Development of a Self-Orienting CubeSat Solar Array

Eric McGill  
University of Dayton

Follow this and additional works at: https://ecommons.udayton.edu/uhp_theses  
Part of the Aerospace Engineering Commons, and the Mechanical Engineering Commons

eCommons Citation
https://ecommons.udayton.edu/uhp_theses/173

This Honors Thesis is brought to you for free and open access by the University Honors Program at eCommons. It has been accepted for inclusion in Honors Theses by an authorized administrator of eCommons. For more information, please contact frice1@udayton.edu, mschlangen1@udayton.edu.
Development of a Self-Orienting CubeSat Solar Array

Honors Thesis
Eric McGill
Department: Mechanical and Aerospace Engineering
Advisors: Andrew Murray, Ph.D.
David Myszka, Ph.D.
May 2018
Development of a Self-Orienting CubeSat Solar Array

Honors Thesis
Eric McGill
Department: Mechanical and Aerospace Engineering
Advisors: Andrew Murray, Ph.D.
David Myszka, Ph.D.
May 2018

Abstract
The sponsor of this conceptual design project was the Air Force Institute of Technology (AFIT) at Wright-Patterson Air Force Base in Dayton, Ohio. AFIT was striving to give CubeSats more capability to conduct research, reconnaissance, and other functions. One of the major barriers for AFIT to overcome to give CubeSats more capability was the ability of the CubeSat to generate usable power while in orbit. All of AFIT’s CubeSats generated the power needed while in orbit with solar panels that are rigidly mounted to the outside of the craft. AFIT believes that a new design for the solar array used on the CubeSat will generate the power needed to increase their capabilities. The design that was deemed the most appropriate at the conclusion of this stage of the project was a design for a two degree of freedom mechanism that is attached to the solar panels to better orient them towards the sun. There are three aspects of the new design coming from this project that will make it unique. 1) Draws no direct power from the CubeSat Energy Storage to perform the movement. 2) Takes up less space on the CubeSat than competing designs. 3) Takes up less of the weight limit of the CubeSat than competing designs. The new Solar Array design should be able to orient four times more solar panel area towards the sun, as compared to the current AFIT design. There will be between 3 to 4 times more energy generation from the new design of solar array as a result, and an increase in the capabilities of the CubeSats.

Acknowledgements
A special thanks to the staff of the Air Force Institute of Technology and Faculty members of the University of Dayton department of Mechanical & Aerospace Engineering.
# Table of Contents

Abstract

1 Introduction Title Page
2 Project Description 1
3 Research 4
4 Orienting Mechanism Conceptual Designs 6
5 Actuation Method 16
6 Final Design 23
7 Discussion 26
8 Recommended Future Work 31
Bibliography 35
Appendix 1 36
Appendix 2 39
1 Introduction

In February 2018, National Geographic ran a story involving the San Francisco based company Planet Labs Inc. [1]. The mission of Planet Labs is to photograph the entire surface of the Earth on a daily basis, Figure 1 shows an example image of San Francisco taken by Planet Labs. To accomplish this feat, Planet Labs owns and operates 250 satellites.

The satellites used by Planet Labs Inc. is a family of small satellites known as CubeSats. The design of CubeSats was developed in late 1990’s by a duo of researchers at the California Polytechnic State University [2]. The purpose of the CubeSat design is to provide a standard vehicle for low Earth orbit satellites with a short service life (approximately two years), low weight (maximum of 4 kg for a 3U CubeSat), a specific mission, and a standard size for ease of deployment. CubeSats have allowed for the entry of small organizations into the realm of space, for the rapid advancement of technology for satellites (as the two year life-span allows for rapid replacement of obsolete technology), and for a greater understanding of the world’s effect on humanity and humanity’s effect on the world.

![Image](image_url)

Figure 1. San Francisco, photo taken from a CubeSat. Courtesy Planet Labs Inc. [3]

One organization which utilizes CubeSats is the Air Force Institute of Technology (AFIT) at Wright Patterson Air Force Base in Dayton, Ohio. AFIT challenged the
University of Dayton’s Design of Innovative Machines Laboratory (DIMLab) to synthesize an original design for a CubeSat solar array which could generate more power by self-orienting to face the sun while in orbit. The following thesis is the final product of two years of research to further define this problem and to propose a solution to the problem.

1.1 CubeSat Design

CubeSats are a family of small satellites which are made up of units. Each unit consists of a cube with sides 10 cm in length. There are multiple CubeSat sizes, ranging from a 1 Unit CubeSat which is a single 10 cm cube, to much larger satellites which are 27 unit or greater. The cube frame of the CubeSat is known as the chassis. The dimensions of 3U CubeSat are shown in Figure 2.

![Figure 2: Dimensions (mm) of a 3 unit (3U) CubeSat [2]](image)

Within the CubeSat Chassis is internal space for the placement of equipment that allows the CubeSats to accomplish their mission. Sensors, batteries, circuit boards, and other components fill this space.

In regards to the sensors, many CubeSats will have sensors that are designed to be trained towards Earth at all times. The Dove Satellite, of Planet Labs Inc. is an example of the set up. An image of the Dove Satellite is shown in Figure 3. The camera of the CubeSat is constantly oriented towards Earth.
1.1.1 CubeSat Deployment

CubeSats must be storable within a fixed volume to allow for storage during launch and for ease of deployment when in space. The container shown in Figure 4 is placed within launch vehicles as secondary or tertiary payload during launches of larger spacecraft. A CubeSat is then placed within the container with the CubeSat rails ensuring a firm fit between the deployment container and the CubeSat chassis.

1.1.2 Attitude Control Systems

Satellites that must maintain an orientation will be subjected to forces that will cause them to fall out of proper orientation. In order for satellites to maintain the proper orientation which allows them to complete their mission, methods and devices for controlling attitude were developed. Attitude is the term used to describe the spatial orientation of satellites. There are two common forms for attitude control systems. Both systems are demonstrated in Figure 5, shown in their proper placement within a 1U
CubeSat chassis. The attitude control mechanisms both work with internal algorithms to recognize and correct differences between expected attitude and actual attitude.

The first device, is the method known as reaction wheels. Reaction wheels utilize the concept of conservation of angular momentum in order to provide a reaction moment to orient the CubeSat. Using three reaction wheels, oriented in each orthogonal direction as shown in Figure 5, to generate a reaction torque to offset unwanted displacements or angular velocities while the CubeSat is in orbit.

The second device, magnetorquers, exerts an electro-magnetic torque against the Earth’s magnetic field to create a reaction moment which reorients the CubeSat. Like the reaction wheels, there are three magnetorquers in each CubeSat, aligned in each orthogonal direction. By aligning reaction torque devices in all three orthogonal directions, misalignment about any axis can be corrected.

2 Project Description

The project discussed in this thesis was initiated by the Air Force Institute of Technology (AFIT) at Wright Patterson Air Force Base in Dayton, OH. AFIT performed research with CubeSats, by sending CubeSats into low-Earth orbit to measure data, very similar to the method used by Planet Labs Inc. as described in the introduction. AFIT believed that some of their mission scopes were being constrained due to a lack of power generated by the Solar Array design which they are currently using. The current AFIT solar array design is shown on a 2U CubeSat in Figure 6. AFIT contacted the University of Dayton DIMLab to perform the design of the CubeSat Solar Array.
2.1 Design Requirements

1. The CubeSat Solar Array should exhibit the quality of tracking the sun. The analogy of a sunflower was used to describe this first design requirement.

