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Analysis of the joint impact of atmospheric turbulence and refractivity on laser beam propagation

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Abstract: A laser beam propagation model that accounts for the joint effect of atmospheric turbulence and refractivity is introduced and evaluated through numerical simulations. In the numerical analysis of laser beam propagation, refractive index inhomogeneities along the atmospheric propagation path were represented by a combination of the turbulence-induced random fluctuations described in the framework of classical Kolmogorov turbulence theory and large-scale refractive index variations caused by the presence of an inverse temperature layer. The results demonstrate that an inverse temperature layer located in the vicinity of a laser beam’s propagation path may strongly impact the laser beam statistical characteristics including the beam wander and long-exposure beam footprint, and be a reason for refractivity-induced spatial anisotropy of these characteristics.

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1. Introduction

Propagation of a laser beam in the Earth’s atmospheric boundary layer can be influenced by refractive index spatial inhomogeneities resulting from complicated dynamics of air masses [1–4]. Under the commonly used in atmospheric optics assumption, the refractive index field \( n(\mathbf{r},t) \) can be represented as a sum of two major components that are associated with atmospheric refractivity \( n_{\text{ref}}(\mathbf{r},t) \) and optical turbulence \( n_{\text{turb}}(\mathbf{r},t) \):

\[
n(\mathbf{r},t) = n_0 + n_{\text{ref}}(\mathbf{r},t) + n_{\text{turb}}(\mathbf{r},t),
\]

where \( \mathbf{r} = \{x,y,z\} \) and \( t \) are correspondingly the coordinate vector, and time. The refractivity term \( n_{\text{ref}}(\mathbf{r},t) \) describes quasi-static, large-scale deviations of the refractive index field from the undistorted value \( n_0 \), which are caused by slowly evolving meteorological processes. This term can be defined as \( n_{\text{ref}}(\mathbf{r},t) = <n(\mathbf{r},t)>_{\text{turb}} - n_0 \), where \( < >_{\text{turb}} \) corresponds to averaging over relatively small scale and rapidly changing turbulence-induced random refractive index inhomogeneities (eddies). The characteristic size of these eddies vary from a few millimeters (inner scale) to tens of meters (outer scale) with a “life-time” ranging from a few to tens of milli-seconds [5,6].

Contrary to the optical turbulence, atmospheric refractivity is associated with spatio-temporal dynamics of large-scale refractive index inhomogeneities (from tens of meters to hundreds of kilometers) that are evolving at a significantly slower (on the order of several hours) pace. Correspondingly, for the typical duration of observations and measurements, the refractivity term in Eq. (1) can be considered as stationary \( n_{\text{ref}}(\mathbf{r},t) \) and hence doesn’t impact laser beam characteristics.

Further simplification can be made for laser systems operating over relatively short (typically a few kilometers) distances and in absence of strong refraction gradients in vicinity of the laser beam propagation path, e.g. caused by inverted temperature layers. In these cases the refractivity term in Eq. (1) can be considered as a constant \( n_{\text{ref}}(\mathbf{r}) = \text{const} \) and hence doesn’t impact laser beam characteristics.

In this paper we discuss more general laser beam propagation scenarios for which atmospheric refraction can play an important role and could significantly alter the major laser beam statistical characteristics.

In principal, for a spatially coherent, monochromatic or quasi-monochromatic laser beam, the impact of both atmospheric turbulence and refraction can be accounted for in the parabolic approximation of the diffraction theory. In this approximation, evolution of the optical field complex amplitude \( A(\mathbf{r},z) \) along the laser transmitter optical axis (oz-axis) can be described by the following parabolic equation [7]:

\[
2ik \frac{\partial A(\mathbf{r},z)}{\partial z} = \nabla_\perp^2 A(\mathbf{r},z) + k^2 \left[ \frac{n^4(\mathbf{r},z)}{n_0^4} - 1 \right] A(\mathbf{r},z).
\]

(2)

Here \( \mathbf{r} = \{x,y\} \), \( k = 2\pi/\lambda \) and \( \nabla_\perp^2 = \{\partial^2 / \partial x^2 + \partial^2 / \partial y^2\} \) are correspondingly a vector in the plane that is orthogonal with respect to the optical axis, wavenumber for wavelength \( \lambda \), and the Laplacian operator over transversal coordinates.

