8-1995

Transient Wave Mixing and Recording Kinetics in Photorefractive Barium Titanate: a Nonlinear Coupled Mode Approach

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Transient wave mixing and recording kinetics in photorefractive barium titanate: a nonlinear coupled mode approach

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Abstract. By using rigorous coupled-wave diffraction theory along with a time-dependent nonlinear formulation, we analyze two- and multiple-wave coupling and the grating kinetics in BaTiO₃ with different boundary interfaces. Effects of electrostatic and optical anisotropy have been included in the analysis. Significant mode conversion to higher orders is observed only when the boundary interfaces are highly mismatched.

Subject terms: photorefractive nonlinear optics; diffraction grating; wave mixing.

Optical Engineering 34(8), 2254–2260 (August 1995).

1 Introduction

The nonlinear dynamical interaction of light with a photorefractive material can be effectively modeled by the Kukhtarev equations. Most theoretical analyses are performed using a linearized approximation, which is adequate when the modulation depth of the light intensity pattern is small, implying a large pump-to-probe intensity ratio. The linearized results predict that the two-beam coupling parameter is independent of the pump-to-probe intensity ratio. However, in many physical cases, this ratio between the two writing beams may not initially be very high, or may decrease during the interaction process in the photorefractive (PR) material due to exchange of energy from the pump to the probe. Experimental results in this regard have been reported by researchers for various PR materials, and an analytical theory, using an empirical form of the space-charge field in the PR material has been proposed and verified by Millerd et al. Ratnam and Banerjee have outlined a steady-state nonlinear theory of the two-beam coupling phenomenon, using participating plane waves, and have reconciled their theoretical results with numerical simulations of the coupling between two (in general) focused Gaussian beams. Similar steady-state solutions have also been obtained by numerical methods. However, the difficulty in providing an analytical nonlinear time-dependent theory stems from the fact that it is virtually impossible to decouple the Kukhtarev equations except in the steady state, and numerical simulations of these coupled equations are also rather formidable.

Recently, Serrano et al. have employed a numerical approach for simulating the recording and erasure kinetics in PR materials for high intensity modulation ratios. The method is based on a pair of coupled equations derived from the Kukhtarev equations under certain approximations and Fourier expansions of the electrostatic field and the free-carrier density. The formulation is thereafter applied to analyze two-wave and multiwave coupling in BSO. The signal-to-pump ratio dependence of buildup and decay rates during two-beam coupling in BaTiO₃ was experimentally and numerically investigated by Horowitz et al., but the effects from possible mismatch at the crystal boundaries were neglected. Furthermore, Brown and Valley have numerically shown the generation of higher diffraction orders during nonlinear two-beam coupling and the resulting profile of the space-charge field in BaTiO₃; however, this analysis is performed in the steady state. The nonlinear dynamics of multiple two-wave mixing and beam fanning in PR materials has been recently numerically analyzed by Snowbell et al.

In a recent study, we have employed rigorous coupled-mode diffraction theory to analyze two-wave and multiwave mixing in a diffusion-controlled photorefractive material that is modeled by the Kukhtarev equations. These equations are first decoupled under a set of approximations to yield a nonlinear partial differential equation for the induced refractive index profile. The transmitted- and reflected-field coupling coefficients are studied for the cases when the incident optical fields are coherent and partially coherent, for different input polarizations, for materials with different gain constants, and for different values of the linear refractive index mismatch. In each case, the exact longitudinal inhomogeneity in the PR material is analyzed using rigorous coupled mode diffraction theory.

In this paper, we exactly decouple the time-dependent Kukhtarev equations to derive a nonlinear partial differential equation for the space-charge field. The resulting equation is thereafter reduced to a more tractable equation under a set of approximations that show the nonlinear evolution of the space-charge field, its dependence on the intensity, and the nonlinear (intensity-dependent) time constant. We next pres-
ent a summary of the rigorous coupled mode approach and describe the numerical algorithm to analyze wave mixing in BaTiO$_3$. This is presented in Sec. 2. In Sec. 3, we show the time evolution of wave mixing when two plane waves of light are incident on a BaTiO$_3$ crystal. We consider three cases: when the crystal is embedded in a refractive index matching medium, the partially mismatched case, and finally when the crystal is in air. The grating profile in the bulk of the material is drawn for each case, and the transmitted and reflected fields are plotted, all for the case when the initial pump-to-probe intensity ratio is 100. Conditions necessary for appreciable generation of higher diffraction orders are described. The differences between using the exact intensity-dependent time constant and a time constant independent of the intensity is also discussed.

