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ALCOHOLISM: A MATHEMATICAL MODEL WITH MEDIA AWARENESS CAMPAIGNS

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KEYWORDS: Alcoholism, Media Campaigns, SIR dynamics

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Abstract. In this paper, we study how media awareness campaigns influence the spread and persistence of drinking behavior in a community. Here, we present a compartmental population model with an additional differential equation to describe the dynamics of media awareness campaigns in combating problem drinking ([10], [12], [21]). Our model indicates a basic reproductive number, R_0 , where there exists an asymptotically stable drinking-free equilibrium if $R_0 < 1$, and a unique endemic state, which appears to be stable when $R_0 > 1$. We found that the following two components affect the basic reproductive number: the strength of peer influence of problem drinkers on susceptibles and the average overall time spent in the problem drinking environment. Furthermore, we conclude that the existence of media awareness programs and effective treatment options does not eliminate a drinking culture in the community, it only temporarily alleviate the issue. To support our findings, we present analytical and numerical approaches.

1. INTRODUCTION

The use of mathematical modeling in the analysis of spread of infectious diseases within communities has a long history, and has been described by [1], [2], and many others. Although SIR models have some naive assumptions, they have played an important role in epidemiology and have built a platform for more sophisticated models. Recently, the wide range of applications of SIR models have been extended to understand the dynamics of smoking, obesity, drug-use, and alcohol abuse problems. The spread of such human social behaviors [3], although different from the spread of infectious diseases, is still highly affected by human contact [4]. Recent applications of SIR models for alcohol abuse and/or problem drinking behaviors have concentrated on the effects of peer influence [5], [6], [7], [8], the impact of imitation mechanism [9], the role of residence time in risky environments [10], and the effectiveness of prevention programs [11], [12].

According to the National Institute of Alcohol Abuse and Alcoholism (NIAAA), 87.6% of people that are 18 years or older have been exposed to alcohol and an estimated 17 million of Americans have an alcohol use disorder [13]. Alcohol related causes are the third leading preventable cause of death in the United States [13]. Utilizing mathematical models to determine a basic reproductive number (R_0), defined as the average number of new problem drinkers generated by a single problem drinker [14], may help public officials develop policies to eliminate and/or reduce the number of problem drinkers and alcohol-abuse related incidences in different communities.

It is difficult to measure how media awareness programs change the response to problem drinking in society. In infectious disease modeling, researchers [15] typically concentrate on the role of media awareness campaigns by encouraging patients to seek doctors, creating awareness among susceptible individuals by advising not to interact with sick individuals, or by calling for hospitalization of sick individuals. Such a conceptual understanding of media awareness campaigns is exemplified by [11] and [15]. Some researchers such as Holder et al. [11] suggest that media awareness campaigns by themselves have very little effect on the prevention of problem drinking, and that therefore, such campaigns should be accompanied by other prevention policies.

As summarized in [16], peer influence in college drinking has been studied from two different perspectives: direct peer influence, being offered a drink at a party, or indirect peer influence, having misconceptions about the social expectations of drinking behavior on campus. In addictive behavior modeling, the effects of peer influence, positive or negative, have been central in studying the spread and persistence of disease [3], [7], [9], [10], and [17]. Understanding effects of peer influence on drinking behavior will help monitor the drinking culture as well as develop control policies.

It is well-known that addiction treatment options may result in relapses. Becker et al. (2008) define relapse as an initiation of problem drinking following a prolonged period of abstinence from alcohol [18]. In [19], the complexity of relapse is attributed to the following two factors: sobriety and the lasting effects of prevention treatment. Moreover, [19] suggest that recovered individuals may either follow an abstinence pattern or revert to heavy drinking shortly after treatment. In [6] and [20], relapse is modeled as a non-linear function of peer influence, based on the frequency of interactions among drinkers and recovered individuals.

In our model, we integrate many of the above dynamics that affect the number of problem drinkers in the community: 1) the existence of media awareness programs, 2) the peer influence of problem drinkers on susceptibles 3) the relapse possibility among problem drinkers. We introduce an SIR model framework with the following population groups, susceptibles $S(t)$, problem drinkers $A(t)$, and recovered individuals $R(t)$. Furthermore, we consider a separate differential equation for the density of media awareness programs, $M(t)$.

The paper is organized as follows. In Section 2, we describe our model and discuss drinking behavior dynamics in detail. We analyze the model, analytically and computationally, and present a threshold value associated with the risk of problem drinking outbreak in Section 3. At the end of this section, we compare our model findings with some particular SIR-type drinking models. To conclude, we will discuss our main results and future directions implicated by our research findings.