Difficulties in utilization of Eq. (2) for numerical analysis of laser beam propagation over extended-range distances (tens of kilometers) and/or in the presence of strong refraction are related with potentially significant deviation of the laser beam propagation trajectory from the optical axis (laser beam bending effect). The laser beam bending may require a significant,
and in many cases unrealistically large, from a computational view point, increase of the numerical grid size to keep the beam footprint inside the numerical simulation region.

For this reason, in analysis of optical wave propagation over long distances the refractive effects are accounted for in the framework of the geometrical optics approximation by computing an optical ray (chief ray) trajectory that is commonly associated with the transmitted laser beam centroid. The ray trajectory – vector-function \( \vec{r} (\vec{r}) \) – can be obtained via integration of first order differential equations, also referred to as ray tracing equations [8,9]:

\[
\frac{d \vec{r}}{dl} = \nabla n_{\text{refr}} (\vec{r}),
\]

(3)

where \( dl = dl(\vec{r}) \) and \( \theta = \theta(\vec{r}) \) are correspondingly, the trajectory’s small element and slope vector. Note that the ray tracing Eqs. (3) don’t account for optical field diffraction that occurs on turbulence-induced refractive index inhomogeneities and hence, cannot be used for analysis of the joint impact of turbulence and refractivity on laser beam characteristics.

In the following section we introduce a computationally efficient mathematical model of laser beam propagation in the presence of both atmospheric turbulence and refractivity, referred to here as the Wave-Optics Ray-Tracing Extension (WORTEX) model.

In the numerical analysis of laser beam propagation using the WORTEX model we utilized an atmospheric refractive index field as being comprised of Kolmogorov turbulence, and inversed temperature layer-induced refractivity, as described in section 3. Accuracy of the introduced propagation model is evaluated in section 4 via comparison of numerical simulation results obtained using both the WORTEX and the wave-optics approaches, which are based on direct integration of the parabolic Eq. (2) using the conventional split-step operator technique [10–12]. The results of numerical analysis of laser beam propagation in turbulent atmosphere with the presence of a localized refractivity structure caused by inversed temperature layer are discussed in section 5.

2. Wave-optics ray-tracing extension (WORTEX) model

Consider the evolution of optical field complex amplitude \( A(r, l) \) along a laser beam centroid trajectory \( \mathcal{L} \) as illustrated in Fig. 1. Here the trajectory is defined by vector \( \vec{r} (\vec{r}) \), \( l \) is a distance along the trajectory and \( r(l) = \{x_l(l), y_l(l)\} \) is a 2D vector in the plane \( P(l) \) orthogonal to a tangent to the trajectory vector \( \theta(l) \). At a relatively short trajectory segment \( \Delta l \), the refractivity term can be considered as spatially uniform \( (n_{\text{refr}}(\vec{r}) = \text{const}) \) and Eq. (2) can be presented in the following simplified form:

\[
2k \frac{\partial A(r, l)}{\partial l} = \nabla_A^2 A(r, l) + 2k^2 n_{\text{turb}}(r, l) A(r, l).
\]

(4)

Note that Eq. (4) describes solely the impact of turbulence on the complex amplitude \( A(r, l) \) along the beam centroid trajectory segment of length \( \Delta l \) a distance \( l \) from the laser transmitter.

In the WORTEX beam propagation model considered, the beam centroid trajectory – vector function \( \vec{r} (\vec{r}) \) – is defined using in the following modified ray-tracing equations:
\[
\left[ n_0 + n_{\text{ref}}(\mathbf{r}) \right] \frac{d(\mathbf{r} + \delta_{\text{turb}})}{dl} = \mathbf{\Theta}, \tag{5}
\]

Impact of turbulence on the trajectory \( \mathbf{r} \equiv \mathbf{r}(l) \) is accounted for in Eqs. (5) using the auxiliary functions \( \delta_{\text{turb}} = \delta_{\text{turb}}(l) \) and \( \Theta_{\text{turb}} = \Theta_{\text{turb}}(l) \). Vector \( \delta_{\text{turb}}(l) \) describes the beam centroid deviation from the trajectory vector \( \mathbf{r}(l) \), which is caused by turbulence-induced changes in laser beam intensity distribution at the transversal plane \( P(l) \):

\[
\delta_{\text{turb}}(l) = W^{-1} \int |A(\mathbf{r},l)|^2 \, d^2 \mathbf{r}, \tag{6}
\]

where \( W = \int |A(\mathbf{r},l)|^2 \, d^2 \mathbf{r} \) is the beam total power that is assumed to be a constant.