2 Theory

2.1 Model for the Photorefractive Material

The Kukhtarev equations are four equations that may be used to model a diffusion-dominated photorefractive material. They comprise the continuity equation, the rate equation, the current equation, and Poisson’s equation. We take the Kukhtarev equations given in Ref. 13, taking care to modify the second equation in accordance with Ref. 1, in order to include the contribution of the thermal excitation rate of charge carriers. Using the continuity equation, the current equation, and Poisson’s equation, it can be shown that the evolution of the space-charge field $E_s(x,y,t)$ satisfies the partial differential equation

$$\frac{\partial E_s}{\partial t} + \frac{e\mu}{\varepsilon} n E_s = - \frac{eD_s}{\varepsilon} \frac{\partial n}{\partial x},$$  \hspace{1cm} (1)

where $e$ is the electronic charge, $D_s$ is the diffusion constant, $\mu$ is the mobility, $\varepsilon_s$ is the effective static permittivity, and $n(x,y,t)$ is the electron concentration. In the above derivation, we have assumed that normal direction of propagation is along the y direction, $x$ is a transverse coordinate, and $t$ denotes time. Note that Eq. (1) shows that $E_s$ evolves according to a relaxation equation with a source term on the right-hand side, where both the relaxation coefficient and the source term depend on $n$. Furthermore, assuming that $N_A\gg (e/\mu)$ $\partial E_s/\partial x$, $N_D(\beta + sI)\gg (e/\mu)$ $\partial E_s/\partial x$ $\partial t$, and $N_D\gg N_D^+$, where $N_A$ is the donor concentration, $N_D$ is the acceptor concentration, $e$ is the electronic charge, $s$ is the ionization cross section, $\beta$ is the thermal excitation rate, and $N_D^+(x,y,t)$ is the ionized-donor density, the rate equation can be simplified to give

$$\frac{\partial E_s}{\partial t} = - \frac{eD_s}{\varepsilon} \frac{\partial n}{\partial x},$$ \hspace{1cm} (2)

Equation (2) suggests that the time constant

$$\tau = \tau(I) = \frac{e}{E_s} \frac{\gamma N_A}{\mu N_D} \frac{1}{\beta + sI}$$ \hspace{1cm} (3)

associated with the growth of the electrostatic field $E_s$, and hence the induced refractive index $\Delta n$ (or the induced permittivity $\Delta \varepsilon = n_0^2 n_s^2 r_{42} E_s$, as explicitly shown below), is inversely proportional to the intensity distribution $I$.

In this paper, we analyze the case of two incident TM waves on a BaTiO$_3$ crystal in which the $c$ axis is oriented at an angle $\theta_c$ to the $-y$ axis [see Fig. 1(a) for the geometry]. This may be accomplished by first resolving the space-charge field parallel and perpendicular to the $c$ axis, and thereafter expressing the resulting dielectric tensor in our chosen coordinate system. After algebra, it is found that the resulting dielectric tensor of the BaTiO$_3$ crystal is given by

$$\varepsilon_{xx} = n_{CO}^2 \cos^2 \theta_c + n_{CE}^2 \sin^2 \theta_c + \Delta \varepsilon(x,y,t) F_{xx},$$

$$\varepsilon_{xy} = (n_{CO}^2 - n_{CE}^2) \sin \theta_c \cos \theta_c + \Delta \varepsilon(x,y,t) F_{xy},$$

$$\varepsilon_{yy} = n_{CO}^2 \sin^2 \theta_c + n_{CE}^2 \cos^2 \theta_c + \Delta \varepsilon(x,y,t) F_{yy},$$ \hspace{1cm} (4)

where

$$F_{xx} = - F_{OE} \left( \frac{r_{13} n_{CO}^2}{r_{42} n_{CE}^2} \sin \theta_c \cos \theta_c + 2 \sin \theta_c \cos \theta_c \right),$$

$$F_{xy} = - F_{OE} \left( \frac{r_{13} n_{CO}^2}{r_{42} n_{CE}^2} \sin \theta_c \cos \theta_c - \cos \theta_c \cos 2\theta_c \right),$$

$$F_{yy} = - F_{OE} \left( \frac{r_{13} n_{CO}^2}{r_{42} n_{CE}^2} \sin \theta_c - 2 \sin \theta_c \cos \theta_c \right),$$

$$F_{OE} = n_{CO}^2 n_{CE}^2 \eta_{CO}^2 \eta_{CE}^2,$$

and

$$\eta_{CO} = n_{CO}^2,$$

$$\eta_{CE} = n_{CE}^2.$$ \hspace{1cm} (5)

