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ABSTRACT

A description of finite element model and analysis of a shell with an infill is performed. A large diameter thin cylindrical shell structure with the edge leaning against compressible foundation soil is analyzed. Different materials are considered individually for the models of each structure shell and infill component (metal or reinforced concrete shell, and granular or elastic infill in a shell and foundation soil loaded by the structure). Contact conditions between 1) the infill and the shell's inner surface and 2) between the foundation material and the shell edge are analyzed. An example of calculating strain conditions in the shell according to the proposed finite element model and tasks of its development process and specification are provided in this paper.

KEY WORDS: Thin shell design; elastic infill; physical modeling; mathematical modeling; proof-of-concept setting; joint deformations; strain conditions.

INTRODUCTION

The economical composite structures facilitating more effective application of positive properties of the component materials' properties are more widely applied in the construction practice. The thin steel shells with the soil infill are used as earth and water retaining structures in marine environment in the hydraulic engineering construction. The structures of large-diameter-shells consist of two basic elements; the shell installed on the foundation/bedding soil and the infill soil (Fig. 1). The shell holds the infill soil as designed, and together they make up a massive composite structure, which serves as a base for the top structures of berths and the retaining structure for the soils behind the berth.

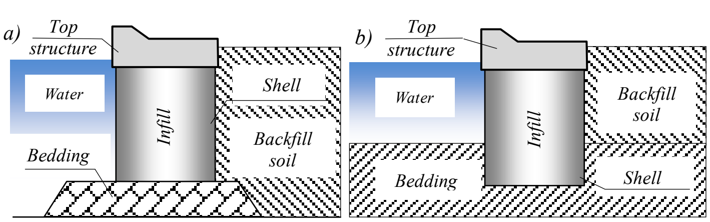

Fig. 1. Layout of a berth made of large-diameter shell: a) the shell on the "rock bedding"; and b) the shell restrained into the existing bedding

The shells with the infill may be utilized for a wide range of design applications, such as the multiple-strand close shells connected to each other by a common cap-plate (Fig. 2a) or the individual shells connected to each other by spans (such as piers or bridges) (Fig. 2b). Such structures are usually designed by special software that allows to include all the loads affecting the shell and to describe behavior of the structure during installation and post-construction.

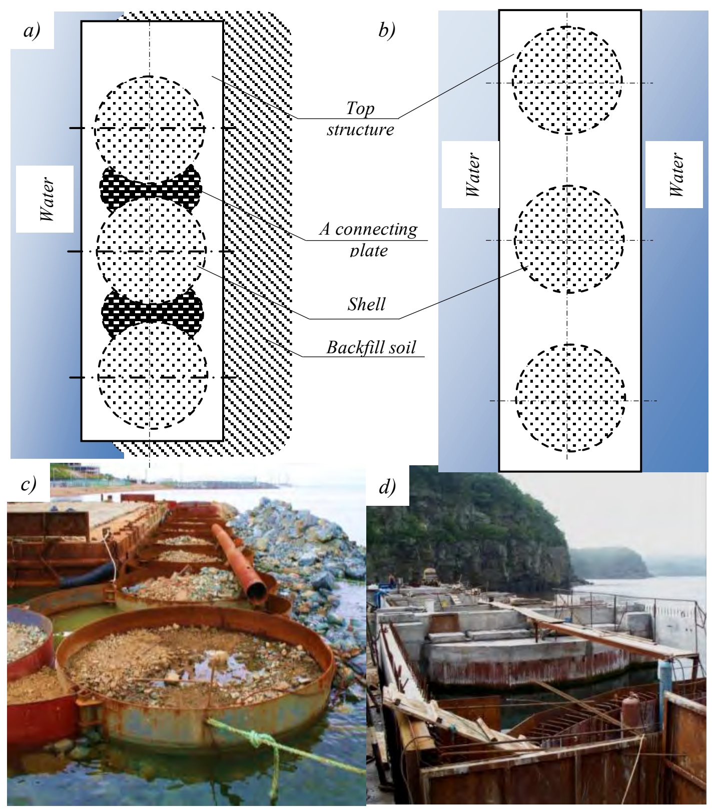

Fig. 2. Examples of large-diameter shell structures: a) a layout of a harbor wall (plan top view); b) a layout of a pier (plan top view); c) construction of a berth in the Fedorov Bay (Vladivostok, Russia) from steel shells with the infill; d) installation of the reinforced-concrete top

structure of the pier from individual shells on Russky Island (Vladivostok, Russia).