2. The Solar Array should be storable in the launch container for a CubeSat shown in Figure 4, and therefore only occupy the space allotted to CubeSats by the California Polytechnic Design Standards.

3. Materials which respond to the effects of sunlight should be used in the actuation of the design.

4. The actuation should be performed in a passive manner, with no internal computerization used to control the position of the solar array.

5. The Solar Array should not interfere with the sensors on the CubeSat. Sensors are located at the end of a CubeSat, as shown by the camera placement in Figure 3.

6. The Solar Array should only be designed for a 3U CubeSat.

7. The Solar Array does not need to counteract the effect on the CubeSat created by its own motion. The counter action will be handled by the CubeSat’s Attitude Control System, which is shown in Figure 5.

8. The number of degrees of freedom should be kept to a minimum, in accordance with Kota and Erdman [5].

9. The Design will be judged to be a success or failure based on Size, Weight, and Power (SWaP). The design will be compared by the amount of extra power generated over a competing design, the amount of Weight the device adds to the system, and the amount of space the Solar Array occupies within the CubeSat.
3 Research

Two areas researched for the project were the state of the art for CubeSat solar arrays and smart materials.

3.1 State of the Art of CubeSat Solar Array

There were many designs for the solar array of CubeSats used in various capacities. Several designs were reviewed to gain knowledge and insight into possible new designs to begin the project.

3.1.1 Static Solar Panels

The design used by AFIT is for the solar panels of the CubeSat to be rigidly attached to outside surfaces of the CubeSat. Solar panels which are rigidly attached to the chassis of the CubeSat, as shown in Figure 6, offer many benefits. First, the simplicity of the design ensures that less can go wrong when the CubeSat is in service. Second, the placement of Solar panels on all of the exterior faces of the CubeSat ensures that there is at least one face pointing towards the sun, and therefore generating power, whenever the CubeSat is exposed to incident solar radiation. Finally, the chassis of CubeSats are already designed to accept these solar panels, and therefore they can be purchased from the same vendors as the chassis [6].

![Figure 6. 2U CubeSat with Rigidly Attached Solar Panels [7]](image)

The disadvantage of the rigidly attached solar panel design is the lack of capability to move the Solar Arrays to an orientation which is better for generating solar energy. As a result, the solar panels are not effectively utilized. Weight is a significant consideration
when launching objects into space. A design of solar array which can generate more power for the weight and size which it occupies would be highly sought after.

### 3.1.2 Static after Non-Reversible Deployment Solar Array

Rather than having rigidly attached solar panels, as shown in Figure 6, there are some CubeSats which have Solar Arrays deploy to a static configuration, as shown in Figure 3. The first advantage to this design is the ability to store more solar panels within the launch container (shown in Figure 4) than the rigidly attached solar panel configuration shown in Figure 6. More solar panels allow for greater generation of solar energy when the CubeSat is oriented in a way that exposes the surface area to the sun. The non-reversible deployment concept demonstrated by the Planet Labs Inc. Dove Satellite in Figure 3 is a commonly used design methodology for CubeSats and spacecraft in general.

The disadvantage of static non-reversible design is the inability of the Solar Panels to orient independent of the CubeSat. For example, the Dove CubeSat in Figure 3 must always be pointing towards Earth to photograph the surface of the Earth. The Dove CubeSat in Figure 3 will always orbit the Earth perpendicular to the vector of incoming solar radiation [8]. Therefore, the only time when the solar panels will be in the orientation to maximize solar power generation will be when they are over Earth’s poles. To maximize the effect of the solar panels shown in Figure 3, the orientation of the CubeSat must be tuned to allow for maximum power generation.

### 3.1.3 Single-Axis Orientable Solar Arrays

A design very similar to the design demonstrated in Figure 3 was utilized on the launch of Mars Cube One (MarCO) in 2018 [9]. The design demonstrated in Figure 7 is named the High Watts per Kilogram (HaWK) [10] solar array, and it is produced by MMA Design LLC out of Boulder Colorado [11].
Figure 7. HaWK Solar Array Deployment [11]

After the deployment, shown in Figure 7, the solar arrays are able to rotate about their common center axis to allow solar array orientation. Rotation about two orthogonal axes is needed to provide the solar array the ability to orient towards the direction of incident solar radiation. One of the axes is provided by the rotation of solar panels. The other, orthogonal axis can be provided by the rotation about the CubeSat’s long axis by using the systems discussed in Section 1.1.2 Attitude Control Systems.

The HaWK Solar Array design does not fulfill the design requirements set by AFIT, specifically design requirement #4 in Section 2.1 Design Requirements. Therefore, this design was not an adequate answer to the project proposal. However, this design was found to be growing in acceptance, as missions to Mars by the MarCO CubeSats featured the HaWK solar array [9], and the single axis orientable solar array design should be considered along with the final design proposed in Section 6 Final Design.

3.1.4 Multi-degree of Freedom Spatial Mechanism Solar Arrays

Another design for a CubeSat solar array has been proposed by Tethers Unlimited, a company out of Bothell, Washington. The design features solar panels which deploy in a manner very similar to the design shown in Figure 7. The solar panels then are orientated by the use of a spatial mechanism.
The mechanism shown in Figure 9 is placed under the solar array shown in Figure 8. The spatial mechanism demonstrated in Figure 8 is a three legged Universal-Revolute-Universal (3 U-R-U) spatial mechanism. The mechanism has three degrees of freedom, and therefore needs three actuators to completely constrain the motion of the device [14]. The three degrees of freedom necessary for this design made it more complex than the two degrees of freedom needed to point the CubeSat in any direction in three dimensional space. The design occupied 70 millimeters of the CubeSat’s internal space when stowed, which was considered excessive [12].
3.1.5 Design of Solar Arrays on the International Space Station

Although not directly related to the project, an investigation into the aiming of solar arrays on larger satellites was conducted. An article by Jeremy Frank states that the solar panels of the International Space Station have two orthogonal axes of rotation. The SARJ, denoted below in Figure 10, is an abbreviation for the Solar Alpha Rotary Joint [15]. The SARJ rotates the entire structure which contains all of the solar arrays shown in Figure 10, providing the first orthogonal axis of rotation to enable three dimensional aiming. The second orthogonal axis of rotation is provided by the BGA, the abbreviation for Beta Gimbal Assembly. The BGA system provided the rotation of each of the eight individual solar arrays [15].

![Figure 10. Visualization of the Movement of the International Space Station solar arrays](image)

Only two degrees of freedom, provided by two orthogonal axes of rotation, are necessary for orienting the solar arrays of the International Space Station. Therefore, the smaller CubeSats should also only need two degrees of freedom. These two degrees of freedom should be provided by orthogonal axes of rotation to give the full range of motion needed to orient the solar arrays toward the direction of incident sunlight.

3.2 Materials Which Enable Self-Orientation

To fulfill design requirement #3 in Section 2.1 Design Requirements, research into sunlight responsive materials needed to occur. The general scope of smart materials was reviewed. “A smart material can sense and respond to an external stimulus such as a change
in temperature, the application of stress, or a change in humidity or chemical environment. Usually a smart material-based system consists of sensors and actuators that read changes and initiate actions.” [16] For an actuator which was to be initiated by a change in the sun’s position relative to the CubeSat, the two most likely stimuli were considered to be a change in temperature and a change in incident light. Two families of smart materials were subjected to further investigation, light activated smart material polymers (LASMPs) and smart metal alloys (SMAs). The desired response to external stimuli for the final choice for a smart material would be a change in length.