The induced permittivity $\Delta \varepsilon$ obeys the differential equation

$$\frac{\partial \Delta \varepsilon}{\partial t} = - \frac{\partial I}{\partial \delta x}$$ \hspace{1cm} (6)

where $\delta = n_{CO}^2 n_{CE}^2 k_0 r_{42}$, and $\eta_{CO}$, $n_{CE}$ defined in Eq. (5), where $\eta_{CO}^2$ represent the lossy parts of the dielectric permittivity. The above equation therefore takes into account both the electrostatic and optical anisotropies in the material.

2.2 Rigorous Coupled Wave Diffraction Theory

In this subsection, we summarize the rigorous coupled wave diffraction theory (RCWDT)\cite{14-16} used to analyze wave mix
ing in a PR material. Details of RCWDT may also be found in Ref. 17.

The solutions of Maxwell’s equations in a diffraction grating (which is the core of RCWDT) when the grating is uniform is carried out by (1) expanding the unknown electric and magnetic fields \( \mathbf{E} \) and \( \mathbf{H} \) in this region in an infinite series of forward- and backward-traveling plane waves (Floquet harmonics) whose exponentials are multiplied by a set of spatially varying amplitudes whose value and form are unknown and are to be determined, (2) expanding the dielectric tensor elements \( (\varepsilon_{xx}, \varepsilon_{yy}, \ldots) \) of the diffraction grating in a Fourier series, (3) substituting the plane-wave expansion of \( \mathbf{E} \) and \( \mathbf{H} \) and the Fourier expansion of \( \varepsilon \) into Maxwell’s equations, and (4) from these substitutions, determining a set of coupled amplitude equations whose solutions determine the set of unknown plane-wave amplitudes that were introduced in step (1). The four equations that result from the last step determine the plane-wave amplitudes of the electric and magnetic field components that are parallel to the diffraction grating interfaces. The equations for the plane-wave amplitudes involve only zero- and first-order derivatives of the plane-wave amplitudes, and thus can be expressed in a state-variable form, which is well known in linear system theory. The numerical solution of these state-variable equations is carried out by determining the eigenvalues and eigenvectors of the state-variable matrix, and using these eigenvectors to express the electromagnetic (EM) eigenmodes of the diffraction grating.

Once the EM fields in all regions of the system are determined, the last major step in the overall analysis is to match the EM field solutions at the front and back interfaces of the diffraction grating with the EM fields that exist inside the grating. The EM fields on the incident side of the diffraction grating consist of the incident plane wave and an infinite sum of propagating and nonpropagating (evanescent) backward-traveling plane waves, which are the result of reflection and diffraction from the grating. The reflection coefficients of the reflected waves in the incident side are as yet undetermined. The EM fields inside the diffraction grating consist of an infinite sum of forward- and backward-propagating and evanescent waves, which are the result of transmission and reflection from the grating. By matching the EM boundary conditions at the diffraction-grating interfaces, the final equations that determine the unknown reflection coefficients in the incident side, the forward and backward eigenmode expansion coefficients in the diffraction grating, and the transmission coefficients in the transmitted region can be determined.

The previous paragraphs have described the diffraction analysis when a uniform diffraction grating is present. The treatment of a nonuniform grating (as encountered in the PR material) is identical to the one described above except that one uses a multilayer diffraction analysis rather than a single-layer analysis.

### 2.3 The Numerical Algorithm

We now describe an algorithm based on the RCWDT that can be used to determine mode coupling and diffraction when two TM polarized incident waves are incident on photorefractive BaTiO\(_3\) in the configuration described above. The temporal variation of the PR material is described by Eq. (4).