The term \( \Theta_{\text{turb}}(l) \) in Eqs. (5) accounts for the turbulence-induced deviation of the trajectory slope vector \( \Theta(l) \). This slope vector deviation can be defined using the angular moment of the far-field intensity:

\[
\Theta_{\text{turb}}(l) = W^{-1} \int |A(\mathbf{k},l)|^2 \, d^2 \mathbf{k}, \tag{7}
\]

where \( \mathbf{k} = \{\kappa_x, \kappa_y\} \) is the angular vector and \( A(\mathbf{k},l) = \int A(\mathbf{r},l) \exp(\mathbf{i} \mathbf{k} \cdot \mathbf{r}) \, d^2 \mathbf{r} \) is the optical field spectral amplitude.

The system of Eqs. (4)-(7) represents the WORTEX model describing laser beam propagation in presence of both atmospheric turbulence and refractivity.

3. Atmospheric turbulence and refractivity models

In this section we introduce turbulence and refractivity models that are used in the numerical simulations described in sections 4 and 5. Consider a stationary atmospheric refractive index field in Eq. (1) comprised of refractivity \( n_{\text{ref}}(\mathbf{r}) \) and turbulence \( n_{\text{turb}}(\mathbf{r}) \) terms.

Under general assumptions, the refractivity term can be represented by the following expression dependent on the temperature profile function \( T(h) \) [13,14]:

\[
n_{\text{ref}}(\mathbf{r}) = n_{\text{ref}}(h) = \frac{A_r P_r}{T(h)} \exp \left[ -B \int_0^h \frac{g(h') \, dh'}{g(0) T(h')} \right], \tag{8}
\]
where \( h = |\mathbf{r}| - R_e \) is height above the ground in a Cartesian coordinate system with the origin at the Earth’s center and \( R_e \) is Earth’s radius. In Eq. (8) \( g(h) = g(0)\left[\frac{R_e}{(R_e + h)}\right]^2 \) is a standard gravity acceleration function, \( P_0 \) is ground atmospheric pressure, and \( B \) and \( A_D \) are constants \((A_D = 7.7911 \times 10^{-3} \text{ K/hPa for } \lambda = 0.53 \mu \text{m} \) and \( B = 3.4177 \times 10^{-2} \text{ K/m} \). 

In the numerical simulations, the temperature profile in Eq. (8) was considered as composed of two terms:

\[
T(h) = T_{\text{MUSA76}}(h) + T_{\text{ITL}}(h),
\]

where \( T_{\text{MUSA76}}(h) = T_0 + \alpha h \) corresponds to standard MUSA76 model describing linear temperature decline with height \( h \) by a rate of \( \alpha = -6.5 \text{ K/km} \) and \( T_0 \) is the temperature on the ground [13,14]. The MUSA76 temperature profile model describes relatively slow temperature decline with the height \( h \) resulting in a smooth refractive index spatial modulation inside the air volume along the propagation path.

The second term in Eq. (9) corresponds to a highly localized, horizontally oriented inverse temperature layer (ITL) a distance \( h_{\text{ITL}} \) above the ground. The ITL can be described by the following temperature profile [13]:

\[
T_{\text{ITL}}(h) = \Delta T \left\{ \frac{1}{1 + \exp\left[-(h-h_{\text{ITL}})/w_{\text{ITL}}\right]} - 1 \right\},
\]

where \( \Delta T \) is the temperature inversion parameter and \( w_{\text{ITL}} \) is the ITL width. Depending on the temperature inversion sign, one can distinguish between desert \((\Delta T < 0)\) and ocean \((\Delta T > 0)\) ITL types [13]. The ITL can occur when, for example, a warmer air mass moves over a cooler one. The lifetime of such structure can vary from a few minutes up to several hours depending on the weather conditions and topography [15].

