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EVALUATION OF DIFFERENT OPTIMAL CONTROL PROBLEM FORMULATIONS FOR SOLVING THE MUSCLE REDUNDANCY PROBLEM

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INTRODUCTION

Since there are many more muscles than degrees of freedom in the human skeleton, muscle forces producing a given motion cannot be uniquely calculated using rigid body dynamics. Optimization methods resolve this redundancy by assuming that human movement is produced by optimizing a performance criterion. Two main approaches are used to solve the resulting optimization problem. The first approach, “static optimization”, neglects muscle-tendon dynamics, whereas the second approach, “dynamic optimization”, takes muscle-tendon dynamics into account. Though dynamic optimization approaches are more consistent with muscle physiology than static optimization approaches, solving the resulting non-convex dynamic optimization problem is challenging.

Two main approaches have been proposed for solving the dynamic optimization problem. The most commonly used approach is direct shooting, which performs forward integration of the dynamic equations to evaluate the cost function. A disadvantage of this approach is the high computational cost of repeated forward integrations, while an advantage is the ability to solve ‘difficult’ systems. Given the discontinuities in many muscle-tendon model descriptions, it is therefore not surprising that shooting methods make up the majority of the proposed methods.

More recently, direct collocation has been proposed as an alternate solution approach [1-3]. Direct collocation is based on a discretization of the dynamic equations. The discretized state equations then act as constraints when optimizing the performance criterion while the discretized states are optimization variables. Collocation methods are often computationally more efficient than are shooting methods. However, solving the underlying non-linear problem using gradient-based optimization methods requires at least first order continuity of the dynamic equations. Therefore, De Groote et al. presented a sequential approach that approximates the discontinuous non-linear dynamic equations by a linear discretization that is updated in every iteration. They applied this approach to calculate muscle excitations that could reproduce inverse dynamic joint torques from gait [3]. Ackerman et al. used direct collocation to solve a trajectory tracking problem during gait for a simple planar musculoskeletal model with continuous dynamics [1].

This study evaluates several possible optimal control problem formulations for solving the muscle redundancy problem with the goal of identifying the most efficient and robust formulation. One novel formulation involves the introduction of additional controls that equal the time derivative of the states, resulting in very simple dynamic equations. The nonlinear equations describing muscle dynamics are then imposed as algebraic constraints in their implicit form, simplifying their evaluation. By comparing different problem formulations for computing muscle controls that can reproduce inverse dynamic joint torques during gait, we demonstrate the efficiency and robustness of the proposed novel formulation.

METHODS

Musculoskeletal model

A simple musculoskeletal model with three degrees of freedom and nine muscles per leg was used in this study (gait10dof18musc) [4].

Activation dynamics was modelled based on [5] using a tanh function to smoothly transition between activation and deactivation:

$$f = 0.5\tanh(b(e-a))$$

$$\frac{da}{dt} = \left[\frac{1}{\tau_a(0.5+1.5a)}(f+0.5) + \frac{0.5+1.5a}{\tau_d}(-f+0.5)\right](e-a)$$

where e is excitation, a is activation, $\tau_a = 0.01s$ is activation time constant , $\tau_d = 0.04s$ is deactivation time constant, and $b = 0.1$ is a parameter determining transition smoothness.

Contraction dynamics was described using the model of Hill [6].

$$l_{MT} = l_T + l_M \cos\alpha$$
$$l_M \sin\alpha = l_M^0 \sin\alpha_0$$
$$F_T = F_M^0 f_t(l_T)$$
$$F_M = F_M^0\left[a f_{act}(l_M) f_v(v_M) + f_{pas}(l_M)\right]$$
$$F_T = F_M \cos\alpha$$

where l_{MT} is muscle-tendon length, l_T is tendon length, l_M is muscle fiber length, v_M is muscle fiber velocity, l_M^0 is optimal fiber length, α is pennation angle, α_0 is optimal pennation angle, F_T is tendon force, F_M is muscle force, F_M^0 is peak isometric muscle force, and a is activation. f_t, f_{act}, f_{pas}, and f_v are the tendon force-length, active muscle force-length, passive muscle force-length, and muscle force-velocity characteristics, respectively. All characteristics are second order continuous.

Experimental data

Experimental data for a gait movement were taken from the example files Gait10dof18musc installed with OpenSim 3.2 [4]. Inverse dynamic joint torques, muscle-tendon lengths, and muscle moment arms were calculated using OpenSim 3.2.

Problem formulations and solution method

The optimization problem was to minimize the integral of the sum of excitations squared over all muscles subject to activation and contraction dynamics and the additional path constraint that the muscle forces should produce the inverse dynamic joint torques. Controls e were bound between 0 and 1. States a were bound between 0.01 and 1. Contraction dynamics was imposed using four different formulations.

1. Using l_M as a state:

$$\frac{dl_M}{dt} = f_1(a, l_M).$$

2. Using F_T as a state:

$$\frac{dF_T}{dt} = f_2(a, F_T).$$

3. Using l_M as a state and introducing u_v as a new control simplifying the dynamic equations:

$$\frac{dl_M}{dt} = u_v.$$

The Hill model was then imposed as a path constraint:

$$f_3(a, l_M, u_v) = 0.$$

4. Using F_T as a state and introducing u_F as a new control:

$$\frac{dF_T}{dt} = u_F.$$

The Hill model was then imposed as a path constraint:

$$f_4(a, F_T, u_F) = 0.$$

All functions f_i were derived from the Hill model stated above. All formulations were mathematically equivalent and thus have the same globally optimal muscle excitations.

The dynamic optimization problems were solved via direct collocation using GPOPS-II with 200 mesh elements. We compared convergence, optimal cost function values, mesh accuracy, CPU times, and robustness against the initial guess (IG). Mesh accuracy was defined as the root mean square (RMS) difference between the excitations calculated using 200 and 400 mesh elements respectively. Robustness against the IG was defined as the RMS difference between excitations calculated using two different IG.

RESULTS AND DISCUSSION

Table 1 Comparison of different problem formulations

Formulation	1	2	3	4
Convergence	NO	**YES**	**YES**	**YES**
Optimal value	-	0.2990	**0.2623**	0.2624
Accuracy	-	2.6e-3	**2.3e-3**	3.6e-3
CPU time [s]	-	**63**	84	76
Robustness IG	-	2.0e-5	**3.2e-8**	2.5e-4

Problem formulation influenced convergence, optimal value, accuracy, and CPU time (Table 1). Using fiber length as a state and introducing extra controls (Formulation 3) resulted in the lowest cost function value, the highest mesh accuracy, and the highest robustness against the initial guess. This formulation, in contrast to the others, did not require inversion of the force-velocity or tendon force-length curves or division by a permitting a lower bound of 0 on activations. In addition, normalized fiber velocity was easy to bound between -1 and 1. Our next step is to investigate this novel approach further in a complex musculoskeletal model.

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