At \( t = 0 \), a probe \( (\theta = -\theta_i) \) and a pump wave \( (\theta = \theta_i) \) are incident on a dark, uniform BaTiO\(_3\) sample. The pump and probe waves interfere, creating a periodic intensity pattern in the PR material. This pattern in turn modifies the optical properties of the PR slab and produces a small periodic modulation \( \Delta \varepsilon \) in the permittivity. During the first time step, no diffraction has occurred, as the dielectric modulation was zero at the beginning of the time step. From the second time step, the \( \Delta \varepsilon \) generated by the optical interference of the first time step will begin to diffract the light. The algorithm calculates the total optical intensity in the medium by (1) calculating the EM fields and diffraction from \( \Delta \varepsilon \) in the PR slab from the pump wave by itself, (2) calculating the EM fields and diffraction from \( \Delta \varepsilon \) in the PR slab from the probe wave by itself, and (3) adding these two EM fields together to find the overall EM field in the PR medium and hence to determine the overall transmitted and reflected EM fields outside the PR material. The algorithm proceeds by substituting the total optical intensity derived above into Eq. (4) to find a new modulation \( \Delta \varepsilon \), which can be used for the next time step, and so on.

The RCWDT calculations for BaTiO\(_3\) were performed using 160 layers and an operating wavelength \( \lambda \) of 633 nm. The crystal length \( L \) was taken to be 1500\( \lambda \), and \( \theta_i = 45 \) deg. The angle of incidence of the two beams was chosen so as to ensure that the fundamental intensity grating spacing was \( \Lambda = 5\lambda \) in the crystal. The optical intensity was calculated at 10 equally spaced points over the layer length, and these values were averaged and substituted in Eq. (4).

We remark that the averaging process represents an important part of the simulation. There are two reasons for this. When the incident waves are perfectly coherent and monochromatic, the EM energy is multiply reflected, in general, from the interfaces, and a strong standing-wave pattern is formed in the PR slab, with a longitudinal periodicity of the order of a wavelength. When the frequency of the light is changed by a small amount, the locations of the extrema along the \( y \) direction will change. When many different frequencies are added together, these individual peaks and nulls will tend to average out, and a longitudinally averaged field will result. Thus spatial averaging simulates the frequency spread of the optical energy. An approximate lower bound for the allowable frequency spread from the source in our case may be calculated to be 0.03%. The other important rationale for the averaging process is that in an actual physical experiment involving a PR crystal, for instance, the quality of the incident and exit surfaces will limit the coherence of the multiply reflected EM waves, or, equivalently, the finesse of the Fabry-Perot cavity (see, for instance, Ref. 13, p. 216).

Other parameters chosen for the numerical simulations are \( r_{s2} = 1640 \, \text{pm/V}, \quad r_{s3} = 8 \, \text{pm/V}, \quad r_{s3} = 28 \, \text{pm/V}, \quad n_O = 2.437, \quad n_p = 2.365, \quad E_{ELO} = 2.42 \times 10^{-6}, \quad \varepsilon_s = 18000, \quad \gamma = 5 \times 10^{-14} \, \text{m}^2/\text{s}, \quad \mu = 0.5 \times 10^{-4} \, \text{m}^2/\text{s}, \quad N_A/N_D = 0.01, \quad s = 0.1 \times 10^{-4} \, \text{m}^2/\text{s}, \quad \beta = 2 \, \text{s}^{-1}. \) All the material parameters have been taken from Refs. 13, 18.

### 3 Computation Results

As shown in the sequence of Figs. 1–3, we determine the results of transient wave mixing and the \( \Delta \) grating profile in the steady state for three cases: (1) when the PR material is surrounded by a refractive index matching medium with \( \varepsilon_1 = \varepsilon_3 = (n_O + n_p)/2 \); (2) when the PR material is sur-
rounded by a partially mismatched medium with $\varepsilon_1 = \varepsilon_3 = (n_0 + n_E)/2$, and (3) when the surrounding medium is air. We also numerically analyze the effect of taking the time constant independent of the intensity and that of taking the exact expression as given in Eq. (3). In each of the three constant independent of the intensity and that of taking the

\[ \Delta t = \frac{\lambda^2}{2c^2} \]

