\Delta H_D = -\frac{0.924}{\left(\frac{1}{107.611} + \frac{1}{16.902}\right)} = -\frac{0.924}{0.068} = -13.588$$

Now we update the head at Node D and proceed to examine Node E using the most up to date heads.

$$H_{D,New} = H_{D,Old} - \Delta H_D = 1175.194 - (-13.588) = 1188.785$$

Node E: We find pipeline flows and then compute the function evaluation for the current node (Node E) using Eq. (76). We also compute the gradient term using the most recent set of pipeline flows for the pipes connected to this node. These values are summarized in the table below.

Table 289 – Summary of Pipeline Flows and Gradients for Node E

Pipe	H(j)	H(k)	Flow (Cfs)	G(Q)	1/G(Q)
P6	1129.312	1120.897	-0.931	16.745	0.06
P7	1129.312	1188.785	0.908	121.249	0.008
		Demand	-1.849		
			$F(H)_E$=-1.871		Σ = 0.068

Now a head correction factor for Node E can be computed:

$$\Delta H_E = -\frac{-1.871}{\left(\frac{1}{16.745} + \frac{1}{121.249}\right)} = -\frac{-1.871}{0.068} = 27.515$$

Now we update the head at Node E and proceed to examine Node F using the most up to date heads.

$$H_{E,New} = H_{E,Old} - \Delta H_E = 1129.312 - (27.515) = 1101.797$$

Node F: We find pipeline flows and then compute the function evaluation for the current node (Node F) using Eq. (76). We also compute the gradient term using the most recent set of pipeline flows for the pipes connected to this node. These values are summarized in the table below.

Table 290 – Summary of Pipeline Flows and Gradients for Node F

Pipe	H(j)	H(k)	Flow (Cfs)	G(Q)	1/G(Q)
P5	1120.897	1188.825	1.444	87.097	0.011
P6	1120.897	1101.797	-1.449	24.415	0.041
		Demand	-1.337		
			$F(H)_F$=-1.341		Σ = 0.052

Now a head correction factor for Node F can be computed:

$$\Delta H_F = -\frac{-1.341}{\left(\frac{1}{87.097}+\frac{1}{24.415}\right)} = -\frac{-1.341}{0.052} = 25.788$$

Now we update the head at Node F. We stop the calculations if all head correction factors are below some specified tolerance. Otherwise we continue with another iteration.

$$H_{F,New} = H_{F,Old} - \Delta H_F = 1120.897 - (25.788) = 1095.109$$

A summary of the head correction factors for each node are presented in the table below. Note that the magnitude of all head correction values are not small – in fact some are quite large. Thus the need for more iterations.

Table 291 – Head Correction Factors Iteration #1

Node	Old Head (Ft)	Head Correction Factor, ΔH (Ft)	New Head (Ft)
A	1201.720	-13.391	1215.111
B	1173.964	-16.875	1190.839
C	1180.811	-8.014	1188.825
D	1175.197	-13.588	1188.785
E	1129.312	27.515	1101.797
F	1120.897	25.788	1095.109

Iteration #2:

Node A: We compute the function evaluation for Node A using Eq. (76) and the most current pipeline flows. We also compute the gradient term using the most recent set of pipeline flows for the pipes connected to Node A. These values are summarized in the table below.

Note that H(j) represents the head at the node being examined - Node A in this case while H(k) represents the head at the other end of the pipe that is connected to Node j. For example, for pipe P_1 H(k) is the head at Reservoir R-1 while for pipe P_2 H(k) is the head at Node B. A positive flow rate indicates that flow enters Node A while a negative flow indicates that discharge is leaving the node.