2. MODEL

Our model modifies two previously studied mathematical frameworks, the SIR dynamics provided in [6] for the spread of alcoholism throughout a community and the dynamics of media awareness programs given in [12] and [21]. Our research combines these aspects into an SIR-type alcoholism model with media awareness campaigns. In our drinking model, we divide the population into three separate groups: susceptible individuals who are light or moderate drinkers, denoted by $S(t)$, problem drinkers who drink heavily, denoted by $A(t)$, and individuals who are in treatment or recently recovered from alcoholism, denoted by $R(t)$. Here, the words alcoholism, problem drinking, and heavy drinking will be used interchangeably as we do not distinguish

between these terms. In order to capture the effects of media in a simple equation, we use media density dynamics similar to those found in [12], [21].

In our model, individuals enter the community as a member of the susceptible population. The departure rate is homogeneous among all groups. The factor of peer influence is modeled as a way to generate new problem drinkers, either by encouraging susceptibles to drink heavily or by causing relapses. Hence, our model only captures the negative direct effects of peer influence in the community. Drinkers may socialize with either susceptibles or recovered people in the population. Recruitment of new problem drinkers occurs by means of social interactions between susceptibles and problem drinkers. Additionally, social interactions between problem drinkers and recovered individuals may result in relapses. Such dynamics will return some proportion of recovered individuals to the alcoholic population. We assume non-linear peer influence proportional to the multiplication of interacting group sizes. Similarly, effects of peer influence in the spread of addictive social behaviors are modeled by a non-linear function of social interactions in [3], [7], [9], and [17].

To further distinguish our model, we propose a novel approach to studying effects of media awareness on problem drinkers. Here, we use media awareness as a motivational tool for problem drinkers to seek treatment options and to move to the recovered group. We assume that the susceptible group continues to socially interact with problem-drinkers even after learning about alcohol-related incidences. Historically, the role of media awareness campaigns is modeled by creating a separate group of aware individuals who avoid social contact with the infected class (based on evidence that education campaigns and media awareness programs can impact individual attitudes and behaviors) ([12] and [21]). In these models, susceptibles exposed to media awareness content only come into contact with problem drinkers once they lose awareness. However, cutting all contact with problem drinkers due to media awareness campaigns does not seem feasible to us. Research suggests that the process of gaining and losing awareness may be a relatively short term phenomenon, but also that more conclusive studies are needed to understand the long term effects of different media awareness campaign types [22]. Therefore, we have integrated media awareness to directly impact drinkers and maintained the social stimulus between the recovered and problem-drinking groups.

In order to study the population of each group and the density of media awareness campaigns our model is represented by a system of nonlinear differential equations,

$$\begin{aligned}
 \frac{dS}{dt} &= \mu - \beta SA - \mu S \\
 \frac{dA}{dt} &= -\lambda AM + \alpha AR + \beta SA - \mu A \\
 \frac{dR}{dt} &= \lambda AM - \alpha AR - \mu R \\
 \frac{dM}{dt} &= \gamma_1 A - \gamma_2 M
 \end{aligned} \tag{2.1}$$

with initial conditions $S(0) \geq 0$, $A(0) \geq 0$, $R(0) \geq 0$, and $M(0) \geq 0$. We look for the solutions in the feasible region $A = \{(S(t), A(t), R(t), M(t)) \in \mathbb{R}^4 : S(t) + A(t) + R(t) \leq 1, M(t) \leq 1\}$ with the assumption that $\gamma_1 \approx \gamma_2$.

All model parameters are assumed to be positive. The departure rate, μ , is equal to the birth rate, and is same in all population groups. The transition parameter, β , measures the strength of peer influence in moving susceptibles to problem drinking group, $A(t)$. Our approach to modeling drinking dynamics with SIR equations is similar to Sanchez et al. [6], with a slight modification of the relapse dynamics. In our model, we limit the relapse transition associated

with non-linearity to be caused by social interactions between recovered individuals and problem drinkers [20]. The relapse parameter, α , represents how much of the social interaction between the two groups will result in moving recovered individuals, $R(t)$, back to the group of problem drinkers, $A(t)$. As peer influence, β , and the relapse possibility, α , cause an increase in the problem drinking behavior, media awareness campaigns, λ , cause an outflow from the problem-drinking stage. Problem drinkers may seek treatment after some exposure to media awareness programs, and decide to move to the recovered group, $R(t)$, with rate λ . The rate of change of density of media awareness programs is affected by the reach of the campaign, γ_1 , and increases as the problem drinkers in the overall population increase. Conversely, the forgetting factor, γ_2 , causes a decay in the density of awareness programs, and decreases as the amount of problem drinkers diminishes.