The turbulence-induced refractive index fluctuations \( n_{\text{wob}}(\mathbf{r}) \) were assumed to obey the Kolmogorov-Obukhov two-thirds power law for the structure function inside the inertial subrange with the Hufnagel-Valley 5/7 (HV-5/7) model for refractive index structure parameter altitude profile \( C_n^2(h) \) [16–19].

Note that the WORTEX approach introduced here is not limited to the ITL-type refractivity that is discussed as an example only. This technique can be easily extended to include large-scale refractive index inhomogeneities such as those obtained using atmospheric computational fluid dynamics [2].

4. Comparative analysis of laser beam propagation in the presence of turbulence and refractivity

For validation of the laser beam propagation model described in section 2, we performed a set of numerical simulations to compare results obtained using WORTEX [Eqs. (4)-(7)] and conventional wave-optics [Eq. (2)] approaches. In both cases we considered atmospheric propagation of a collimated Gaussian beam of radius \( a = 1.5 \text{ cm} \) which is transmitted horizontally from elevation \( h_0 = 25 \text{ m} \). We assumed a coordinate system with its origin at the ground with the laser transmitter location defined by vector \( \mathbf{r}_0 = \{x_0, y_0, z_0\} = \{0, h_0, 0\} \), where the \( oz \)-axis coincides with the laser beam transmission direction (optical axis), and \( ox, oy \)- are correspondingly the axes parallel (horizontal) and orthogonal to ground. For simplicity, we neglected here the Earth surface curvature.
The atmospheric refractivity was represented by Eq. (8) with \( T(h) = T_{\text{MUSA76}}(h) \) and \( T_0 = 288.15 \, \text{K} \). The turbulence-induced refractive index fluctuations were considered as obeying the Kolmogorov power spectrum with \( C_{\text{v}}^2 = 5 \times 10^{-18} \, \text{m}^{-2/3} \).

In the numerical simulations, we computed a set of intensity distributions \( I(r,z) = |A(r,z)|^2 \), using both direct numerical integration of the parabolic Eq. (2), and the system of Eqs. (4-7).

Numerical integrations of Eq. (2) and Eq. (4) were performed using the standard split-step-operator method [10–12]. In this technique the turbulence-induced refractive index perturbations along the propagation path are represented by a set of \( M \) thin, two dimensional, statistically independent phase screens \( \{ \psi_m(r,z) \} \), \( (m = 1, \ldots, M) \). Each set of phase screens is referred to here as a turbulence realization. In the simulations described here, we used \( M = 20 \) equidistantly located phase screens. The statistical averaging (long-exposure) laser beam characteristics were computed using an ensemble of \( N_{\text{turb}} = 5000 \) statistically independent turbulence realizations.

For the numerical integration of Eq. (2), each phase screen included both turbulence- and refractivity-induced phase components:

\[
\psi^{(m)}(r) = k \int_{z_m - \Delta z/2}^{z_m + \Delta z/2} \left[ n_{\text{turb}}(r, z) + n_{\text{refr}}(r, z) \right] \, \text{d}z , \tag{11}
\]

where \( m = 1, \ldots, M, \, z_m = m \Delta z, \, \Delta z = L/M \) and \( L \) is distance from laser transmitter.

In the numerical simulations based on the WORTEX model [Eqs. (4-7)], the term \( n_{\text{refr}}(r, z) \) in Eq. (11) was omitted, having been accounted in Eqs. (5) for beam centroid trajectory.

The results of numerical simulations based on the conventional wave-optics technique [Eq. (2)] are presented in Fig. 2 by instantaneous (short-exposure) laser beam intensity distributions \( I(r,z) \) corresponding to a single turbulence realization. As expected, due to the presence of standard (MUSA76 model based) refractivity the beam footprint undergoes gradual vertical shift (bending) toward the ground with propagation distance increase, and approached the boundary of the numerical grid at the propagation path end (at \( L = 10 \, \text{km} \)).