Table 292 – Summary of Pipeline Flows and Gradients for Node A

Pipe	H(j)	H(k)	Flow (Cfs)	G(Q)	1/G(Q)
P1	1215.111	990.000	7.688	29.335	0.034
P2	1215.111	1190.839	-3.907	11.504	0.087
P3	1215.111	1188.825	-2.011	24.210	0.041
		Demand	0.000		
			$F(H)_A$=1.770		Σ = 0.162

A head correction factor for Node A can be computed:

$$\Delta H_A = -\frac{1.770}{\left(\frac{1}{29.335}+\frac{1}{11.504}+\frac{1}{24.210}\right)} = -\frac{1.770}{0.162} = -10.926$$

Now we update the head at Node A and proceed to examine Node B using the most up to date heads.

$$H_{A,New} = H_{A,Old} - \Delta H_A = 1215.111 - (-10.926) = 1226.037$$

Node B: We find pipeline flows and then compute the function evaluation for the current node (Node B) using Eq. (76). We also compute the gradient term using the most recent set of pipeline flows for the pipes connected to this node. These values are summarized in the table below.

Table 293 – Summary of Pipeline Flows and Gradients for Node B

Pipe	H(j)	H(k)	Flow (Cfs)	G(Q)	1/G(Q)
P2	1190.839	1226.037	4.776	13.649	0.073
P4	1190.839	1188.825	-0.192	19.458	0.051
P8	1190.839	1188.785	-0.573	6.642	0.151
		Demand	-1.159		
			$F(H)_B$=2.852		Σ = 0.275

Now a head correction factor for Node B can be computed:

$$\Delta H_B = -\frac{2.852}{\left(\frac{1}{13.649}+\frac{1}{19.458}+\frac{1}{6.642}\right)} = -\frac{2.852}{0.275} = -10.371$$

Now we update the head at Node B and proceed to examine Node C using the most up to date heads.

$$H_{B,New} = H_{B,Old} - \Delta H_B = 1190.839 - (-10.371) = 1201.210$$

Node C: We find pipeline flows and then compute the function evaluation for the current node (Node C) using Eq. (76). We also compute the gradient term using the most recent set of pipeline flows for the pipes connected to this node. These values are summarized in the table below.

Table 294 – Summary of Pipeline Flows and Gradients for Node C

Pipe	H(j)	H(k)	Flow (Cfs)	G(Q)	1/G(Q)
P3	1188.825	1226.037	2.426	28.408	0.035
P4	1188.825	1201.210	0.511	44.873	0.022
P5	1188.825	1095.109	-1.719	100.995	0.01
		Demand	-0.835		
			$F(H)_C$=0.384		Σ = 0.067

Now a head correction factor for Node C can be computed:

$$\Delta H_C = -\frac{0.384}{\left(\frac{1}{28.408} + \frac{1}{44.873} + \frac{1}{100.995}\right)} = -\frac{0.384}{0.067} = -5.731$$

Now we update the head at Node C and proceed to examine Node D using the most up to date heads.

$$H_{C,New} = H_{C,Old} - \Delta H_C = 1188.825 - (-5.731) = 1194.556$$

Node D: We find pipeline flows and then compute the function evaluation for the current node (Node D) using Eq. (76). We also compute the gradient term using the most recent set of pipeline flows for the pipes connected to this node. These values are summarized in the table below.

Table 295 – Summary of Pipeline Flows and Gradients for Node D

Pipe	H(j)	H(k)	Flow (Cfs)	G(Q)	1/G(Q)
P7	1188.785	1101.797	-1.115	144.428	0.007
P8	1188.785	1201.210	1.514	15.204	0.066
		Demand	0.000		
			$F(H)_D$=0.398		Σ = 0.073

Now a head correction factor for Node D can be computed:

$$\Delta H_D = -\frac{0.398}{\left(\frac{1}{144.428}+\frac{1}{15.204}\right)} = -\frac{0.398}{0.073} = -5.452$$

Now we update the head at Node D and proceed to examine Node E using the most up to date heads.

$$H_{D,New} = H_{D,Old} - \Delta H_D = 1188.785 - (-5.452) = 1194.237$$

Node E: We find pipeline flows and then compute the function evaluation for the current node (Node E) using Eq. (76). We also compute the gradient term using the most recent set of pipeline flows for the pipes connected to this node. These values are summarized in the table below.