3.2.1 Light Activated Smart Material Polymers (LASMPs)

Research into smart materials began with the prime consideration given towards materials that respond to light as the external stimulus. A class of smart materials, called light activated smart material polymers was discovered. The behavior of these smart materials is detailed by Hamel et al [17]. The materials have a different bond structure depending on the wavelength of light which is striking the material. Figure 11 demonstrates the process known as photo-induced bonding and cleaving [17]. To activate and to disengage one of the chemical bonding states, light of a uniform wavelength would have to be incident upon the material. The symbol $\lambda_c$ stands for the critical wavelength of light, the threshold wavelength for engaging or disengaging the molecular structure [17].

![Figure 11. Example of Light Induced Bonding [17]](image)

The large scale implications of the molecular changes shown in Figure 11 can be seen in Figure 12. Figure 12 details the method for fixing the chemical bonding structure to create the shape memory effect in the polymers [17]. The Shape memory effect is used to create the responsive movement needed to create a light sensitive actuator.
Ultimately, LASMPs were not identified as a viable method of actuation for the self-orienting CubeSat application for the following three reasons:

1. The light incident had to be of a uniform wavelength and high intensity. Requiring a method for filtering out all other wavelengths in the solar spectrum, and then intensifying the remaining light.

2. The smart material must be subjected to another incident light wavelength to return to its initial material properties. Therefore, another method of illuminating the smart material, when it is in the shade would have to be developed.

3. Information by AFIT Staff stated that plastics that have strong bonds, such as Teflon and a few others, are the only space worthy plastics. Plastics with weak molecular bonding structures deteriorate in the environment of space. The smart material properties of light activated polymers come from a change in molecular bonding from one state to another, and therefore, the material needs to have weak bonds at some point in the materials life.

Reasons 1 and 2 would make for a very complicated actuation system, and the 3rd reason meant that it was very likely the LASMP would rapidly degrade when exposed to the environment of outer space. Research for a practical actuation system design which utilized LASMPs yielded no such system.

3.2.2 Smart Metal Alloys (SMAs)

An alloy of nickel and titanium called nitinol, is a common smart material used in the aerospace industry as an actuator. The shape-memory effect property of Nitinol gives
the capability of being used as an actuator. “The shape memory effect is a unique property possessed by some alloys that undergo the martensitic reaction. These alloys can be processed using a sophisticated thermomechanical treatment to produce a martensitic structure.” [16] The martensitic structure is the crystal structure that the nitinol manifests at low temperatures. The martensitic structure is soft and pliable, with a needle shaped crystal structure that can easily be reshaped due to applied forces, as shown in Figure 13.

Figure 13. The Phase Transformation of Nitinol [18]

Nitinol undergoes a phase change when it is heated above its transition temperature. The transition temperature of nitinol is heavily influenced by its chemical content, ranging from 0° Celsius to 115° Celsius [19]. When the nitinol is heated above the phase transition temperature, the crystal structure transforms from the needle-shaped martensite crystal structure to the cubic shaped austenite. The shape memory effect of nitinol is manifest from this phase transformation, the nitinol returns to the crystal structure which has been programmed into its austenite phase, no matter how it is deformed in the martensite phase. The shape change caused by the shape memory effect can take the form of extension, contraction, the rotation of a leaf spring, or the return to an arbitrary shape.

Heating the nitinol actuators can be accomplished in several different ways. One way is to expose the nitinol to ambient temperatures that are above the transition temperature of nitinol. Heat transfer into the nitinol raises its internal temperature and causes the phase change in the nitinol. Another way is by passing an electric current directly through the nitinol actuator. Nitinol wires of differing sizes have different resistances, ranging from 36.2 ohms per inch for 0.001 inch diameter wire, down to 0.11 ohms per inch for 0.020 inch diameter wire [20]. Passing electric current through the resistive wire generates heat which increases the temperature of the wire above the transition temperature and initiates actuation.
Different actuator configurations exist for nitinol. These actuators can be in the shape of tension springs that pull when heated, compression springs that push when heated, and wires that contract and pull when heated. The wires that make up all of these actuator shapes can be of differing diameters, from 0.001 inches to 0.020 inches according to Dynalloy, a nitinol actuator supplier [20]. The actuating force is determined by the diameter of the wire, with the least force applied by the smallest diameter wire and the most force applied by the largest diameter wire [20].

Figure 14 below shows the stress strain curve for Nitinol, with the lower curve representing the behavior of Nitinol in the martensite phase, and the upper curve representing the behavior of Nitinol in the austenite phase. The various curves connecting the lower curve to the upper curve represent the unloading of superelastic nitinol, as nitinol with large strains, returns to no strain without plastic deformation. The yield point of the austenite phase was found to be the point where the bottom curve becomes horizontal. The horizontal region on the top curve does not represent yielding, but rather the superelastic behavior of nitinol in austenite phase.

![Stress-Strain Curve of Nitinol](image)

**Figure 14. Stress-Strain Curve of Nitinol [21]**

A major concern for the design of actuators made from nitinol is the fatigue of the nitinol material. The life of the Nitinol actuator depends on the strain amplitude of the nitinol during its loading and unloading [22]. Nitinol, deflected between 1% and 2% strain can undergo between 1000 and 10000 cycles before failure [22]. Any use of nitinol as a long-term actuator should take into account the fatigue of nitinol as it is cycled.
3.2.2.1 Nitinol Used as an Actuator on a Satellite Solar Array Mechanism

The use of Nitinol as an appropriate actuator for spacecraft and for the application of solar array orientation was confirmed by an article by Iwata et al. [23]. The Iwata et al. design featured a single axis of rotation which was controlled by 6 nitinol spring actuators, the top and side views of the actuator system are shown in Figure 15. The nitinol was heated by a concave mirror, which directed incident sunlight at the nitinol springs. The design reveals an innovative way to heat nitinol in an extraterrestrial application, and it also demonstrates a previous attempt at using nitinol in an application to track the sun in a passive way.

3.2.2.2 Nitinol for the Application of a Light-Seeking Glider

Nitinol actuators were used for a very lightweight application in the design of a microglider by Kovac et al. [24]. The microglider design used nitinol wires to actuate its rudder to guide the craft towards a light source to which the glider is flown close. The glider used active sensors which register information about the lights position, and control the electrical current through the Nitinol actuators. The glider is shown in Figure 16.

Figure 15. Iwata et al. Solar Tracking Array Mechanism Side View (Left) and Top view Reacting to Sunlight (Center and Right) [23]

Figure 16. The SMA Actuated Microglider Developed by Kovac et al. [24]
4 Orienting Mechanism Conceptual Designs

Many different design routes for the orientation strategy were developed before the final design was decided. Some of the conceptual designs presented were not adherent to all of the design requirements listed in Section 2.1 Design Requirements, and were not candidates for the final design.