The main loads affecting the shells can be grouped in three main categories, as listed in the following:

Permanent loads:
- infill soil (in the simplified calculation models, such as Jansen (1895) method, the infill soil is simulated by the external load);
- active loads from the shore soil held by the shell;
- passive loads from soil on the front side of the shell (if the shell is restrained);
- bedding soil (if the shell is buried); and
- permanent engineering structures above the berth.

Temporary long-term loads:
- transportation vehicles and fuel-handling machines located on the berth;
- stackable cargo; and
- lateral loads from the soil affected by the temporary loads on the berth.

Short-term loads:
- mooring line tension, when ship is affected by wind and/or current;
- ship leant on the berth in the process of mooring;
- moored ship (affected by wind and/or current) on the berth;
- waves when the wave crest or trough is approaching;
- ice;
- wind; and
- transport and installation of individual structure elements and the whole assembled block during the construction.

TRADITIONAL METHODS OF CALCULATION

Mathematical modeling of shell and infill interaction began in 1940-50s (Vlasov, 1949; Goldenveizer, 1953; Lurie, 1947; Timoshenko, 1966). Main calculation models were developed to solve the tasks of missile engineering; in general, the open rotary shells were applied as large-span shells in the construction up to the 1970s.

In view of the difficult calculations, application of the closed rotary shells with the infill started by the end of the 20th century, when the computers with high performance had appeared. Experimental researches were also carried out in order to specify design layout of a structure, to determine a possibility of the in-plane shear inside the infill and the influence of different factors on the structure stability.

The traditional structural design of a shell with the infill is based on many assumptions, such as the basic:
- simulation of the internal infill as the external load for a shell (Jansen method) (in most cases, in order to describe the mode of deformation of the infill, a model of the elastic body is applied, where the infill is considered as the uniform medium with the linear dependence between stress and strain);
- hypothesis on the uniform distribution of pressure all over the facility sole;
- simplification of conditions on contact between the wall and the shell in case of strained system;
- averaging of stresses in the shell on contact with the soil foundation.

The introduced assumptions often result in substantive divergences between the computational data and the experimental research data (Tsimbelman and Chernova, 2012). This, and they may inaccurately limit application for such cost-efficient structures, as well. There are rather limited choices of designs allowing to increase the efficiency of the infill under various operational conditions.

The indicated issues confirm that the behavior of the shell structures with the elastic infill in the mode of deformation is studied insufficiently both in natural conditions and in simulation. Special constructive measures required to limit deformations may significantly increase the cost of the structure and may only serve as a solution for a small group of problems. Therefore, the development of a mathematical apparatus model for the shell structure and elastic infill is needed and would advance the design of these structures.

DESCRIPTION OF MATHEMATICAL MODEL

The infill soil is the main element for the described structure. The mathematical formulation of the determining equations of the theory of soil plasticity should show some specificity of mechanical behavior for the soil materials. Plastic shear deformations of the soils are accompanied by the volumetric strains. The volumetric strains may be both positive (loosening/dilatancy) and negative (compaction). The presence of the residual volumetric deformations in the soil predetermines a closed form of the loading surface (Zaretskiy and Lombardo, 1983).

The infill is considered as the elastic cylinder, and the bedding is considered as the elastic half-space. The soil infill and the bedding are considered as the plastically compressed elastoplastic bodies.

At the elastic stage, the soil looks like the linear-elastic material, and its behavior is defined by the modulus of general deformation *E,* and the Poisson's ratio, *ν*. The model with the Mohr-Coulomb flow criterion is the classic model of the soil plastic deformation widely applied in the engineering (Neto, 2008; Semenov, 2008) and it is given as:

$$\left(\sigma_{\max}-\sigma_{\min}\right)+\left(\sigma_{\max}+\sigma_{\min}\right)\sin\varphi-2c\cos\varphi=0\,, \tag{1}$$