Fig. 2. Short-exposure intensity distributions of a Gaussian beam along the propagation path in turbulent atmosphere in the presence of standard (MUSA76-type) refractivity at the distance \( z = 0 \) (left column), \( z = 5 \, \text{km} \) (middle column) and \( z = 10 \, \text{km} \) (right column). The top row images show intensity distributions inside the entire (~3.6 \times 3.6 \, \text{m}^2) computational area, the bottom row images present the same intensity distributions displayed inside 0.2 \times 0.2 \, \text{m}^2 squares centered relative to the corresponding beam centroids.
To maintain computational accuracy along the entire propagation path and prevent grid boundary induced errors due to beam footprint vertical shift, in the numerical integration of Eq. (2) we used a large size (4096 x 4096 pixels) grid that corresponded to a 3.584m x 3.584m square in the physical domain. Correspondingly, generation of phase screens and calculation of 2D Fourier transforms – a part of the split-step-operator technique - were also performed at 4096 pixel-wide area, resulting in extremely computationally expensive simulations.

The requirement for a larger size numerical grid that has the capacity to accommodate refractivity-induced laser beam trajectory bending, poses a major problem for the conventional wave-optics approach in analysis of laser beam propagation in presence of strong atmospheric refraction.

In the numerical simulation example in Fig. 2, an increase of the propagation distance from 10 km to 20 km required at least twice larger numerical grid to preserve a comparable simulation accuracy, and correspondingly significantly longer computational time.

In contrast, the large size grid was not required for numerical integration of the propagation Eq. (4) in the WORTEX model. A comparable simulation accuracy in laser beam parameters estimation was in this case achieved using an 8-fold smaller size numerical grid (512x512 pixels).

Since in the WORTEX technique the refractivity-induced beam footprint shift is accounted for in the modified ray tracing Eqs. (5), the grid size can be chosen based solely on the turbulence-induced beam widening but not on the beam centroid trajectory bending. As shown in the following section, this allows analysis of laser beam propagation over significantly longer distances and/or in the presence of strong refractivity such as induced by the stratified atmospheric layers.

To compare results obtained with the conventional wave-optics and WORTEX techniques the following two beam characteristics were computed using both methods: averaged over turbulence realizations (long-exposure) beam centroid displacement, and its standard deviation, referred to here as beam wander. Note that long-exposure beam centroid displacement primarily depends on refractivity, while beam wander is a commonly used characteristic of turbulence-induced effects.

The beam centroid displacement from optical axis ($oz$) can be defined as

$$\langle r(z) \rangle_{\text{turb}} = W^{-1} \left\langle \int r \left| A(r,z) \right|^2 d^2r \right\rangle_{\text{turb}}, \quad (12)$$

where $r(z) = \{x(z), y(z)\}$ is the instantaneous (short-exposure) beam centroid vector corresponding to a single turbulence realization, and $\{ \}_{\text{turb}}$ denotes averaging over an ensemble of $N_{\text{turb}}$ statistically independent turbulence realizations.

Note that in the described in section 3 atmospheric refractive index model [see Eqs. (8), (9)] the refractivity component varies solely in vertical in respect to the ground direction that, in our case, coincides with $oy$-axis. For this reason, only $y$-components $\{ y(z) \}$ of the centroid vectors $\{ r(z) \}$ in Eq. (12) were computed.

The obtained $N_{\text{turb}}$ values of the short-exposure beam centroid components $\{ y(z) \}$ were used to estimate both the long-exposure beam centroid vertical displacement $\Delta_y(z) \equiv \langle y(z) \rangle_{\text{turb}}$ and the standard deviation:

$$\sigma_y(z) = \sqrt{\left\langle \left( y(z) - \Delta_y(z) \right)^2 \right\rangle_{\text{turb}}} / \Delta_y(z). \quad (13)$$

In the case of the WORTEX based simulations, the corresponding to $\Delta_y(z)$ and $\sigma_y(z)$ beam trajectory characteristics, denoted as $\Delta_y(z) \equiv \langle y(z) \rangle_{\text{turb}}$ and $\sigma_y(z)$, were estimated...
using the \( y \)-components of the trajectory vectors \( \{ \mathbf{F} \} \) obtained by numerical integration of the modified ray tracing Eqs. (5).

We assumed here that the refractivity-induced trajectory slopes are relatively small and we can substitute the trajectory length \( l \) by distance \( z \). In the more general case one can use the dependence \( l(z) \) that can be directly obtained from integration of the ray tracing Eqs. (5).