4.1 Saturn Ring Design

The first design generated to create the ability to orient the CubeSat solar panels was named the Saturn ring design, due to its resemblance to the planet Saturn. As shown in Figure 17, the black ring would contain all of the solar panels needed for the CubeSat’s operation. The green cube is representative of a 1 unit CubeSat. The blue component is called a yoke, and both the grey parts attached to the yoke provide 360 degrees of motion about two orthogonal axes of rotation. The solar panels of the Saturn ring concept have the ability to perform complete spherical motion around the CubeSat, and align with the oncoming rays from the sun, wherever the sun was in space.

![Figure 17. The Saturn Ring Concept](image)

The Saturn ring design was formulated prior to a formal agreement on the design requirements. The design was not pursued further because it conflicted with several of the design requirements listed in Section 2.1 Design Requirements. Design requirement #2 was violated because the device, as shown, would not be storable within the launch container for a 3U CubeSat. The Saturn ring design, by being designed around a 1U rather than a 3U CubeSat also violated design requirement 6.

Though the design was not consistent with the design requirements, it did lead to a greater appreciation of a number of ways it is possible to cause an object to orient towards
the sun. The Saturn ring design and many other failed early designs led to a deeper appreciation of the design problem and helped to generate the design requirements listed in Section 2.1.

4.2 Serial Universal Joint Mechanism

The Saturn ring design was intended to give the CubeSat the ability to point anywhere in space, with all of its solar panels. The difficulties associated with storing the design inside the launch vessel brought about the need for a new concept that was collapsible to the necessary size, and also did not compromise on the ability to point its solar arrays at any direction in space.

A design resolving the need to adhere to design requirement #2 in Section 2.1 Design Requirements is called the Grashof Double Rocker design. The design features a Grashof double rocker mechanism, seated atop a single axis of rotation, which had the ability to rotate 180 degrees. The design has the ability to collapse to fit within the designated volume of the CubeSat launch container. Then once the CubeSat is deployed into orbit, the solar arrays would deploy away from the CubeSat and begin to orient toward the sun. The first axis of rotation was a fixed revolute joint at the attachment of the four bar mechanism and the CubeSat chassis. The second axis of rotation was the movement of the four bar mechanism, which could rotate over a 180° range of motion. The combination of these two axes and the solar arrays on the top and bottom of the CubeSat guaranteed at least half of the CubeSat solar array area would be facing the sun at all times.

![Figure 18. Grashof Double Rocker Design Stowed (left) and Deployed (right)](image)

The Grashof double rocker design opened the idea of using non-reversible degrees of freedom to deploy the CubeSat solar array once it arrived in orbit. In order to make the
four bar mechanisms store within the volume designated for the CubeSat, the mechanism had to fold flat. To accomplish this, the fixed points of the Grashof double rocker mechanism were mounted on a link with a non-reversible prismatic joint. The joint is non-reversible for the sake of stowing the CubeSat, after the CubeSat is deployed it may assume any size and shape that is possible. The stowed CubeSat configuration is shown in Figure 20. In Figure 19 and Figure 20, the blue link represents the stationary segment of the non-reversible prismatic and the orange links represent the segments of the non-reversible prismatic that are meant to travel toward the blue link during the deployment process. Once the Grashof double rocker mechanism is in the deployed state, as shown in Figure 20, the mechanism assumes one degree of freedom and the revolute joints on the orange link are fixed joints within the plane of the mechanism.

![Figure 19. Non-reversible Prismatic Joint extended and the Mechanism Laying Flat](image1)

![Figure 20. Non-Reversible Prismatic Joint Contracted and the Mechanism Deployed](image2)

The Grashof double rocker design incorporated a 1U CubeSat, as shown in Figure 18, and did not adhere to design requirement #6 in Section 2.1 Design Requirements. Design requirement #5 was clarified by the AFIT staff as a result of this design. The design should not obscure the sensors which perform the mission of the CubeSat. Having full spherical motion of the solar array was no longer considered as part of the project objective.
The sensors are generally trained towards Earth and only point from one end of the CubeSat, see Figure 3 for the Dove Satellite camera as an example of how other sensors were considered for the design. Therefore, the opposite end of the CubeSat would hold the Solar Array and only hemispherical range of motion would be needed to provide power to the solar array, because there is no way to gather sunlight while the CubeSat is on the night side of the Earth.

4.3 Parallel Universal Joint Design

The Grashof double rocker design was a type of Universal joint design, with two orthogonal axes of rotation. The first axis was provided by the motion of the Grashof double rocker mechanism, the other axis was provided by rotating the entire mechanism about the centerline of the CubeSat. The movement of the Grashof was made complex by the alignment of one of the orthogonal axes of the universal joint to cause full 360° rotation about the centerline of the CubeSat. A universal joint can cause hemispherical range of motion regardless of the two orthogonal axes chosen. The design of a universal joint structure that held both orthogonal axes parallel to the top of the CubeSat when the solar array is level relative to the top of the CubeSat was considered and ultimately decided to be the best option for continued design development.

Figure 21. The Kinematic Structure of the Parallel Universal Joint Design with Overlaid Coordinate System
The simplicity of the parallel universal joint design allows for the replacement of rotary actuators with linear actuators. Many smart materials were known to act as linear actuators [20], therefore the base design assumption for smart materials was that the actuation created by the smart materials would be a linear contraction. Therefore, the design shown in Figure 21 became the first concept generated which allowed for the use of linear actuators in a simple enough regime to warrant further investigation and development of the possibility of using smart materials as preferred by AFIT design requirement #3 in Section 2.1 Design Requirements.

### 4.4 Sarrus Linkage

The main issue presented by the parallel universal joint design shown in Figure 21 is the need to create clearance between the universal joint and the chassis of the CubeSat. If no clearance is created the universal joint will interfere with the chassis of the CubeSat, causing the entire mechanism to become seized. At the same time the solar array mechanism must be able to store within the volume of the CubeSat as stated by design requirement #2 in Section 2.1 Design Requirements.

The solution for providing the needed clearance was identified in the implementation of a Sarrus linkage as a non-reversible deployment mechanism, similar to the non-reversible prismatic joint presented in Figure 20. The Sarrus linkage is a straight-line mechanism which converts the rotational motion of the bottom links of the linkage into a vertical translation of the top of the Sarrus linkage. The overall motion is very similar to the action seen on the serial prismatic joints of a collapsible umbrella. The Sarrus linkage was chosen over a serial prismatic joint to maximize the use of horizontal space, and to minimize the use of vertical space within the CubeSat chassis.