rounded by a partially mismatched medium with $\varepsilon_1 = \varepsilon_3 = (n_0 + n_E)/2$, and (3) when the surrounding medium is air. We also numerically analyze the effect of taking the time constant independent of the intensity and that of taking the exact expression as given in Eq. (3). In each of the three cases, the initial pump-to-probe intensity ratio is set equal to 0.014. This corresponds to a peak-to-peak variation of the electric modulation: two different dielectric slanted gratings have resulted from the photorefractive-optical interaction, and each extends over about 750 wavelengths inside the PR material, and qualitatively seem to agree with the results in Ref. 10, derived using a scalar, unidirectional beam propagation method, and assuming a steady state with no time evolution throughout the computations. However, the grating with one slant angle is sandwiched between gratings of a different slant angle; also, the grating envelope appears to approximate a hyperbolic-secant function, in agreement with Ref. 19. Furthermore, Figs. 1 (c) and 1 (d) show that, at least in the matched case, there is no significant generation of higher-order diffraction even in the presence of two slanted gratings, which is different from the results in Ref. 10; also, as shown in Fig. 1(c), the pump is totally depleted. We can explain the absence of the higher-order diffraction by noting, as stated above, that the grating with a different slant angle appears in the middle of the PR material, so that any higher-order diffraction, even if produced, will not likely survive at the output face of the crystal. Figure 1(c) shows that in our case, there is virtually no conversion of the incident energy into reflected fields due to the matched boundary conditions. Note that the difference between taking the time constant intensity independent and intensity dependent is clear from Figs. 1(c) and 1(d); in the latter case the evolution of the

\[ I_{inc} = 7.49 \times 10^7 \text{ W/m}^2 \]

\[ \tau(I_0) = 16.9 \text{ msec} \]

\[ \Delta t = 2 \tau(I_0) = 3.38 \text{ msec} \]
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Partially Mismatched Case

\[ \varepsilon_1 = \frac{(n_0 + n_e)}{2} \]

\[ \varepsilon_2 = \varepsilon_1 \]

\[ n_0 = 2.437, n_e = 2.365 \]

\[ \lambda = 633\mu m \]

\[ \Lambda = 5\lambda = \text{grating spacing} \]

\[ \Delta \varepsilon_{\text{max}} = 0.0543 \]
\[ \Delta \varepsilon_{\text{min}} = -0.0611 \]

\[ t = 230 \text{ sec} (108 \text{ time steps}) \]

**Fig. 2** (a) Wave coupling geometry in PR BaTiO\(_3\) for the partially mismatched case; (b) the induced permittivity profile in the steady state; (c) evolution of the transmitted orders; (d) evolution of the reflected orders.

Diffracted transmitted and reflected orders is slower than in the former, but both cases yield the same steady states. Hence, in the totally matched case, computation time may be minimized by using a uniform time constant. We would also like to comment that the kinetics of the grating recording has a characteristic time constant identical to that of \( T_0 \) and \( T_1 \).

Figure 2(b) depicts the steady-state permittivity profile when the surrounding medium has a permittivity \( \varepsilon_1 = \varepsilon_3 = (n_0 + n_e)/2 \). Note that, once again, the dielectric grating is similar to the case discussed above; once again, there is almost total depletion of energy from the pump into the probe, and some reflected orders result. Also, comparing Fig. 2(b) with 1(b), we note that the two different slanted gratings in the former case are not as well defined as in the latter case, due to reflection effects from the boundaries. Again, there are no higher orders, since the second slanted grating occupies a middle region of the material. We should point out that once again, there is no difference, with respect to the steady state, between the computations performed with a uniform and an intensity-dependent time constant.

Figure 3(b) shows the simulation results for the dielectric grating (also in the steady state) when the medium surrounding the PR material is air, and using a uniform time constant. In this case, the grating profile is not as distinct as in the two previous cases, possibly due to reflection effects from the boundary. A significant amount of the higher order is generated; in fact, in the steady state, as shown in Fig. 3(c), there is no energy in the zeroth and first orders. Also, as evident from Fig. 3(d), there is substantial reflected power in the zeroth- and second-order directions.

One significant difference between this set of plots and those presented above is in the evolution of the diffracted orders when we use the intensity-dependent time constant in our simulations. As shown in the sequence of Figs. 3(g) and 3(h), the transmitted and reflected orders evolve quite differently than in Figs. 3(c) and 3(d), although the general nature of the time dependence seems to be qualitatively preserved. This is also noticeable in differences in the plots for the induced permittivity [compare Fig. 3(f) with 3(b)], possibly due to the spatiotemporal evolution of the intensity-dependent time constant, shown in Fig. 3(e) at \( t = 0.239 \) s.

The simulations show that in general, there may be significant conversion of energy into higher transmitted and reflected orders in the case of refractive index mismatch between the PR material and the surrounding medium, and that these results cannot be predicted without employing the rigorous coupled wave approach. Also evident is the fact that while a uniform time constant can minimize computation time and yield accurate results for the perfectly matched and partially mismatched cases, the more exact expression for the time constant must be used to analyze the totally mismatched case.