Table 296 – Summary of Pipeline Flows and Gradients for Node E

Pipe	H(j)	H(k)	Flow (Cfs)	G(Q)	1/G(Q)
P6	1101.797	1095.109	-0.822	15.066	0.066
P7	1101.797	1194.237	1.153	148.524	0.007
		Demand	-1.849		
			$F(H)_E$=-1.518		Σ = 0.073

Now a head correction factor for Node E can be computed:

$$\Delta H_E = -\frac{-1.518}{\left(\frac{1}{15.066}+\frac{1}{148.524}\right)} = -\frac{-1.518}{0.073} = 20.795$$

Now we update the head at Node E and proceed to examine Node F using the most up to date heads.

$$H_{E,New} = H_{E,Old} - \Delta H_E = 1101.797 - (20.795) = 1081.002$$

Node F: We find pipeline flows and then compute the function evaluation for the current node (Node F) using Eq. (76). We also compute the gradient term using the most recent set of pipeline flows for the pipes connected to this node. These values are summarized in the table below.

Table 297 – Summary of Pipeline Flows and Gradients for Node F

Pipe	H(j)	H(k)	Flow (Cfs)	G(Q)	1/G(Q)
P5	1095.109	1194.556	1.774	103.791	0.01
P6	1095.109	1081.002	-1.230	21.238	0.047
		Demand	-1.337		
			$F(H)_F$=-0.793		Σ = 0.057

Now a head correction factor for Node F can be computed:

$$\Delta H_F = -\frac{-0.793}{\left(\frac{1}{103.791} + \frac{1}{21.238}\right)} = -\frac{-0.793}{0.057} = 13.912$$

Now we update the head at Node F. We stop the calculations if all head correction factors are below some specified tolerance. Otherwise we continue with another iteration.

$$H_{F,New} = H_{F,Old} - \Delta H_F = 1095.109 - (13.912) = 1081.197$$

A summary of the head correction factors for each node are presented in the table below. Note that the magnitude of all head correction values are not small – in fact some are quite large. Thus the need for more iterations.

Table 298 – Head Correction Factors Iteration #2

Node	Old Head (Ft)	Head Correction Factor, ∆H (Ft)	New Head (Ft)
A	1215.111	-10.926	1226.037
B	1190.839	-10.371	1201.210
C	1188.825	-5.731	1194.556
D	1188.785	-5.452	1194.237
E	1101.797	20.795	1081.002
F	1095.109	13.912	1081.194

Table 299 – Selected Results Single Node Adjustment Method Example #2

	Iteration #0					
	Node A	Node B	Node C	Node D	Node E	Node F
Old Head	1200	1160	1150	1155	1130	1140
Q1	8.190					
Q2	-5.117	5.235				
Q3	-2.845	-0.455	2.898			
Q4			0.730			
Q5			-0.513			1.097
Q6					1.022	-1.059
Q7				-0.569	0.783	
Q8		-0.926		1.902		
Demand	0	1.159	0.835	0	1.849	1.337
F(H)	0.277	2.695	2.280	1.333	-0.044	-1.299
∆H	-1.720	-13.964	-30.811	-20.194	0.688	19.103
New Head	1201.720	1173.964	1180.811	1175.197	1129.312	1120.897
	Iteration #1					
	Node A	Node B	Node C	Node D	Node E	Node F
Old Head	1201.72	1173.964	1180.811	1175.197	1129.312	1120.897
Q1	8.134					
Q2	-4.201	5.196				
Q3	-1.777		2.322			
Q4		0.371	0.456			
Q5			-1.350			1.444
Q6					-0.931	-1.449
Q7				-0.790	0.908	
Q8		0.435		1.714		
Demand	0	1.159	0.835	0	1.849	1.337
F(H)	-13.391	-16.875	-8.014	-13.588	27.515	25.788
∆H	1215.111	1190.839	1188.825	1188.785	1101.797	1095.109
New Head	1201.72	1173.964	1180.811	1175.197	1129.312	1120.897
	Iteration #2					
	Node A	Node B	Node C	Node D	Node E	Node F
Old Head	1215.111	1190.839	1188.825	1188.785	1101.797	1095.109
Q1	7.688					