The design called for a non-reversible Sarrus linkage, after the link is deployed it will never have to return to its original stowed configuration. The non-reversible deployment of the Sarrus linkage would best be actuated by torsion springs, or a method that generates a similar torque at one joint at each of the four legs of the Sarrus linkage. One possible alternate method was the use of compliant revolute joints with a designed torsion spring effect at each joint.
4.5 Universal Joint Arithmetic

The orientation of the Universal Joint can be modelled using 3-dimensional position vector arithmetic. The two inputs needed to solve for the two degrees of freedom are the two angular displacements of the solar panel from being parallel with the top of the CubeSat. The coordinate system used for this problem is shown as the overlaid coordinate system in Figure 21. The two angular displacements were designated \( \Psi \) (Psi) for rotation about the \( X_{\text{CubeSat Chassis (CC)}} \) and \( \theta \) (Theta) for rotation about the \( Y_{\text{CC}} \), both axes shown Figure 21. For the two parallel axes of the universal joint, the two rotational matrix for each axis is as follows:

For the axis which is in the \( X_{\text{CC}}-Z_{\text{CC}} \) plane shown in Figure 21, \( Y_{\text{CC}} \) is unaffected by any rotation about only this axis, the rotation matrix is:

\[
\begin{bmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{bmatrix}
\]  (1)

For the axis which lies in the \( Y_{\text{CC}}-Z_{\text{CC}} \) plane shown in Figure 21 of the universal joint reference frame, \( X_{\text{CC}} \) is unaffected by any rotation about only this axis, the rotation matrix is:
\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \Psi & -\sin \Psi \\
0 & \sin \Psi & \cos \Psi
\end{bmatrix}
\] (2)

When rotation occurs about both axes, a transformation matrix is used to define the new absolute position of relative points. The transformation matrix is created by multiplying the two rotational matrices together, with the axis which does not change its absolute position coming first in the multiplication:

\[
\begin{bmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \Psi & -\sin \Psi \\
0 & \sin \Psi & \cos \Psi
\end{bmatrix}
= \begin{bmatrix}
\cos \theta & \sin \theta \sin \Psi & \sin \theta \cos \Psi \\
0 & \cos \Psi & -\sin \Psi \\
-\sin \theta & \cos \theta \sin \Psi & \cos \theta \cos \Psi
\end{bmatrix}
\] (3)

When matrix (3) is multiplied by the \( X_{U-Joint}, Y_{U-Joint}, \) and \( Z_{U-Joint} \) coordinates of a point relative to the universal joint, the solution is the coordinates of the point relative to the CubeSat chassis:

\[
\begin{bmatrix}
\cos \theta & \sin \theta \sin \Psi & \sin \theta \cos \Psi \\
0 & \cos \Psi & -\sin \Psi \\
-\sin \theta & \cos \theta \sin \Psi & \cos \theta \cos \Psi
\end{bmatrix}
\begin{bmatrix}
X_{U-Joint} \\
Y_{U-Joint} \\
Z_{U-Joint}
\end{bmatrix}
= \begin{bmatrix}
X_{CC} \\
Y_{CC} \\
Z_{CC}
\end{bmatrix}
\] (4)

See Appendix 1: MATLAB Code for Universal Joint and Nitinol Actuator Vector Diagram
5 Actuation Method

The actuation system for the final design must be able to heat the nitinol actuators, which were chosen as a result of the research conducted in Section 3.2.2 Smart Metal Alloys (SMAs). The actuation system was designed as a set of two antagonistic pairings of nitinol springs. Several methods were considered for the heating of the nitinol, with the final design direction being the use of static, auxiliary solar panels.

5.1 Heat Due to Direct Solar Insolation

The idea of using a nitinol actuator with a low transition temperature that would be sufficiently heated by the directly radiant energy of the sun was one which was considered. The use of direct sunlight would require for the nitinol actuators to be placed on the outside of the CubeSat, near the end opposite the solar array. With the actuators at the far end of the CubeSat from the solar array, there is less likelihood that the solar panels will block the sunlight from reaching the nitinol actuators and causing the solar panel to return to its resting position.

The method was dismissed due to concerns that the heat of the CubeSat itself might reach a temperature where all of the actuators have undergone phase change into austenite phase. A more comprehensive thermal analysis may reveal the use of direct insolation to be a viable option.

5.2 Solar Intensifying Mirrors

The use of mirrors to reflect solar radiation into the nitinol such as the design demonstrated by Iwata et al. was considered. The mirror designs proposed would fold flat against the side of the CubeSat during launch, and then when the CubeSat is deployed, the mirrors would assume their final shape. The ability to morph solar collection mirrors to a shape which is most beneficial for power generation resulted in the consideration of collector shapes such as the one shown in Figure 23.

The shape in Figure 23 was a shape of solar collector which was described as not needing any form of solar tracking to ensure that all directions of incident light are reflected with high intensity upon the focal point of the shape. The solar mirror collector design was not pursued further due to the concerns of a complex, morphing kinematic structure adding
to the unreliability of the CubeSat actuation system. The addition of a morphing structure with many degrees of freedom would have also violated the design requirement #8 in Section 2.1 Design Requirements.

![Figure 23. Compound Parabola Solar Collector [25]](image)

5.3 Static Solar Panel

The use of the energy of solar radiation to heat the nitinol, whether by direct solar insolation or by radiation which was intensified by a mirror or lens, was considered a difficult method for nitinol actuation. The use of an electric current internal to the nitinol actuator was the method advocated by dynalloy to actuate nitinol wires and springs [20]. Two iterations of the concept of electric current actuation were derived, each drawing power from a different source. One drew power directly from the internal power supply of the CubeSat. The flow of current through a nitinol actuator was dependent upon a photoresistor on the opposite end of the CubeSat from the Solar panel end. If light was incident upon the photoresistor, a circuit would pass an electric current through the nitinol actuator, if the light did not shine on the photoresistor, the circuit would prohibit current from passing through the nitinol actuator.

The second concept did not directly withdraw any of the internal power supply for use in actuating the movement of the solar array mechanism. Instead the second concept used auxiliary solar panels, placed in the same location on the CubeSat as the photoresistors on the first concept. The auxiliary solar panels would stay fixed to the chassis as the rest of Solar panels would orient on the solar array mechanism towards the direction of
incident solar radiation. When light was incident upon the solar panel, a circuit would allow for the passing of a current through the nitinol actuators, thereby heating the nitinol actuator and causing the solar array to orient in the direction of the sun. The auxiliary solar panel design would be used for the final design.

5.4 Proof of Concept Model

The other key feature of the actuation system was the use of martensite phase nitinol springs as a reverse biasing element which would return the Solar Array to the neutral position. A proof of concept model was produced which tested the ability of a martensite phase nitinol spring to center bias a platform which was undergoing actuation by a pair of antagonistic Nitinol springs. The Proof of concept is shown in Figure 24.

Figure 24. The proof of concept model for an antagonistic pair of Nitinol Actuators

When an electric current is passed through the nitinol springs shown in Figure 24, the nitinol through which the current passed is transformed to austenite phase by temperature rise and the spring contracts, deflecting the top platform from horizontal. Numerous runs of the proof of concept demonstrated deformation in the nitinol spring which was extended by the actuation, resulting in a permanent angular deflection after the voltage is removed. The solar array must return to center to ensure full capture of the incident solar radiation by the solar array, when the sun is directly along the Z-axis in Figure 21.
6 Final Design

The final design combined all of the knowledge gained through design concepts and research into CubeSats, materials, and actuation techniques.

6.1 Deployment Stages

The deployment of the final CubeSat solar array design consists of 5 stages which are as follows:

Stage 1: The CubeSat is in stowed in the volume of a 3U CubeSat to be ready for launch. At this stage the entire CubeSat is to fit within the volume of the Deployment container shown in Figure 4. Figure 25 demonstrates the configuration of the CubeSat solar array when the CubeSat is stowed within its launch container.

