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Research exercise: Mathematical Study of the Foot and Mouth Outbreak Model

Foot and Mouth Disease:
Mathematical Study of the Foot and Mouth Outbreak

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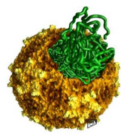
Abstract

The foot and mouth outbreak in the UK in 2001 was a disastrous event for the country and the economic. The disease did not only cost UK government so much to stop the disease, but it also affected the tourism industry. Mathematical epidemic models can provide clear strategy for minimizing the effect of such a disease, determining the expected manner of its progression in the event of a future outbreak based upon the latest available data on the epidemic. This project is to explore how to minimize the cost, how to contain the disease in minimal time, and how realistic these models will be considering the limitation of the model. Numerical and qualitative tools such as MATLAB's built in ode solver will be used.



History

Foot-and-mouth disease (FMD) is a severe, highly contagious viral disease of cattle and swine. It also affects sheep, goats, deer, and other cloven-hooved ruminants. The virus responsible for the disease is a picornavirus, (RNA-containing viruses of the family Picornaviridae, infectious to humans and other animals, and including the polioviris and the rhinoviruses that cause the common cold) the prototypic member of the genus Aphthovirus. (A genus in the family Picornaviridae that includes foot and mouth disease viruses and equine rhinitis A virus) . Infection occurs when the virus particle is taken into a cell of the host. The cell is then forced to manufacture thousands of copies of the virus, and eventually bursts, releasing the new particles in the blood. The virus is highly variable, which limits the effectiveness of vaccination. There are seven known types and more than 60 subtypes of the FMD virus. Immunity to one type does not protect an animal against other types.

The virus causes a high fever for two or three days, followed by blisters inside the mouth and on the feet that may rupture and cause lameness. Foot-and-mouth disease or hoof-and-mouth disease (Aphtae epizooticae) is an infectious and sometimes fatal viral disease that affects cloven-hoofed animals

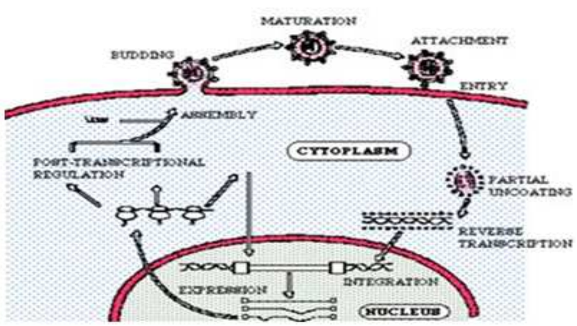
The outbreak can occur when,

- Animals carrying the virus are introduced into susceptible herds.
- Contaminated facilities and vehicle are used to hold or transport susceptible animals.
- Raw or improperly cooked garbage containing infected meat or animal products is fed to susceptible animals.
- Contaminated clothes or footwear, or using contaminated equipment, can pass the virus to susceptible animals.
- Susceptible animals are exposed to materials such as hay, feedstuffs, hides, or biologics contaminated with the virus.
- Susceptible animals drink common source contaminated water, and inseminated by semen from an infected animal.

Although, FMD is harmless to human, it can infect people. The virus can exist in a person's nose for up to 28 hours after exposure to infected animals and be spread through the air.

FMD is one of the most difficult animal infections to control, because FMD can be found in all secretions and excretions from acutely infected animals, including expired air, saliva, milk, urine, feces and semen. Pigs, in particular, produce large quantities of aerosolized virus. Animals can shed FMD for up to four days before the onset of symptoms. Also, the virus mutates so quickly, vaccination isn't always effective. This disease could spread rapidly to all sections of the country by routine livestock movements unless it was detected early and eradicated immediately. If FMD were to spread unchecked, the economic impact could reach billions of dollars in the first year. Deer and other susceptible wildlife populations could become infected and potentially serve as a source for reinfection of livestock.

In this project, I explored the Foot and Mouth disease research done by Trevor Wood, using numerical technique.



The Nondimensionalised form of SIR model

A simple model for an epidemic like Foot and Mouth is the S-I-R model. SIR Model is one of the most common and classic method for modeling the infectious diseases. This SIR model was created by Kermack and McKendrick in 1927. This model breaks down the population into 3 categories.

Susceptible	(S)	May catch the disease.
Infected	(I)	Have the disease, and may spread it to susceptible.
Removed	(R)	Have died or have recovered and are immune.
Reproduction	(R0)	The rate of the infection reproducing itself.
Beta	(β)	The constant rate of infection.
Gamma	(□)	The constant rate of removal.
a		The rate of the vaccination.
b		The rate of the culling of susceptible.
c		The rate of the culling of infectives.