Our unique approach concentrates on λ , a measure of the efficiency of media awareness programs in convincing individuals to recognize their problem drinking symptoms and to seek treatment options. Distinct to traditional models, we allow social interactions among recovered individuals and problem-drinkers. Similar, yet distinctly different differential equation approaches to modeling media awareness programs have previously been studied by [12] and [21], where an isolated class of aware individuals were created. To more realistically capture the dynamics of drinking in a community, we preserve the possibility of socialization between recovered individuals and drinkers. By integrating media awareness programs into an SIR drinking model, we hope to better understand the true dynamics of problem-drinking in communities, and thus inspire the development of more effective prevention and control policies.

3. MODEL ANALYSIS

Given that the initial values $S(0)$, $A(0)$, $R(0)$ are positive and that $M(0) > 0$, the total population, $N(t)$, with $dN/dt = \mu - \mu N$, $\lim_{t \rightarrow +\infty} N(t) = 1$. With the assumption $\gamma_1 \approx \gamma_2$, we can establish an upper bound for the media awareness, $M(t)$, $M(t) \leq 1 - M(0)e^{\gamma_1 t}$. The stability theory of differential equations was used to study 2.1. By using a linearization approach, we investigated the behavior of the above system around the disease-free equilibrium point $(1, 0, 0, 0)$. At this equilibrium point, the population is made up of all susceptible individuals. The non-zero elements of the Jacobian matrix, $a_{11} = -\mu$, $a_{12} = -\beta$, $a_{22} = \beta - \mu$, $a_{33} = -\mu$, $a_{44} = -\gamma_2$, and $a_{42} = \gamma_1$, led to a set of eigenvalues $-\gamma_2$, $-\mu$, $-\mu$, and $\beta - \mu$. The drinking-free equilibrium point is thus asymptotically stable if $\beta < \mu$. Therefore, we propose a basic reproductive number,

$$R_0 = \frac{\beta}{\mu} \quad (3.1)$$

where β is the transmission rate due to peer influence of problem drinkers on susceptible individuals and $1/\mu$ is the overall total average time spent in the problem state. When $R_0 < 1$, the disease-free equilibrium is stable, and it is not possible to establish a drinking culture in the population. A decrease in R_0 can be caused by two factors:

1. lower transmission rate, β
2. less time spent in the problem drinking environment

Our model presents an alcohol-present equilibrium at (S^*, A^*, R^*, M^*) , given by

$$\begin{aligned} S^* &= \frac{\mu}{\beta A^* + \mu} \\ R^* &= \frac{\lambda(A^*)^2}{\alpha A^* + \mu} \\ M^* &= A^* \end{aligned} \quad (3.2)$$

when A^* is a solution of the quadratic equation

$$a(A^*)^2 + b(A^*) + c = 0$$

, with coefficients

$$\begin{aligned} a &= 1 \\ b &= \frac{\alpha}{\beta + \lambda} \left[\frac{1}{R_0} \left(1 + \frac{\beta + \lambda}{\alpha} \right) - 1 \right] \\ c &= \frac{1}{\beta(\alpha + \lambda)} (1 - R_0) \end{aligned} \quad (3.3)$$

When $R_0 > 1$, we find that there is a unique alcohol-present equilibrium point, $A^* = \frac{-b + \sqrt{b^2 - 4c}}{2}$ in $(0, 1)$. $R_0 > 1$ implies that $c < 0$, therefore, $b^2 - 4c \geq 0$, and all roots are real numbers. In addition, $\sqrt{b^2 - 4c} > |b|$. We consider $1 < R_0 < 1 + \frac{\beta + \lambda}{\alpha}$, which implies that $-b < 0$. If $R_0 > 1 + \frac{\beta + \lambda}{\alpha} > 1$ then $-b > 0$. Either case yields that $-b + \sqrt{b^2 - 4c} > 0$, therefore $A^* = \frac{-b + \sqrt{b^2 - 4c}}{2} > 0$. The positivity of λ , α , and μ guarantees that A^* is bounded by 1. The alcohol-present equilibrium point, A^* , is

$$A^* = \frac{-\frac{\alpha}{\beta + \lambda} \left[\frac{1}{R_0} \left(1 + \frac{\beta + \lambda}{\alpha} \right) - 1 \right] + \sqrt{\left[\frac{\alpha}{\beta + \lambda} \left(\frac{1}{R_0} \left(1 + \frac{\beta + \lambda}{\alpha} \right) - 1 \right) \right]^2 - 4 \frac{1}{\beta(\alpha + \lambda)} (1 - R_0)}}{2}$$