![Figure 25. Stage 1: The Solar Array is Stowed for Launch](image)

Stage 2: After the CubeSat has exited the deployment container shown in Figure 4, it has entered low Earth orbit. Once in orbit, there is no restriction for the size of CubeSats. The CubeSat undergoes a series of non-reversible deployments. The first in the series of non-reversible deployments is the unfurling of the side solar panels to be in the same plane as the top solar panel. The auxiliary Solar Panels are revealed as a result. Figure 26 demonstrates the second deployment stage. The non-reversible motions performed can be accomplished by the use of torsional springs, and methods to seize moving structures until the time for deployment.
Figure 26. Stage 2: The Side Solar Panels Unfurl

Stage 3: Then to allow for the actuation of the Solar Array, the first Sarrus linkage undergoes a non-reversible deployment. The first Sarrus linkage deployment ensures that the Solar Array will not interfere with the CubeSat chassis. Figure 27 demonstrates deployment stage 3.

Figure 27. Stage 3: The First Sarrus linkage deployment

Stage 4: The Solar array also had an issue with interference with the universal joint. A second Sarrus linkage was used to create clearance between the universal joint and the solar array. Figure 28 demonstrates the fourth stage of deployment.

Figure 28. Stage 4: The Second Sarrus linkage deployment
After stage 4, the CubeSat is able to articulate about its universal joint and the nitinol actuators can perform their intended function. One example of the actuations which can be performed is shown in Figure 29.

![Figure 29. The Solar Array Free to Perform Intended Actuation](image)

### 6.2 Components of Mechanism

The complex movements needed to accomplish the deployment and articulation of the Self-orienting CubeSat solar array required the interaction of many mechanical components. These components, and the two main actions which the components perform, stowing and deploying, and shown in Figure 30 and Figure 31. In each figure, the solar panels were made transparent to allow for viewing of the mechanism contained beneath the array. Each of the components were described in greater detail in the following paragraphs.

The design of the solar array mechanism module was inspired by the tethers unlimited design listed in Section 3.1.4 Multi-degree of Freedom Spatial Mechanism Solar Arrays. The module can be seen in both Figure 30 and Figure 31. The module will attach rigidly to the chassis of the CubeSat and contain all of the components which are involved with the movement of the solar array (apart from the auxiliary solar panels.) Attachment points for the nitinol springs and the base of the first Sarrus linkage will be located on the module.

In stowed configuration, the entire volume of the mechanism will be contained within the solar array mechanism module, as is shown in Figure 30. The module will also
have additional space to store the nitinol spring actuators while stowed. Nitinol can be plastically deformed to fit within the space provided by the module. Then when the deployment of the mechanism occurs, the nitinol being in its compliant state, will be deformed into a new shape, which can then be actuated by the passing of current through the nitinol.

![Figure 30. Solar Array Mechanism when Stowed](image)

The design of the first Sarrus linkage was meant to take up a minimum amount of vertical and horizontal space within the module while stowed, (See Figure 30) and allow for maximum clearance between the solar array and the CubeSat after deployment. The Sarrus linkage formed an “X” shape within the module (See Figure 30) to allow for the links to be of the greatest possible length. The long links allow for the greatest amount of displacement between the CubeSat chassis and the solar array when deployed. (See Figure 31) The component which formed the top of the Sarrus linkage also contained the first axis of rotation of the universal joint. Possible methods of non-reversible actuation for the Sarrus linkage were discussed in Section 4.4 Sarrus Linkage.
The design of the universal joint contained only three components, the inner of these components also functioned as the top of the first Sarrus linkage. The inner universal joint component also held the first orthogonal axis of rotation. The middle universal joint component rotated about the first orthogonal axis and contained the second orthogonal axis. The outer universal joint component rotated about the second orthogonal axis and contained the base of the second Sarrus linkage. Without the second Sarrus linkage the Universal joint would interfere with the Solar Array, seizing all movement of the universal joint. The actuation of the solar array with antagonistic pairs of nitinol springs required a way to center bias the position of the solar array. (See Section 5.4 Proof of Concept Model) The use of torsion springs within the universal joint was determined to be one possible way of center biasing the Solar Array.

The second Sarrus linkage was originally not considered for the design, but the necessity of having a low-profile universal joint required the use of some mechanism for creating the clearance between the universal joint and the solar array. The second Sarrus linkage should undergo a non-reversible deployment from the stowed configuration shown in Figure 30 to the deployed configuration shown in Figure 31.
6.3 Actuation

The actuation of the design is provided by passing an electric current through antagonistic nitinol actuators. The electric current for the actuation would be provided by four static solar panels, called auxiliary solar panels, located on the end of the CubeSat opposite the solar array. When struck by solar radiation, an electric current would pass through the nitinol actuator on the same side of the CubeSat as the auxiliary panel which was exposed to the incident solar radiation. The nitinol would then contract and pull the solar array towards the direction of the incoming sunlight. Thereby achieving the passive sun tracking motion prescribed by design requirements #1 and #4. The auxiliary solar panels are demonstrated in Figure 29 as the solar panels on the opposite end of the CubeSat from the articulating solar array. The use of center biasing springs return the solar array to center after actuation was discussed in Section 6.2 Components of Mechanism.

7 Discussion

Two analyses were undertaken to determine the benefit and feasibility of the design. The size, weight, and power (SWaP) analysis is a standard for aerospace application equipment. A SWaP analysis will differentiate the Self Orienting Solar Array Mechanism from other, similar designs. A feasibility analysis was conducted on the nitinol actuator to determine if the actuation concept could be performed with the available energy.

7.1 SWaP Analysis and associated concerns

The size occupied by the Self Orienting Solar Array Mechanism was 17 mm of the total 300 mm length of the CubeSat chassis. In terms of volume, 170 cubic centimeters were sacrificed of the 3000 cubic centimeters available in the CubeSat, about 6% of the internal volume. The size of the mechanism is subject to change as the structural analyses to perform the final sizing of the mechanism components was not completed as part of the project. The components of the final design were miniscule, and it is uncertain if the components would withstand the structural demands placed on the mechanism during operation. The answer of how much size is taken up by the Mechanism is heavily dependent on the structural analysis of the components, which still needs to be undertaken.
Another factor which could negatively affect the size analysis result of the Self Orienting Solar Array Mechanism is the inability of the design to house an extra volume “Tuna Can” [2]. Figure 32 shows the dimensions of the CubeSat extra volume. The extra volume is a cylindrical housing which is able to expand the internal volume of the CubeSat, in order to house more equipment. Many 3U CubeSat applications have this feature applied to them, as can be seen on the right end of the Dove Satellite shown in Figure 3. The design of the Self-orienting Solar Array mechanism would not allow for the extra volume shown in Figure 32 and would therefore occupy more than the 6% of the internal volume stated in the size analysis.