where σ_{max} and σ_{min} are the maximum and the minimum principal stresses, ϕ is the internal friction angle, *c* is the specific cohesion. In the space of principal stresses, the Mohr-Coulomb condition is an improper hexagonal pyramid. Advantage of the model consists of clear physical meaning of its parameters, which are determined rather simply. Eq. 1indicates that the intermediate principal stress does not influence on soil's strength, which is the main reason Mohr-Coulomb strength does not reflect real soil behavior. Drucker and Prager (1952*)* proposed the criteria of plastic flow as:

$$\sqrt{J_2}=A+BI_1\,, \tag{2}$$

where $I_1=\sigma_{11}+\sigma_{22}+\sigma_{33}$ is the linear invariant of the stress tensor, J_2 is the quadric invariant of a deviator of the stress tensor given as:

$$J_2=\frac{1}{6}\left(\left(\sigma_{11}-\sigma_{22}\right)^2+\left(\sigma_{22}-\sigma_{33}\right)^2+\left(\sigma_{33}-\sigma_{11}\right)^2\right)+\sigma_{12}^2+\sigma_{23}^2+\sigma_{13}^2\,. \tag{3}$$

and *A* and *B* are experimentally determined parameters.

The Drucker-Prager flow condition takes into account influence of the middle principal strain on the soil strength which helps to better describe triaxial deformations (Zaretskiy and Lombardo, 1983). The flow surface given by Drucker and Prager (1952) in the space of principal strains looks like a proper circular cone relative to the axis of hydrostatic pressure $\sigma_1 = \sigma_2 = \sigma_3$, and it is an approximation of the

Mohr-Coulomb flow surface (Neto, 2008). The parameters A and B in (Eq. 2) can be stated by the cohesion c and the internal friction angle ϕ as (Wang, 2004; Shashenko, 2010):

$$A = \frac{6c\cos\phi}{\sqrt{3}(3-\sin\phi)} \quad \text{and} \quad B = \frac{2\sin\phi}{\sqrt{3}(3-\sin\phi)}\,. \tag{4}$$

As the mathematical model of the structure, the membrane barrel shell of finite sizes was accepted. The shell material is modeled as the isotropic-linear-elastic material. Its behavior is determined by two experimentally defined parameters, such as namely the modulus of elasticity, E, and the Poisson ratio, v. Then the stress-strain dependence looks like as follows:

$$\sigma_{ij} = \frac{\nu E}{(1+\nu)(1-2\nu)}\delta_{ij}e_{kk} + \frac{E}{2(1+\nu)}e_{ij}\,,$$

where σ_{ij} is the stress component, e_{ij} is the strain component, δ_{ij} is the Kronecker symbol.

A model may be applied with the help of modern computer systems based on the finite element method. As an example, it is proposed that a shell with the infill was modeled and analyzed using ANSYS finite-element software. The model and the results obtained are given below.

CALCULATION EXAMPLE

The bedding and the shell infill are modeled using 8-noded finite elements that allow applying Drucker and Prager's model. Finite elements with 4-node are selected for the steel shell. The contact surface between the soil and the steel shell is modeled with 0.1 – 0.4 friction factors. Type of the contact surface for the ANSYS software is "CONTACT_AUTOMATIC_SURFACE_TO_SURFACE" using the soft contact option. In order to take into consideration the deflection of the bedding affected by the shell with the infill, the densities of the infill and of the shell material are taken into account. The bedding is simulated as a parallelepiped fixed at the bottom.

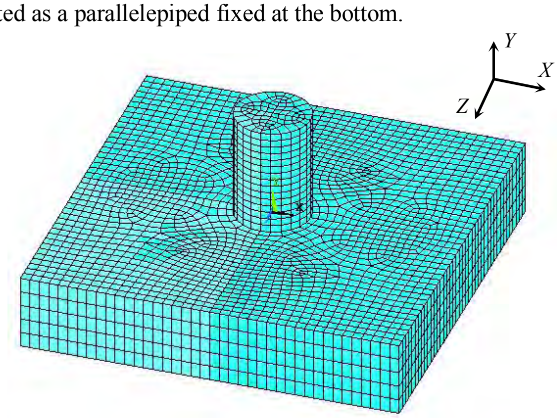

Fig. 3. Finite-element model of a shell with the infill

The proposed model includes a cylinder shell with outer diameter D=10 m and height H=14.5 m, and the shell wall thickness of 0.02 m width. The elastic modulus of the metal shell E=200 GPa, Poisson ratio of ν=0.3, and the shell material density ρ=7800 kg/m^3 are used for the metal shell. The infill soil has the following properties: elastic modulus E=16 MPa, ν=0.3, c=0, φ=26°, soil density ρ=1000 kg/m^3. Parameters of the bedding are as follows: the modulus of general soil deformation E=23 MPa, Poisson's ratio ν=0.3, specific cohesion c=0, inner friction angle ϕ=32°.