Q2	-3.907	4.776				
Q3	-2.011		2.426			
Q4		-0.192	0.511			
Q5			-1.719			1.774
Q6					-0.822	-1.230
Q7				-1.115	1.153	
Q8		-0.573		1.514		
Demand	0	1.159	0.835	0	1.849	1.337
F(H)	-10.926	-10.371	-5.731	-5.452	20.795	13.912
ΔH	1226.037	1201.210	1194.556	1194.237	1081.002	1081.197
New Head	1215.111	1190.839	1188.825	1188.785	1101.797	1095.109
	Iteration #3					
	Node A	Node B	Node C	Node D	Node E	Node F
Old Head	1226.037	1201.21	1194.556	1194.237	1081.002	1081.197
Q1	7.308					
Q2	-3.955	4.531				
Q3	-2.216		2.474			
Q4		-0.365	0.597			
Q5			-1.904			1.951
Q6					0.122	-0.345
Q7				-1.286	1.334	
Q8		-1.108		1.782		
Demand	0	1.159	0.835	0	1.849	1.337
F(H)	-7.100	-9.834	-5.172	-7.873	1.146	-1.818
ΔH	1233.137	1211.044	1199.728	1202.110	1079.856	1083.015
New Head	1226.037	1201.21	1194.556	1194.237	1081.002	1081.197
	Iteration #4					
	Node A	Node B	Node C	Node D	Node E	Node F
Old Head	1233.137	1211.044	1199.728	1202.11	1079.856	1083.015
Q1	7.052					
Q2	-3.714	4.257				
Q3	-2.289		2.515			
Q4		-0.487	0.639			
Q5			-1.935			1.990
Q6					0.548	-0.471
Q7				-1.340	1.378	
Q8		-1.267		1.756		
Demand	0	1.159	0.835	0	1.849	1.337

F(H)	-6.358	-7.431	-6.295	-6.5	-0.78	-1.578
ΔH	1239.495	1218.475	1206.023	1208.610	1080.636	1084.593
New Head	1233.137	1211.044	1199.728	1202.11	1079.856	1083.015
	Iteration #5					
	Node A	Node B	Node C	Node D	Node E	Node F
Old Head	1239.495	1218.475	1206.023	1208.610	1080.636	1084.593
Q1	6.817					
Q2	-3.615	4.092				
Q3	-2.291		2.484			
Q4		-0.513	0.635			
Q5			-1.977			2.020
Q6					0.619	-0.431
Q7				-1.374	1.405	
Q8		-1.336		1.730		
Demand	0	1.159	0.835	0	1.849	1.337
F(H)	-5.417	-6.056	-4.968	-5.477	-1.934	-2.032
ΔH	1244.912	1224.531	1210.991	1214.087	1082.570	1086.625
New Head	1239.495	1218.475	1206.023	1208.610	1080.636	1084.593
	Iteration #6					
	Node A	Node B	Node C	Node D	Node E	Node F
Old Head	1244.912	1224.531	1210.991	1214.087	1082.57	1086.625
Q1	6.610					
Q2	-3.556	3.951				
Q3	-2.308		2.465			
Q4		-0.536	0.634			
Q5			-2.002			2.038
Q6					0.627	-0.410
Q7				-1.394	1.421	
Q8		-1.378		1.698		
Demand	0	1.159	0.835	0	1.849	1.337
F(H)	-4.394	-4.933	-4.159	-4.606	-2.211	-2.256
ΔH	1249.306	1229.464	1215.150	1218.693	1084.781	1088.881
New Head	1244.912	1224.531	1210.991	1214.087	1082.57	1086.625
	Iteration #7					
	Node A	Node B	Node C	Node D	Node E	Node F
Old Head	1249.306	1229.464	1215.15	1218.693	1084.781	1088.881