Figure 32. Extra Volume "Tuna Can" optional feature of a CubeSat [2]

The weight of the Self Orienting Solar Array Mechanism was calculated to be 0.1 kilograms using the SolidWorks mass properties tool. Aluminum 7075-T6 material was assigned to all of the Components of the mechanism. The weight of the auxiliary solar panels and the wires associated with the implementation of the solar panel was assume to be 0.075 kilograms per solar panel, based on the mass of a 1 unit CubeSat solar panel [26]. For the four added auxiliary solar panels, the total added weight was assumed to be 0.3 kilograms. The 0.1 kilograms added by the mechanism and the 0.3 kilograms added by the solar panels resulted in a total added weight of 0.4 kilograms. The total weight allowance for a 3U CubeSat is 4 kilograms, or 1.33 kilograms per unit [2]. Before further structural analysis to refine the size and weight of the Self Orienting Solar Array Mechanism, the components added to the CubeSat would contribute 10% of the allowable mass of the CubeSat.
The amount of power added by the Self Orienting Solar Array Mechanism was estimated by the total area of solar panel which is pointed towards the sun compared to the rigid attachment of solar panels as shown in Section 3.1.1 Static Solar Panels. Based on images of two orientations the CubeSat could attain during orbit, an average power generation was estimated. The two orientations considered are shown in Figure 33, the top pair of images is representative of the CubeSat’s orientation over the Polar Regions or high temperate regions of Earth, and shows about two times more power generation from the self-orienting design. The bottom pair of images of Figure 33 is representative of the CubeSat’s orientation over the equator of Earth, and shows about 13 times more power generation from the self-orienting design. Figure 33 shows a greater percentage increase in the power harvested over the equator than over the Polar Regions due to the use of the Self Orienting Solar Array Mechanism. The average power benefit over the entire orbital period was estimated to be 3 times to 4 times for the Self Orienting Solar Array Mechanism over the rigidly attached solar array.

Figure 33. Comparison of Power Added by Self Orienting Solar Array Mechanism over the Rigidly Attached Solar Array for Two Orientations of CubeSats. The CubeSat over the Polar Regions of Earth (Top) and the CubeSat over the Equator (Bottom)

A better estimate of the benefit for the Self Orienting Solar Array Mechanism will require more complex analytical techniques which incorporate orbital mechanics, power
consumption by reaction wheels to counteract motion of Solar Array, and orientation of the CubeSat.

7.2 Base Nitinol Thermal Analysis and associated Concerns

The nitinol actuator was modeled as a resistor acting within an electrical circuit for the investigation of the actuator feasibility. The nitinol would generate heat, which would cause the nitinol to perform the transition from martensite to austenite phase necessary for actuation. The nitinol would lose heat due to the effects of conduction into the CubeSat chassis and also due to radiation away from the CubeSat into outer space. The effective heating of the nitinol would increase the nitinol’s temperature above its phase transition temperature. Heating would need to cease before the nitinol is above its shape memory setting temperature. The transition temperature of nitinol is subject to change based on the chemical content of the nitinol [19]. The shape memory setting temperature of the nitinol can be as low as 400 degrees Celsius [27]. If the nitinol is heated above the shape memory setting temperature for any amount of time, the ability of the nitinol to actuate will be compromised and the Self Orienting Solar Array Mechanism will become seized. The transition temperature of the nitinol chosen for the actuator sets the minimum temperature that must be reached in order to induce actuation by the nitinol. The shape memory setting temperature of the nitinol sets the upper limit for the heating of the nitinol.

An analysis of the heat transfer from a heated nitinol actuator was performed at steady state. Two conditions were considered, the heat transfer of a nitinol actuator due only to radiation and the heat transfer of a nitinol actuator due only to conduction into the CubeSat chassis. The calculations associated with this analysis can be found in Appendix 2.

The analysis concluded that the dominant mode of heat transfer for a large nitinol actuator would be conduction, and the dominant mode of heat transfer for a small nitinol actuator would be radiation. Radiation would be preferable to conduction heat loss due to the behavior of the nitinol actuators observed on the proof of concept model discussed in Section 5.4 Proof of Concept Model. The nitinol losing heat mainly due to conduction would revert to martensite phase close to the attachment points for the nitinol, leading to uneven deformation of the nitinol actuators. When the nitinol was exposed to convection,
a heat transfer mode that is uniform across the surface of the nitinol, the deformation after
return to martensite phase is much more uniform. Radiation is a form of heat transfer which
is uniform over the surface of the nitinol, therefore design efforts should be made to ensure
radiation is the dominant form of heat transfer on the design.

8 Recommended Future Work

For the continuation of design of the Self Orienting Solar Array Mechanism as it
stands now, the first step must be to create an accurate energy analysis of the nitinol heating
and cooling. From the analysis of the thermal behavior of the actuator, the size of nitinol
actuator can be determined. The size of nitinol actuator determines the dynamic effects on
the Self Orienting Solar Array Mechanism during its movement. The weightless
environment of low Earth orbit results in structural forces from the actuators and dynamic
effects from movement being the only considerations for structural design. A lifecycle
analysis of the nitinol should be performed as part of the structural and thermal design,
reference the fatigue data by Robertson et al. [22].

Also the design of actuation and seizing methods for the movements detailed in
Section 6.1 Deployment Stages should be performed as a next step. Designs featuring non-
reversible deployment motions, such as the Planet Labs Inc. Dove Satellite shown in Figure
3 should be considered in the research portion of the design task.

An appropriate design of the center biasing mechanism discussed in Section 6.2
Components of Mechanism should be undertaken. More information on designing spring
biased nitinol actuators can be found in Swensen and Dollar [18].

If a different route is sought other than this design, a good route to pursue would be
the use of piezoelectric actuators. A report on a gimbal produced for the purpose of using
piezoelectric actuators was written by Tschaggeny, Jones, and Bamberg [28]. Piezoelectric
actuators were also considered to have lower power requirements and greater force output
than nitinol actuators by the group designing the light seeking microglider [24].

The use of piezoelectric actuators may require the internal computation of the
CubeSat, leading to the redefining of several of the design requirements listed in Section
2.1 Design Requirements.
Bibliography


Appendix 1: MATLAB Code for Universal Joint and Nitinol Actuator Vector Diagram

clear; close all; clc;

% Script for the displacement of the Nitinol in CubeSat

% PICK TWO ANGLES; Called Theta and Psi

Theta = 0; % Degrees
Psi = 0; % Degrees

Origin = [0;0;0]; %mm % Absolute Origin at the point where the two U-
% joint Axes cross

% Points of Attachment on the Solar Panels, based on the solar Panel % Coordinate system
P_1_R = [25;0;0]; %mm % Point of attachment on the CubeSat Solar Array, % relative to fixed reference frame on the Solar Panels
P_2_R = [0;25;0]; %mm % Point of attachment on the CubeSat Solar Array, % relative to fixed reference frame on the Solar Panels
P_5_R = [-25;0;0]; %mm % Attachment point directly opposite P_1_R
P_6_R = [0;-25;0]; %mm % Attachment point directly opposite P_2_R

% Points of Attachment on the CubeSat Chassis, based on the Chassis % (Absolute) Coordinate System
P_3 = [20;0;-94]; %mm % Point of attachment on the CubeSat Chassis, % absolute... not relative
P_4 = [0;20;-94]; %mm % Point of attachment on the CubeSat Chassis, % Absolute... not relative
P_7 = [-20;0;-94]; %mm % Point of Attachment Directly Opposite to P_3
P_8 = [0;-20;-94]; %mm % Point of Attachment Directly Opposite to P_4

% To find the lengths of the Nitinol Actuators; the Points on the Solar % Array must be transformed to be in the Absolute Reference Frame

% The Matrix which the Points on the Solar Array must be multiplied % into
% is \([\cosd(\Theta) \ 0 \ \sin(\Theta); 0 \ 1 \ 0; \ -\sin(\Theta) \ 0 \ \cosd(\Theta)] \times [1 \ 0 \ 0; 0; 0 \ \cosd(\Psi) \ -\sin(\Psi); 0 \ \sin(\Psi) \ \cosd(\Psi)]\)