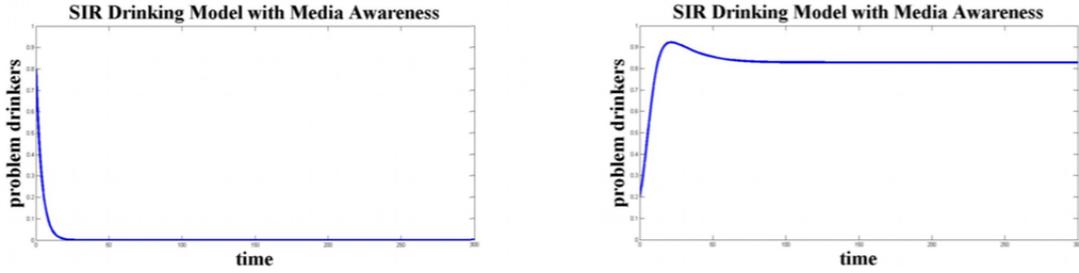
We note that when $R_0 = 1$, we have the case of an alcohol-free equilibrium, where the one-to-one ratio between β and μ yields two roots, one resulting in $A^* = 0$, and the other in a negative value for A^* (thus, an invalid root since A^* must be positive).

4. NUMERICAL RESULTS

Next, we present the numerical results for the stability of drinking-free equilibrium vs. permanent drinking. In Figure 1, we illustrate the stability of the disease-free equilibrium when $R_0 < 1$, as well as the stability of the alcohol-present stage, when $R_0 > 1$. Our results show that the transmission rate (β) of susceptibles becoming problem drinkers due to peer influence, and the overall average time that individuals spent as problem drinkers, play an important role in the establishment of the drinking culture in the community. In Figure 1(a), we show that although initially the alcohol presence is strong in the community, under the condition $R_0 < 1$ the population will eventually become sober. However, as Figure 1(b) suggests, in the long-run when $R_0 > 1$, around 82% of the population will become problem drinkers.

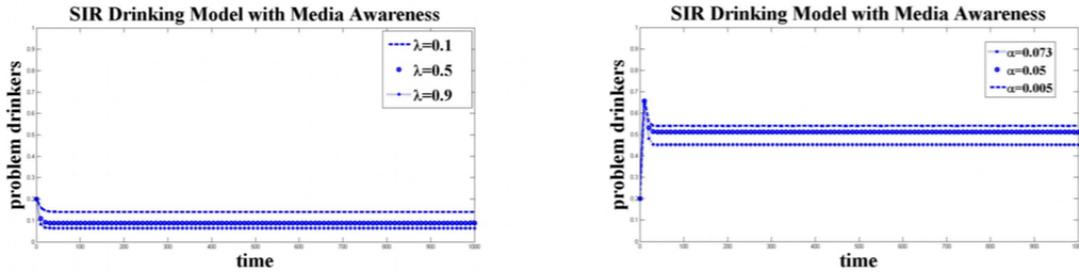
Additionally, we explore the roles of media awareness, λ , and relapse, α . We note that when $1 < R_0 < 1 + \frac{\beta + \lambda}{\alpha}$, a drinking culture is established (in the long-run), with a lower percentage of problem drinkers as compared to when $R_0 > 1 + \frac{\beta + \lambda}{\alpha} > 1$. The results suggest that once a drinking-culture is established, media awareness campaigns and treatment options are less likely to impact a significant percentage of the population. Consequently, their effects are expected to be diminished. In environments where peer influence is effective in recruiting new problem drinkers in the community, there need to be treatment options with low relapse rates and media awareness campaigns with large λ values. Then, to some extent, we could potentially dampen the final percentage of problem drinkers. However, we would not be able to eliminate the problem completely. In Figure 2, we explore the effects of λ and α on the final percentage of problem-drinking in a population where drinking is part of the culture ($R_0 > 1$). Figure 2(a) shows that different levels of λ (the measure of efficiency of the media awareness campaigns) have an impact on the final percentage of problem drinkers in the population. As we increase the efficiency of the media-awareness programs, it may be possible to reduce the number of problem drinkers in the overall population. When $R_0 > 1$, we can explore the effects of change in the relapse rate, α , on the final percentage of problem drinkers (Figure 2(b)). As we create treatment options with lower rates of relapse, the percentage of problem drinkers in the overall population will be reduced. The initial population proportions began with 80% of the population as susceptibles (s_0) and 20% as problem drinkers (a_0), in order to examine the effects of λ and α in a drinking culture ($R_0 > 1$). We assumed no individuals started as recovered, thus r_0 (initial proportion of recovered individuals) was set to zero, displaying the impact of an increase in α as time progressed. Lastly, the density of media awareness programs was initiated (m_0) at zero to exhibit the effects of an increase in λ over time.