Q1	6.439					
Q2	-3.505	3.836				
Q3	-2.316		2.446			
Q4		-0.553	0.632			
Q5			-2.019			2.049
Q6					0.631	-0.395
Q7				-1.408	1.430	
Q8		-1.401		1.664		
Demand	0	1.159	0.835	0	1.849	1.337
F(H)	-3.614	-4.039	-3.556	-3.821	-2.382	-2.391
ΔH	1252.920	1233.503	1218.706	1222.514	1087.163	1091.272
New Head	1249.306	1229.464	1215.15	1218.693	1084.781	1088.881
	Iteration #10					
	Node A	Node B	Node C	Node D	Node E	Node F
Old Head	1258.348	1239.641	1224.277	1228.499	1092.062	1096.176
Q1	6.075					
Q2	-3.395	3.594				
Q3	-2.313		2.389			
Q4		-0.574	0.621			
Q5			-2.034			2.053
Q6					0.632	-0.387
Q7				-1.422	1.435	
Q8		-1.427		1.585		
Demand	0	1.159	0.835	0	1.849	1.337
F(H)	-2.085	-2.385	-2.188	-2.348	-2.461	-2.444
ΔH	1260.433	1242.026	1226.465	1230.847	1094.523	1098.620
New Head	1258.348	1239.641	1224.277	1228.499	1092.062	1096.176
	Iteration #20					
	Node A	Node B	Node C	Node D	Node E	Node F
Old Head	1270.325	1253.517	1238.076	1242.882	1113.381	1117.217
Q1	5.564					
Q2	-3.204	3.267				
Q3	-2.246		2.269			
Q4		-0.576	0.590			
Q5			-1.972			1.978
Q6					0.609	-0.455
Q7				-1.383	1.387	

Q8		-1.392		1.443		
Demand	0	1.159	0.835	0	1.849	1.337
F(H)	-0.616	-0.734	-0.788	-0.8	-1.598	-1.563
ΔH	1270.941	1254.251	1238.864	1243.682	1114.979	1118.780
New Head	1270.325	1253.517	1238.076	1242.882	1113.381	1117.217
	Iteration #30					
	Node A	Node B	Node C	Node D	Node E	Node F
Old Head	1274.778	1258.836	1243.875	1248.764	1125.48	1128.947
Q1	5.364					
Q2	-3.114	3.145				
Q3	-2.194		2.206			
Q4		-0.566	0.573			
Q5			-1.919			1.922
Q6					0.577	-0.501
Q7				-1.347	1.349	
Q8		-1.351		1.376		
Demand	0	1.159	0.835	0	1.849	1.337
F(H)	-0.296	-0.35	-0.373	-0.39	-0.792	-0.764
ΔH	1275.074	1259.186	1244.248	1249.154	1126.272	1129.711
New Head	1274.778	1258.836	1243.875	1248.764	1125.48	1128.947
	Iteration #50					
	Node A	Node B	Node C	Node D	Node E	Node F
Old Head	1277.892	1262.558	1247.908	1252.888	1133.928	1137.044
Q1	5.220					
Q2	-3.049	3.056				
Q3	-2.159		2.161			
Q4		-0.560	0.561			
Q5			-1.882			1.883
Q6					0.544	-0.528
Q7				-1.321	1.321	
Q8		-1.322		1.327		
Demand	0	1.159	0.835	0	1.849	1.337
F(H)	-0.062	-0.074	-0.087	-0.088	-0.17	-0.16
ΔH	1277.954	1262.632	1247.995	1252.976	1134.098	1137.204
New Head	1277.892	1262.558	1247.908	1252.888	1133.928	1137.044
	Iteration #80					

	Node A	Node B	Node C	Node D	Node E	Node F
Old Head	1278.669	1263.489	1248.914	1253.909	1135.965	1138.994
Q1	5.184					
Q2	-3.033	3.033				
Q3	-2.150		2.150			
Q4		-0.558	0.558			
Q5			-1.873			1.873
Q6					0.536	-0.534
Q7				-1.315	1.315	
Q8		-1.315		1.316		
Demand	0	1.159	0.835	0	1.849	1.337
F(H)	-0.005	-0.005	0	-0.013	-0.02	-0.019
ΔH	1278.674	1263.494	1248.914	1253.922	1135.985	1139.013
New Head	1278.669	1263.489	1248.914	1253.909	1135.965	1138.994