% The Resultant Transforming Matrix is
\[T = [\cosd(\Theta) \ \sin(\Theta) \ \sin(\Theta) \ \sin(\Psi); 0 \ \cosd(\Psi); \ -\sin(\Theta) \ \cosd(\Theta) \ \sin(\Psi)] \times [\cosd(\Theta) \ \cosd(\Psi) \ \sin(\Theta) \ \cosd(\Psi)]\]

% If both \(\Psi\) and \(\Theta\) are 0, \(T\) is a 3x3 identity matrix

% Use the matrix \(T\) to transform relative points \(P_{1\ R}, P_{2\ R}, P_{5\ R}, P_{6\ R}\) to absolute
% points
\[
P_{1\ R} = T \times P_{1\ R}; \quad \text{\% Absolute point for } P_{1\ R}
\]
\[
P_{2\ R} = T \times P_{2\ R}; \quad \text{\% Absolute point for } P_{2\ R}
\]
\[
P_{5\ R} = T \times P_{5\ R}; \quad \text{\% Absolute point for } P_{5\ R}
\]
\[
P_{6\ R} = T \times P_{6\ R}; \quad \text{\% Absolute point for } P_{6\ R}
\]

% Calculations for Nitinol
\[
L_{13} = P_{1} - P_{3}; \quad \text{\% Vector Notation for Nitinol}
\]
\[
L_{24} = P_{2} - P_{4}; \quad \text{\% Vector Notation for Nitinol}
\]
\[
L_{57} = P_{5} - P_{7}; \quad \text{\% Vector Notation for Nitinol}
\]
\[
L_{68} = P_{6} - P_{8}; \quad \text{\% Vector Notation for Nitinol}
\]

% Lengths of vectors
\[
L_{L_{13}} = \text{norm}(L_{13}) \quad \text{\% Length of vector } L_{13}
\]
\[
L_{L_{24}} = \text{norm}(L_{24}) \quad \text{\% Length of vector } L_{24}
\]
\[
L_{L_{57}} = \text{norm}(L_{57}) \quad \text{\% Length of vector } L_{57}
\]
\[
L_{L_{68}} = \text{norm}(L_{68}) \quad \text{\% Length of vector } L_{68}
\]

% Plotting
plot3([P_{1}(1) \ P_{3}(1)], [P_{1}(2) \ P_{3}(2)], [P_{1}(3) \ P_{3}(3)])
hold on; grid on; axis equal
plot3([P_{2}(1) \ P_{4}(1)], [P_{2}(2) \ P_{4}(2)], [P_{2}(3) \ P_{4}(3)])
plot3([P_{5}(1) \ P_{7}(1)], [P_{5}(2) \ P_{7}(2)], [P_{5}(3) \ P_{7}(3)])
plot3([P_{6}(1) \ P_{8}(1)], [P_{6}(2) \ P_{8}(2)], [P_{6}(3) \ P_{8}(3)])
plot3([P_{3}(1) \ P_{5}(1) \ P_{7}(1) \ P_{8}(1) \ P_{3}(1)], [P_{3}(2) \ P_{5}(2) \ P_{7}(2) \ P_{8}(2) \ P_{3}(2)], [P_{3}(3) \ P_{5}(3) \ P_{7}(3) \ P_{8}(3) \ P_{3}(3)])
plot3([P_{1}(1) \ P_{2}(1) \ P_{5}(1) \ P_{6}(1) \ P_{1}(1)], [P_{1}(2) \ P_{2}(2) \ P_{5}(2) \ P_{6}(2) \ P_{1}(2)], [P_{1}(3) \ P_{2}(3) \ P_{5}(3) \ P_{6}(3) \ P_{1}(3)])
plot3([Origin(1) \ P_{1}(1)], [Origin(2) \ P_{1}(2)], [Origin(3) \ P_{1}(3)])
plot3([Origin(1) \ P_{2}(1)], [Origin(2) \ P_{2}(2)], [Origin(3) \ P_{2}(3)])
plot3([Origin(1) \ P_{5}(1)], [Origin(2) \ P_{5}(2)], [Origin(3) \ P_{5}(3)])
plot3([Origin(1) \ P_{6}(1)], [Origin(2) \ P_{6}(2)], [Origin(3) \ P_{6}(3)])
plot3([Origin(1)], [Origin(2)], [Origin(3)], 'go', 'linewidth', 10)
Appendix 2: Thermal Analysis of Nitinol Actuator

Radiation:
Assume: The nitinol is a 15 inch long straight wire. The first diameter chosen for the nitinol’s wire diameter is 0.008 inches. The power generated from the 1U solar panel for the static solar array was determined to be 1.62 Watts, with the solar radiation being 1360 W/m², 80% of the solar panel is occupied by Solar cells which generate power, The solar panel has an efficiency of 25%, and the solar panel is offset from direct sunlight by 45° meaning only 70% of the total possible sunlight is striking the Solar Panel.
The nitinol dissipates all energy generated by the solar panel as heat to increase its temperature to steady state for actuation.
The temperature of the surroundings is 0K.

Find – the steady state temperate for radiative cooling of the nitinol wire.

Solution –
Governing Equations: First Law of Thermodynamics and the Stefan-Boltzmann Law

First law of thermodynamics:
\[ \dot{Q}_{in} = \dot{Q}_{out} \]

Stefan-Boltzmann Law:
\[ \dot{Q}_{out} = \sigma \varepsilon A (T^4_{\text{system}} - T^4_{\text{surroundings}}) \]
\[ \sigma = 5.67 \times 10^{-8} \frac{W}{m^2 \cdot K^4} \]
\[ \varepsilon = 0.8 \text{ (assumption)} \]
\[ A = 2.394 \times 10^{-4} m^2 \text{ (surface area of a 0.008 inch diameter cylinder)} \]
\[ T_{surroundings} = 0K \]

The calculated Temperature of the Nitinol (the system) was 621.3K or 348.2°C, which was thought to be too close to the Nitinol setting temperature.

The next calculation assumed a constant temperature of 400K, which is just over the threshold temperature for nitinol actuation. The goal was to find how large the diameter of the wire would be to effectively cool the nitinol by radiation given an assumed power input of 1.5 watts.

Solution –
\[ 1.5W = \left(5.67 \times 10^{-8} \frac{W}{m^2 \cdot K^4}\right) \times 0.8 \times A \times ((400K)^4) \]

Solving for the area, and then the diameter of the wire yielded a wire which is 0.043 inches in diameter, which is very thick and not commercially available.

Conclusion – For radiative only heat transfer the 1U solar panel may be generating too much power input.
Conduction:

The next case considered was heat loss by conduction:

\[ \dot{Q}_{out} = -kA \left( \frac{dT}{dx} \right) \]

\( k \), the conduction coefficient of nitinol was assumed to be 180 W/m-K

\( A \), the cross sectional area varied with the two wire diameters chosen for the analysis

\( \left( \frac{dT}{dx} \right) \), the temperature gradient was held constant at \( \left( \frac{400K}{0.254m} \right) \)
The nitinol would once again lose heat to 0K surroundings and the temperature of the nitinol would be heated to 400K for the actuator to function.

Solutions:
For the case of 0.043 inch diameter wire, the heat loss was found to be 2.645W.

For the case of 0.008 inch diameter wire, the heat loss was found to 0.089W.

Conclusion – there is an incredibly fine balance that nitinol size, power input, and mode of heat loss play in designing the actuator.