As an example, a simplified model calculation situation (for an individual shell) is considered. The active pressure of the soil (from the shore side) of 2,000 kg/m^3 density per half area of the shell wall is applied. The lateral earth pressure acting on the shell is 0 kPa at the upper edge of the cylinder and 95.7 kPa at 14.5 m depth (shell height). The lateral pressures acting on the shell are shown in Fig. 4.

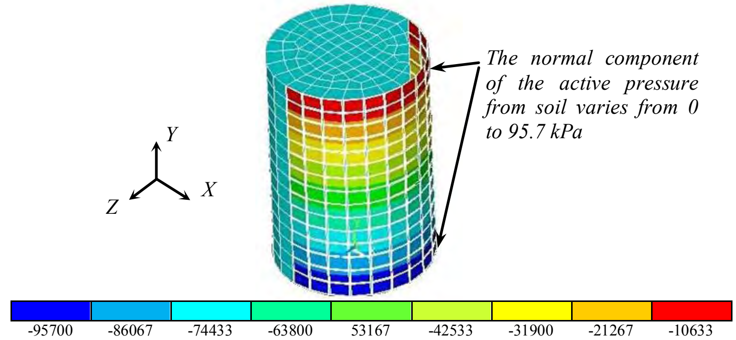

Fig. 4. Soil pressures acting on the shell (in Pa)

The analysis results show that the shell affected by the active earth pressures applied and its own weight, and moved both vertically (Fig.5) and horizontally (Fig. 6). Fig. 6 also shows that the horizontal deformations are larger at the top of the shell compared to the bottom, indicating bending of the shell. Total displacements of the shell are given in Fig. 7 and the figure shows that maximum displacement of the structure from the original position does not exceed 0.2 m.

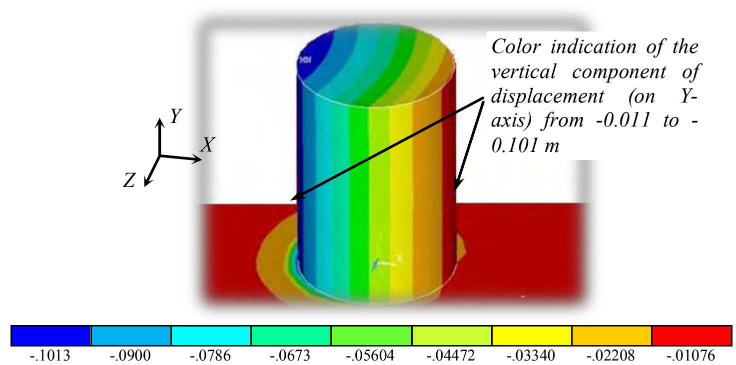

Fig. 5. Vertical displacements (in m)

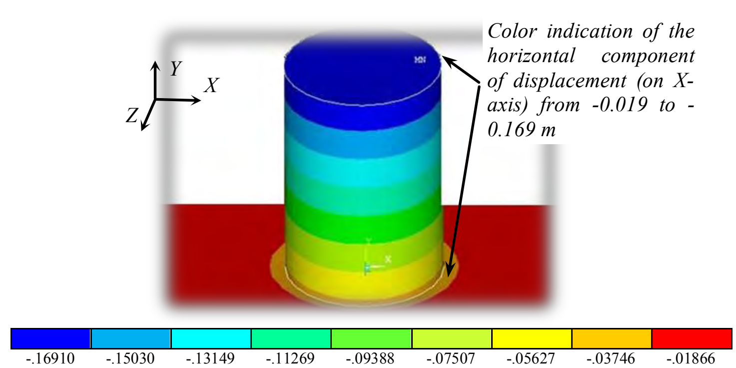

Fig. 6. Horizontal displacements (in m)