The assumption made here is that there is a constant rate of infection. The susceptible are the animals which have no immunity to the disease in question and might be infected if they are exposed. Infectious are those animals which are currently infected with the disease and most likely transmit the disease to the susceptible population of the animals. Removed are those animals which are immune from the disease, recovered from the disease (The disease may not kill the adult animals), have the disease and are isolated, or died from the disease. The S + I + R represent the total population of the animals and it is equal to a constant N. We are going to explore how these categories change over time, for how fast can FMD may contained will decide how much the cost will be.

$$\frac{dS}{dt} = -SI - aSI - bI$$

$$\frac{dI}{dt} = SI - \frac{1}{R0}I - cI$$

$$\frac{dR}{dt} = \frac{1}{R0}I + cI$$

$$\frac{dV}{dt} = aSI$$

$$R0 = \frac{\beta S0}{\gamma}$$

Constants and Supporting Functions

N=100	S(0)=99
a= varies	I(0)=1
b=varies	R(0)=0
c=varies	R0=3
S(t)+I(t)+R(t)=N	

$$S(t+\Delta t) = S(t) + \Delta t(t(i), S(t))$$

$$I(t+\Delta t) = I(t) + \Delta t(t(i), I(t))$$

$$R(t+\Delta t) = S(t) - \Delta t(t(i), R(t))$$

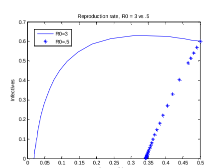
$$V(t+\Delta t) = S(t) - \Delta t(t(i), V(t))$$

Results

R0 3 vs .5

R0 is a reproduction rate of the virus. Naturally, if the reproduction rate is bigger than 1, then there will be epidemic

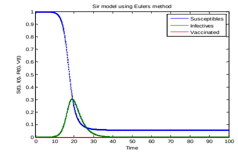
If R0<1, then there will be no epidemic.
If R0>1, then there will be epidemic.



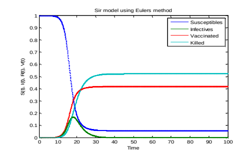
The following graphs show the inter action between the virus and vaccination, and culling.

The figure 1 of each set shows the graph when no culling or vaccination is used.
The figure 2 is when high level of vaccination is used.
The figure 3 is when high culling of susceptible and infected is used.
The figure 4 shows the use of vaccination and culling.

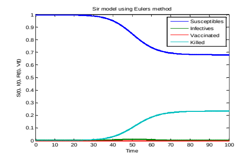
Euler's method



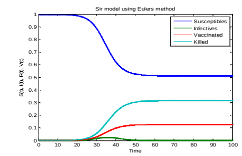
a=0 b=0 c=0



a=0.8 b=0 c=0

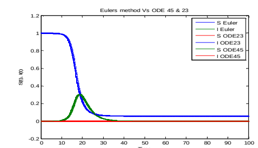


a=0 b=0.3 c=0.5

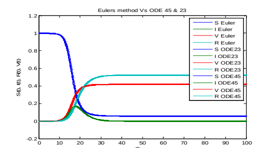


a=0.4 b=0.1 c=0.4

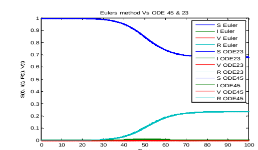
Euler's method and ode45 and ode23



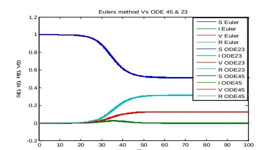
a=0 b=0 c=0



a=0.8 b=0 c=0



a=0 b=0.3 c=0.5



a=0.4 b=0.1 c=0.4

Conclusion

There are some limitations to this model. There is incubation period when the animals can infect others, but show no symptom, it is impossible to cull at the desired speed and it is impossible to know the number of the infected animals at any given moment. These graphs showed that culling and vaccination leaves the greatest number of the susceptible animals. Also, this model assumes the perfect mixing of susceptible and infected animals but in reality, this is not the case. The mathematical model used does not take into account the age structure (homogeneous population) and geographical distributions. These graphs shows the differences in high vaccination, high culling, and using both methods. It is apparent that some amount of vaccination before the outbreak may help to prevent the epidemic, and if the epidemic is current, the most effective way to contain the epidemic will be the swift culling of infected animals. The vaccination of non-infected animals during the epidemic is current, will not be as effective. These models neglect factors that may be important to the way epidemic spreads. Euler's method is a first-order numerical procedure for solving ordinary differential equations with a given initial value. It serves as the basis to construct more accurate numerical techniques. It also suffers from stability problems. For this reason, higher order method should be used to solve the system of ordinary differential equations. A comparison is made between approximate solutions obtained using Euler's and Runge Kutta methods.

References and Acknowledgments

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