The long-exposure beam centroid displacement \( \Delta_y(z) \) and its standard deviation (beam wander) \( \sigma_y(z) \) computed using numerical integration of the parabolic Eq. (2) are compared in Fig. 3 with the corresponding trajectory characteristics \( \Delta_y(z) \) and \( \sigma_y(z) \) obtained based on the WORTEX model.

The results show that the WORTEX model provides accurate estimation of both the refractivity-induced long-exposure beam centroid displacement and beam wander characteristics.

The numerical simulations also show that the instantaneous (short-exposure) beam centroid displacements \( \{ y_z(z) \} \) and the corresponding trajectory displacements \( \{ y_z(z) \} \), obtained using conventional wave-optics and WORTEX models are nearly coincided (with < 1\% error) for all turbulence realizations examined.

The results presented in this section demonstrate that the introduced beam propagation model can provide computationally efficient and accurate evaluation of laser beam characteristics in propagation scenarios where both atmospheric turbulence and refraction-induced effects play an important role in defining major laser beam parameters.

In the following section, we apply the WORTEX approach for analysis of laser beam characteristics along an extended-range atmospheric propagation path in the presence of both turbulence and strong refractivity effects.

5. Impact of a localized refractive layer on laser beam propagation characteristics

Consider propagation of a Gaussian collimated beam in turbulent atmosphere with the presence of a horizontally oriented inverse temperature layer (ITL). The beam parameters are similar to those in section 4 (radius \( a = 1.5 \) cm, wavelength \( \lambda = 0.53 \mu m \)), but propagation distance is extended from 10 km to 20 km. We assume that the laser beam is transmitted from an elevation \( h_0 = 25 \) m at a small angle \( 0 \leq \alpha \leq 3 \) mrad in respect to the horizon.

The ITL-induced refractivity is described by Eqs. (8)-(10). The ITL parameters selected in simulations, including height above the ground (\( h_{ITL} = 45 \) m), width (\( w_{ITL} = 4.0 \) m), and
temperature inversion (\( \Delta T = 5.0 \) K for ocean- and \( \Delta T = -5.0 \) K for desert-type ITLs) are within the commonly considered in literature range [13].

The turbulence-induced refractive index fluctuations are described by the Kolmogorov power spectrum with \( H V \cdot 5/7 \) altitude profile.

The numerical simulations were based on the WORTEX technique described in section 4. Numerical integration of the propagation Eq. (4) with the split-step-operator technique was performed at a 2048-pixel-wide square numerical grid with ~2 mm pixel size. The turbulence-induced phase aberrations were modelled using \( M = 40 \) equidistantly located thin phase screens.

A set of \( N_{\text{turb}} = 5000 \) laser beam propagation numerical simulation trials corresponding to statistically independent turbulence realizations were performed to compute the following laser beam characteristics:

(a) Instantaneous beam centroid trajectory components \{ \( x_T(z) \) \} and \{ \( y_T(z) \) \};

(b) Instantaneous beam centroid trajectory elevations (heights) above the ground \{ \( h_T(z) \) \}, obtained from \{ \( y_T(z) \) \} values with accounting for the Earth curvature (\( R_E = 6371 \) km);

(c) Ensemble-average (long-exposure) beam centroid trajectory displacements along horizontal \( \Delta_T^x(z) = \langle x_T(z) \rangle_{\text{turb}} \) and vertical \( \Delta_T^y(z) = \langle h_T(z) \rangle_{\text{turb}} \) axes;

(d) Instantaneous deviations of the beam centroid trajectories along horizontal \{ \( \delta_T^x(z) \) \} and vertical \{ \( \delta_T^y(z) \) \} axes, from the corresponding ensemble-average values \( \Delta_T^x(z) \) and \( \Delta_T^y(z) \);

(e) Standard deviations \( \sigma_T^x(z) \) and \( \sigma_T^y(z) \) of beam trajectory deviations referred to as beam centroid wanders in horizontal and vertical directions.

The numerical simulation results obtained for two laser beam transmission angles \( \alpha = 1.0 \) mrad and \( \alpha = 3.0 \) mrad are shown in Fig. 4 and Fig. 5 correspondingly for the ocean- and desert-type ITLs. Except the sign of the temperature inversion \( \Delta T = 5.0 \) K for the ocean- and \( -5.0 \) K for the desert-type ITL) all remaining ITL parameters are identical for both cases.