We compared our results to other SIR-type drinking models. In [12], an SIR-type model with an additional, separate compartment consisting of aware individuals (in the presence of media awareness), researchers proposed a basic reproduction number that is equal to our own when their parameter variables p and q are set to θ . In their model, p is defined as the parameter describing the linear dynamics of relapse and q denotes the parameter related to the linear dynamics of problem drinkers going into treatment. Additionally, we compare our results with SIR-type drinking models that do not include a media awareness component, such as models presented in [6], [10], and [17]. Our result for the basic reproductive number is similar to the R_0 found in [6], with the assumption that the direct relapse effect is zero. However, our model yields a unique alcohol-present state rather than two alcohol-present equilibrium points, as found in their model. Similar to our outbreak condition, [17] indicated that peer influence directly affects



(a) $R_0 < 1$, $\lambda = 0.02$, $\gamma_1 = \gamma_2 = 0.05$, (b) $R_0 > 1$, $\gamma_1 = \gamma_2 = 0.05$, $\alpha = 0.0003$, $\alpha = 0.0003$, $s_0 = 0.2$, $a_0 = 0.8$, $r_0 = 0$, $s_0 = 0.8$, $a_0 = 0.2$, $r_0 = 0$, and $m_0 = 0$ and $m_0 = 0$

FIG. 1. Stability Results with different levels of R_0



(a) $s_0 = 0.8$, $a_0 = 0.2$, $r_0 = 0$, $m_0 = 0$, (b) $s_0 = 0.8$, $a_0 = 0.2$, $r_0 = 0$, $m_0 = 0$, $\gamma_1 = \gamma_2 = 0.05$, $\beta = 0.6$, $\mu = 0.05$, $\gamma_1 = \gamma_2 = 0.05$, $\beta = 0.6$, $\mu = 0.05$, $\alpha = 0.073$ and $\lambda = 0.1$

FIG. 2. Effect of λ and α on the levels of alcohol-present equilibrium.

the threshold value for alcohol abuse on campus. Mubayi et al. (2010) explore the influence of heterogenous drinking environments on the spread of alcoholism [10]. In their setting, the average time spent in high-drinking environment may increase the number of heavy drinkers in the community. Additionally, [20] suggest that reducing the time individuals spend in drinking environments might be the most effective strategy in reducing the size of the problem drinking population.

5. CONCLUSION

In this paper, we provided a compartmental mathematical model to assess the impact of public media awareness programs on the spread of problem-drinking in the community. The analysis of the model shows that there exists an asymptotically stable equilibrium point $(I, 0, 0, 0)$ and a unique endemic state (S^*, A^*, R^*, M^*) . The basic reproductive number is defined as $R_0 = \frac{\beta}{\mu}$. Our analysis of the equilibrium points indicate that when $R_0 < 1$, a drinking culture is not established in the population. Conversely, when $R_0 > 1$, we observe an endemic state

$(S^*(t), A^*(t), R^*(t), M^*(t))$, and we study and hypothesize about its stability. Our analysis using stability theory of differential equations, as well as the numerical simulations, suggest that if $R_0 > 1$, problem drinking is established as part of the culture and can only be eliminated by decreasing the rate of transmission, β or reducing the time spent in drinking environment, $1/\mu$. If indeed a drinking culture has already been established, preventive measures such as robust media awareness programs and well-designed treatment options can only help control, or mitigate the spread of the problem drinking, but not fully eliminate it. Even in the presence of such treatment options and/or media awareness programs, problem drinking cannot be completely eliminated in the community due to new recruits, susceptibles moving to problem-drinking stage, and the longevity of the average time spent in the drinking environment. In the future, we plan to extend our model to investigate the effects of media awareness components in heterogenous social mixing environment, where peer influence may not be uniform in each class.

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