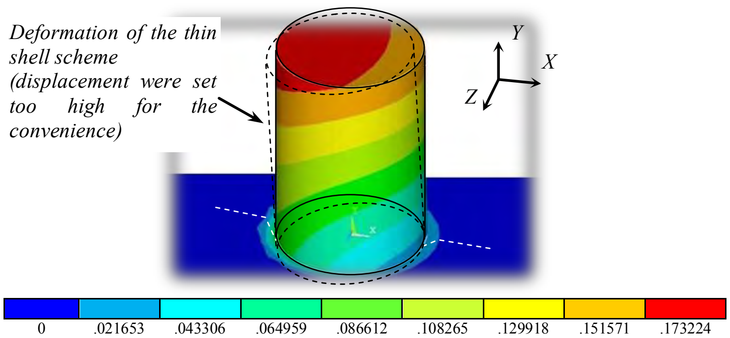

Fig. 7. Total displacement (in m)

The unsymmetrical loading of the structure causes the differential settlement of the shell. In this example, the settlement of the structure changes from 0.011m up to 0.1 m (Fig. 8).

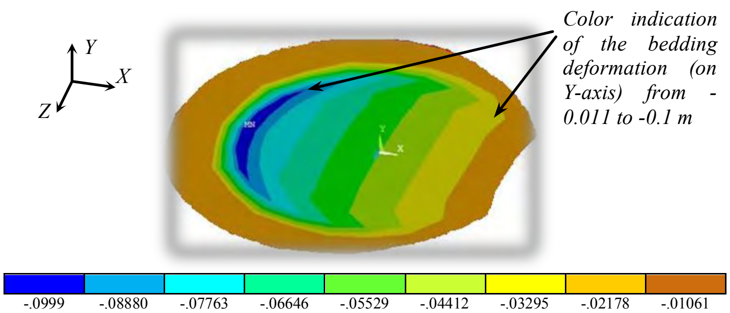

Fig. 8. Bedding deformation (in m)

The stresses appeared in the steel shell are represented in Fig.9. Under the stated loads such stresses do not exceed 100 MPa.

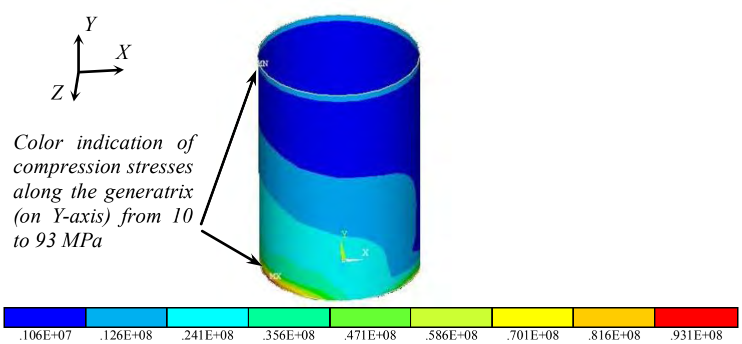

Fig. 9. Stresses in a shell (along the generatrix) (in Pa)

The considerable significant deformations indicate insufficient resistance of the bedding soil. The design has to be improved to obtain reasonable outcomes (to allowable value of careen and settlement of the structure, as well as allowable stresses in the shell).

EXPERIMENT

As the first stage of verification of the proposed computational model, a physical model was built and a laboratory experiment is carried out. The described experiment proposes studying of a structure behavior in laboratory conditions. Thus the researched model and conditions of its operation should match (be similar) to real structures and the phenomena happening in nature. Conditions of simulation of a material and a shell, as well as material of an infill, should ensure the complete similarity of tension in nature and in the model. In order to solve this task, the similarity theory and the dimensional analysis methods (Kirpichev, 1953) are applied.

The concepts of scale factors α connecting all values included into the equations for nature (X_n) and into a set of equations for the model (X_m) (Florin, 1961), where $X_m = \alpha \cdot X_n$, are entered for the simulation. Thus, a scale of volume forces of the environmental dead load α_γ; a scale of an angle of internal friction of the environment α_φ ($\sin \phi_m = \alpha_\varphi \cdot \sin \phi_n$); and a scale of the environment cohesion α_c are entered. The applicable scale factors, such as scale of lengths α_l and scale of powers α_p, are also entered for the sizes, loads and stresses.