Consider first the impact of the ocean-type ITL. As seen in Fig. 4(a) the presence of an ITL causes bending of the long-exposure beam centroid trajectory toward the ground. Note, that in the considered propagation geometry, even a relatively small (2.0 mrad) deviation of the transmission angle \( \alpha \), triggered a significant change in the beam centroid trajectories shape resulting in approximately 37 m displacement of the beam footprints over 20 km distance [compare the long-exposure trajectories \( \Delta_T^x(z) \) in Fig. 4(a) corresponding to different \( \alpha \)].

At the transmission angle \( \alpha = 1.0 \) mrad, the ITL-induced bending effect resulted in a rapid decline of the trajectory slope with the propagation distance increase, leading to the slope angle sign reversal at distance ~16.5 km. This effect resembles laser beam “reflection” off ITL layer.

With the transmission angle increase to \( \alpha = 3.0 \) mrad, the trajectory geometry dramatically changes. In this case the beam centroid trajectory passes through the ITL effected zone with a relatively smooth slope angle decline inside it.

In the case of desert-type ITL in Fig. 5(a) the beam trajectories corresponding to both transmission angles cross the ITL affected region. The presence of the ITL causes in this case the trajectory slope angle increase – an opposite effect if compared with the ocean-type ITL in Fig. 4(a).
The presence of ITL also affects turbulence-induced laser beam statistical characteristics such as beam centroid fluctuations. Consider the sets of instantaneous trajectory deviation curves shown in Fig. 4(b) and Fig. 4(c). These curves correspond to the beam centroid vertical deviations \( \{ \delta^{(v)}_T(z) \} \) from the turbulence-averaged beam centroid trajectory obtained for different turbulence realizations. These curves represent examples of a few (20) trajectory deviations selected of the 5000 realizations used for computation of the long-exposure beam centroid trajectories shown in Fig. 4(a,d).

The results show that the instantaneous beam centroid deviation curves \( \{ \delta^{(v)}_T(z) \} \) can be strongly affected by presence of ITL: the beam centroid deviations are enhanced when the long-exposure beam trajectory enters the ITL affected region, and could also be reduced when the trajectory is trapped in close vicinity of the ITL center as in Fig. 4(b).

This behavior can be explained by the influence of ITL-induced and dependent on beam trajectory phase aberrations inside the propagation beam footprint. These aberrations are dominated by the wavefront vertical tilt at the ITL periphery, and by a cylindrical aberration (vertical astigmatism) within the ITL center area. The simulations show that, as expected, the presence of ITL doesn’t impact the beam centroid fluctuations in the horizontal direction (along x-axis).

The standard deviations of the beam centroid trajectory fluctuations (beam centroid wanders) in horizontal \( \sigma^{(x)}_T(z) \) and vertical \( \sigma^{(v)}_T(z) \) directions are shown in Fig. 4(d) as...
functions of distance \( z \) for both transmission angles examined. The numerical analysis shows that for both angles the beam centroid wanders in the horizontal direction practically coincide (< 1\% error), and for this reason, are presented in Fig. 4(d) by a single (dotted) curve \( \sigma_T^{(x)}(z) \).

The situation is different for the beam centroid vertical wander \( \sigma_T^{(h)}(z) \). The presented in Fig. 4(d) results show that presence of ITL in the vicinity of a laser beam propagation path can significantly affect vertical beam centroid wander \( \sigma_T^{(h)}(z) \). Comparison of the standard deviations in the horizontal \( \sigma_T^{(x)}(z) \) and vertical \( \sigma_T^{(h)}(z) \) directions for identical transmission angles in Fig. 4(d) indicates that ITL can cause strong beam centroid wander anisotropy. This anisotropy depends not only on ITL parameters, but also on the laser beam transmission angle \( \alpha \) and propagation distance \( z \) – compare \( \sigma_T^{(h)}(z) \) plots in Fig. 4(d) corresponding to different \( \alpha \).