In passing, we would like to briefly compare the results presented above with the case presented in Ref. 12 with a set...
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Mismatched Case

\[ \varepsilon_1 = 1, \quad \varepsilon_3 = 1, \quad \Delta \varepsilon(x, y, t) \]

\[ \lambda = 633 \mu m \]

\[ \Lambda = 5 \lambda = \text{grating spacing} \]

\[ n_0 = 2.437, \quad n_e = 2.365 \]

\[ \Delta \varepsilon_{\text{max}} = 0.00553, \quad \Delta \varepsilon_{\text{min}} = -0.00618 \]

\[ I_{\text{inc}} = 1.79 \times 10^7 W/m^2 \]

\[ \tau(I_0) = 5.98 \text{msec} \]

\[ \Delta t = 2 \tau(I_0) = 1.19 \text{msec} \]

\[ l = 0.129 \text{sec (108 time steps)} \]

\[ \Delta \varepsilon_{\text{max}} = 0.00321, \quad \Delta \varepsilon_{\text{min}} = -0.0053 \]

\[ t = 0.239 \text{sec (200 time steps)} \]

\[ \varepsilon(x, y, t) \]

\[ t = 0.239 \text{sec (200 time steps)} \]

\[ \tau^{-1}(x, y, t) \]

\[ \Delta \varepsilon(x, y, t) \]

\[ I_{\text{inc}} = 1.79 \times 10^7 W/m^2 \]

\[ \tau(I_0) = 5.98 \text{msec} \]

\[ \Delta t = 2 \tau(I_0) = 1.19 \text{msec} \]

\[ \varepsilon_{\text{max}} = 1, \quad \varepsilon_{\text{min}} = 0.5 \]

\[ t = 239 \text{ sec (200 time steps)} \]

\[ \Delta \varepsilon_{\text{max}} = 0.00321, \quad \Delta \varepsilon_{\text{min}} = -0.0053 \]

\[ t = 0.239 \text{ sec (200 time steps)} \]

Nonuniform \( \tau \)

Fig. 3 (a) Wave coupling geometry in PR BaTiO\(_3\) for the totally mismatched case; (b) the induced permittivity profile in the steady state; (c) evolution of the transmitted orders; (d) evolution of the reflected orders. Part (e) is a plot of the intensity-dependent time constant, and (f)--(h) are plots similar to (b)--(d) but using the intensity-dependent time constant.
of different input conditions, viz., equal pump and probe incident intensity for BaTiO$_3$ in air. Due to the nonlinear coupled nature of the problem, it should not be assumed a priori that the latter results can be entirely predicted from those given in Fig. 3 from the moment the transmitted zeroth- and first-order intensities become equal. In fact, with equal input intensities, the temporal evolution follows quite a different path, but the steady state has qualitative similarities with that discussed above, in the sense that only the higher-order transmitted diffraction survives the competition. However, in the reflected direction, there is about equal power in the zeroth, first, and higher orders.

4 Conclusion

By using rigorous coupled wave diffraction theory along with a time-dependent nonlinear formulation, we have analyzed two- and multiple-wave coupling (including the grating dynamics) in BaTiO$_3$ with different boundary interfaces. Effects of electrostatic and optical anisotropy have been included in the analysis. Significant mode conversion to higher orders is observed only when the boundary interfaces are highly mismatched, and a substantial part of the incident energy is reflected. Our computations also depict the grating envelopes in the PR material, and show that gratings with two different slant angles may exist in the PR material. The generation of higher orders could, for instance, be used to simultaneously address or connect two or more different optical elements in an optical interconnection scheme. Finally, our computations show that an exact intensity-dependent time constant must be used to analyze wave mixing in the cases when the PR material is surrounded by a mismatched medium (e.g., air), whereas a uniform time constant can be assumed to analyze wave mixing in the matched and partially matched cases.

References


Partha P. Banerjee: Biography and photograph appear with the special section guest editorial in this issue.

John M. Jarem received his BS, MS, and PhD degrees in electrical engineering from Drexel University, Philadelphia, in 1971, 1972, and 1975, respectively. From 1975 to 1981, he worked as an assistant professor of electrical engineering at the University of Petroleum and Minerals in Dhahran, Saudi Arabia. He worked for the University of Texas at El Paso as an associate professor of electrical engineering from 1981 to 1987, and has been a professor of electrical and computer engineering at the University of Alabama in Huntsville since 1987. His research interests are electromagnetics, antenna theory, and optics.