Simulation of an infill. Conditions of soil simulation are determined by the task analysis of a stress state of the structure that at any plastic strain rate can be introduced by the applicable solutions of the mixed task of both the theory of linearly strained environment and the theory of the limit equilibrium (Florin, 1961): $\alpha_p = \alpha_c = \alpha_l \cdot \alpha_\gamma$; $\alpha_\varphi = 1$. In the study, the incoherent (granular) environment is supposed, where conditions of the simulation are as follows: $\alpha_p = \alpha_l \cdot \alpha_\gamma$; $\alpha_\varphi = 1$.

Provided that the granular environment is simulated as the infill for the model, the sand with specific gravity γ_m = 16 kN/m^3 and an angle of internal friction ϕ_M = 32° is accepted. Physical, deformation and strengthening properties are determined according to standard procedures, such as the method of in-plane shear, uniaxial compression and triaxial compression of a sample under the National Standards (GOSTs 25100-95, 5180-84, 12536-79, and 23906-79).

As a result, necessary scale factors α connecting values of a computational model and a physical model (Table 1) are indicated for the given scale of lengths 1:20 (α_l=0.05).

Table 1. Scale factors

Factor	α_l	α_γ	α_p	α_φ
Shell	0.05	0.089	0.0045	-
Infill	0.05	1.6	0.08	1
Bedding	0.05	0.8	0.04	1

Shell simulation. In the simulation it is provided that the length of all units of a model should be diminished proportionally to simulation scale (Florin, 1961). Parameters of the model of the thin shell with an infill can be obtained from: $J_m = \alpha_l^4 \cdot J_n$, where J_m is a moment of inertia of sectional view of a model, and J_n is a moment of inertia of sectional view of nature.

The equation of unit rigidity dependence from properties of the soil environment is obtained from condition of equality of the ratio of model-and-nature rigidity to the ratio of stresses for a model and nature (Dovgalenko, 1977):

$$\frac{J_m \cdot E_m}{J_n \cdot E_n \cdot \alpha_l^4} = \frac{\sigma_m}{\sigma_n} = \alpha_p = \alpha_l \cdot \alpha_\gamma,$$

where E_m is a model elastic modulus, and E_n is a nature elastic modulus. The required geometric performances of sectional view of the model are determined from:

$$J_m = \frac{J_n \cdot E_n \cdot \alpha_l^5 \alpha_\gamma}{E_m}.$$

The plexiglass is used as the model material. Elastic modulus E_m of the model material is defined by laboratory standard procedures (GOST 11262-80, and GOST 4648-71). The scale factors α and for the given scale of lengths α_l were defined for the model beforehand, and as a result, the necessary sizes of the model (such as the height, diameter, and wall thickness) which provide the required J_m are selected. The main model sizes are shown in Fig. 10.

The model of the filled barrel shell was fixed by its end face on the compressible bedding. The experimental device allowed non-central loading of the shell by application of the horizontal force to its cap. The schematic of the experimental setup is given in Fig. 10. Value of applied force is computed from the condition of equality of the moments forwarded by the shell to the bedding for the physical/computational model (including the scale factor):

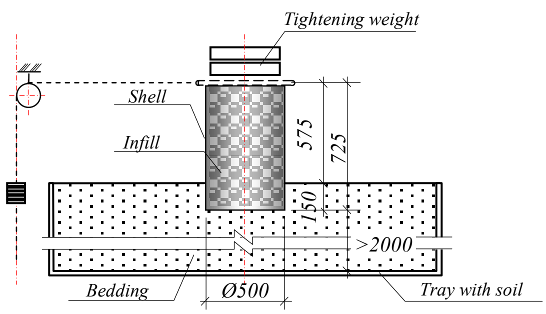

Fig. 10. The schematic and dimensions of the experimental setup device (all dimensions are in millimeters)

The actual experimental setup is shown in Fig. 11. The experimental setup (Fig. 11) was also equipped with clock-type displacement detectors capable to indicate shell displacements; with strain-gauge system which makes it possible to monitor increase of pressure inside the shell. The main task of this investigation phase was the analysis of the shell displacements and stress distribution in its bedding. Comparison of the outcomes from the experiment with the calculation data from the proposed model will allow to draw preliminary conclusions on reliability of the calculation and to determine directions of the computational model improvement.