In the numerical simulations we also observed the ITL-induced anisotropy in the long-exposure beam footprint. The circular-shape long-exposure beam footprint in absence of ITL becomes elliptical. For the propagation conditions corresponding to Fig. 4 the ratio \( R_{h/x} \) of the long-exposure beam width in the vertical \( h \) and horizontal \( x \) directions were correspondingly \( R_{h/x} = 0.88 \) for \( \alpha = 1.0 \) mrad and \( R_{h/x} = 1.1 \) for \( \alpha = 3.0 \) mrad. The anisotropy was also observed in focal spot centroid wander – a characteristic that is commonly used for atmospheric turbulence strength evaluation (\( C_n^2 \) measurements).

![Fig. 5. Impact of desert-type ITL on the Gaussian laser beam characteristics at distance \( z \) along the optical axis for the transmission angles \( \alpha = 1.0 \) mrad and \( \alpha = 3.0 \) mrad: (a) long-exposure beam centroid vertical displacement \( \Delta_T^{(h)}(z) = \langle h'(z) \rangle_T \); (b) and (c) are examples of instantaneous beam centroid trajectory deviations \( \delta_T^{(h)}(z) = h'(z) - \Delta_T^{(h)}(z) \) in vertical direction from the turbulence-averaged trajectory \( \Delta_T^{(h)}(z) \) for \( \alpha = 1.0 \) mrad (b), and \( \alpha = 3.0 \) mrad (c); and (d) standard deviations of beam centroid fluctuations along vertical \( \sigma_T^{(x)}(z) \) and horizontal \( \sigma_T^{(h)}(z) \) directions. The ITL height is indicated in (a) by dashed line. The standard deviations \( \sigma_T^{(h)}(z) \) for the transmission angles \( \alpha = 1.0 \) mrad and \( \alpha = 3.0 \) mrad are coincide and shown in (d) by a single dotted line.](image-url)
Appearance of anisotropy in laser beam statistical characteristics such as beam wander and long-exposure beam footprint is not a surprise. Located in the vicinity of laser beam propagation path ITL-induced refractivity creates highly spatially anisotropic low-order spatially distributed phase aberrations that affect the turbulence-induced beam centroid trajectory fluctuations and result in anisotropy of laser beam statistical characteristics in the orthogonal to ITL direction.

Quite similar behavior of the laser beam characteristics was also observed in analysis of laser beam propagation in presence of the desert-type ITL. The corresponding results obtained for identical propagation geometry and ITL parameters (except the sign of the temperature inversion) are shown in Fig. 5.

6. Conclusion

In this paper we introduced a physics-based WORTEX model for numerical analysis of laser beam propagation in atmosphere which accounts for the combined effects of turbulence and refractivity. The model accuracy was evaluated via direct comparison of numerical simulation results obtained using the conventional wave-optics and WORTEX methods for laser beam propagation over limited by 10 km a distance in atmosphere with refractive index field composed of Kolmogorov turbulence and standard (MUSA76) refractivity components.

This comparison demonstrated that the WORTEX model can provide accurate estimation of both short- and long-exposure beam centroid displacements and beam wander characteristics.

The WORTEX technique was further extended for analysis of laser beam propagation in turbulent atmosphere over extended-range distances (up to 20 km) in the presence of strong refractivity structures which are originated from elevated above the ground inverse temperature layers (ITLs).

The numerical simulations show that turbulence and refractivity effects on laser beam propagation can be strongly coupled. Presence of turbulence affects the refractivity-induced beam centroid trajectory bending. On the other hand, refractivity layers in the vicinity of laser beam propagation path could strongly impact the turbulence-induced laser beam statistical characteristics such as long-exposure beam footprint, focal spot and beam centroid wander resulting in these characteristics anisotropy in respect to the horizontal and vertical directions. This anisotropy depends not only on parameters of the refractivity layer, but also on the laser beam transmission angle and propagation distance.

Note that both beam and focal spot centroid wander are commonly utilized for turbulence strength characterization via $C_n^2$ measurements. The presented results illustrate that the presence of a spatially localized refractivity layer in the vicinity of the laser beam propagation path, could significantly affect such measurements. Without taking into account the potential impact of refractivity effects, the anisotropy observed in such measurements can be easily misinterpreted as atmospheric turbulence anisotropy and/or as deviation from the classical Kolmogorov atmospheric turbulence model (anisotropic and / or non-Kolmogorov turbulence).

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