2.8.4. Single Node Adjustment Method – Example #3

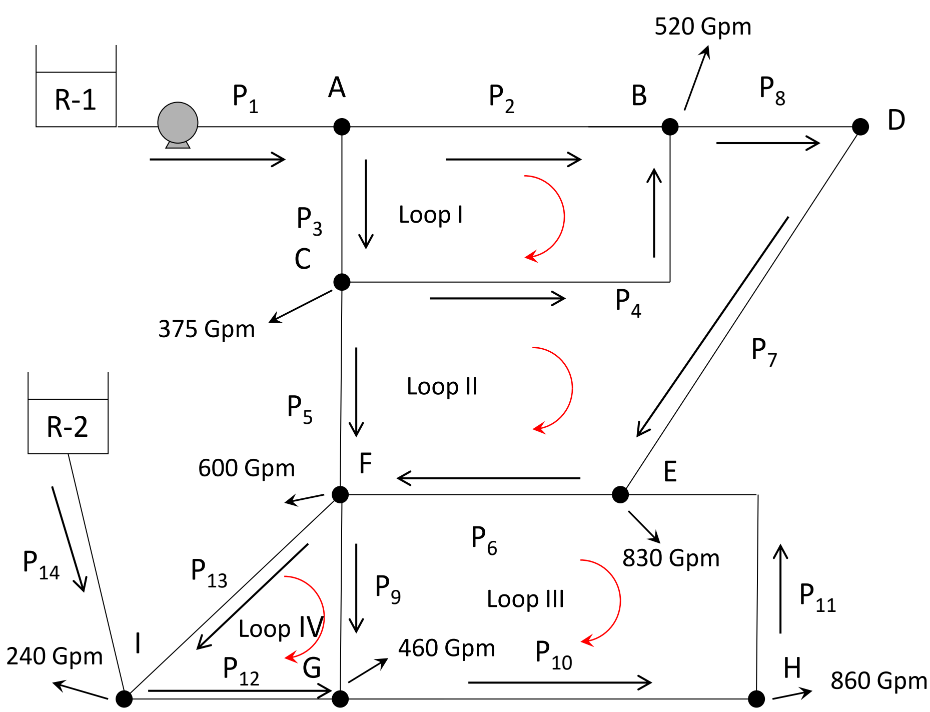

Figure 122 – Initial Flows for Four Loop, One Path System

The water use or nodal demands are given in Table 61. The pipe data for the 14-pipe system is shown in Table 62 below. Also provided are the initial flow and flow directions. The initial set of

flows satisfy continuity at each junction node as required by the Hardy-Cross method. Note that the initial flow direction travels from the Start Node to the End Node.

Table 300

Node	Demand (Gpm)	Elevation	Initial Head (Ft)
A	0	925	
B	520	950	
C	375	980	
D	0	880	
E	830	910	
F	600	860	
G	460	850	
H	860	950	
I	240	900	
R-1	N/A	860	
R-2	N/A	1020	

Table 301

Pipe Label	Start Node	End Node	Length (Ft)	Diameter (In)	C-Factor	Minor Loss Coeff
P_1	R-1	A	800	12	120	10
P_2	A	B	1500	10	120	15
P_3	A	C	1250	10	120	0
P_4	C	B	2560	6	120	0
P_5	C	F	890	8	120	0
P_6	E	F	1490	6	120	60
P_7	D	E	2400	8	120	0
P_8	B	D	760	8	120	0
P_9	F	G	1560	8	120	0
P_{10}	G	H	3200	6	120	0
P_{11}	H	E	1860	10	120	0
P_{12}	I	G	600	10	120	0
P_{13}	F	I	1180	8	120	0
P_{14}	R-2	I	450	10	120	20

Pump Curve

Pump Head (Ft)	Pump Discharge (Gpm)	Pump Discharge (Cfs)
250	0	0.000
235	1500	3.342
190	3000	6.684