Fig. 11. Physical modeling and laboratory experiment setup

During the experiment, the load was applied stage by stage. Each loading stage involved applying in shares making 1/10 from value of an expected maximum load. The application of the next loading was performed after the fading of strains from the previous stage was stabilized. At each stage of loading, shell displacements and value of stresses were recorded in the base zone of the shell (Fig. 12).

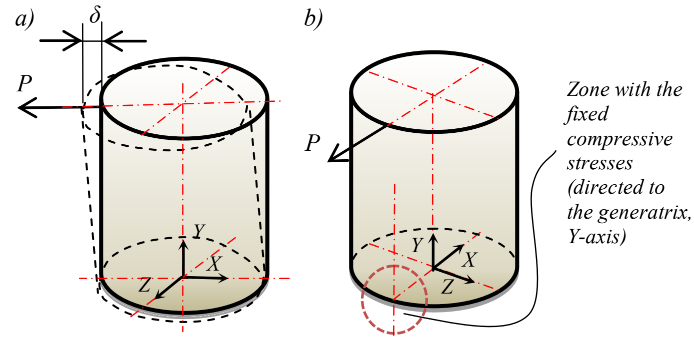

Fig. 12. The fixing parameters a) displacement of the shell top; b) zone with the compressive stresses fixed by strain gauges

The results obtained from the experiment and the calculation numerical modeling are given in Table 2.

Table 2. Comparative analysis of results from numerical and physical modeling

	Experiment	Including scale factors α	Numerical Model	Divergence Δ, %
Cap displacement δ (mm)	9.65	193.0	173.0	10.4
Stress, σ (MPa)	0.39	87.44	93.1	6.1

The analysis and the results presented in Table 2 indicate that:

- The analysis of indications from strain gauges confirms that the compression stresses concentrate at the level of the shell bedding into the soil (the compressed area);
- The accepted in calculation hypothesis of corruption of the "shell-soil" system completely matches with the actual activity of such structure under non-central load;
- The difference between the experimental data and the outcomes from calculation according to the proposed numerical model made 10 % for displacements and 6 % for stresses.

CONCLUSIONS

This study presents a finite-element model of a thin-wall barrel shell with the infill installed by an end face on the compressible base. The model allows defining structural deformation and stresses in a body of the thin shell, and bedding deformation as well, for given conditions of loading and properties of materials of the shell, the infill, and the bedding.

In order to tentatively estimate applicability of the presented model, the model experiment of the shell with the infill is carried out. All components of the model and their parameters (the sizes and the shell material, performance of the infill, etc.) are got under the rules of similarity theory with the given scale factor. During the experiment under non-central loading of the shell the most critical parameters were fixed, namely displacement of the shell top and stresses in the compressed base of the structure.

Comparison of outcomes from the experiment with the values of strains and stresses computed on the proposed model shows good enough outcomes with a permissible variation (about 10 % for the displacements and about 6 % for the stresses). Afterwards the computational model may be applied to solution of some engineering tasks originating during design of different hydraulic engineering structures.

The model may be modified and complicated, and more components may be added in order to better describe properties of certain structures (such as additional loads and structural elements). By carrying out calculations by the program system and numerical software and including various loads (from wind, waves, shocks, and seismic loads) it is possible to predict behavior of the structures during construction and operation. This would help to give recommendations on its construction, to simulate emergency condition, and search for the preventive measures.

Among the important advantages of the described computational model presented is the ability to analyze the behavior of the compressible (compliant) soil bedding of the shell with the infill. It is possible to investigate and determine the stresses in the bedding of the structure taking into consideration of pressing of the sharp shell edge in the soil, and it is also possible to predict structure movements and settlements.

The investigations represented in this study may increase the use of large-diameter shells with the infill. It will be possible to apply such economically efficient structures on soft foundation soils with rocky beddings or not. The computational model makes it possible to consider the settlement limits of the foundation, and the parametric study can be performed to determine the permissible loads on the facility to keep the settlements within the tolerable limits.

The authors intend to advance the mathematical model to better capture the interaction between a shell with the infill and the foundation soil by including the infill friction and cohesion against the shell's inner walls, which may also be affected in case of any "roughness" due to shelves, edges, and membranes.

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