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The Kou Jump-Diffusion Model for Option Pricing

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Why Kou’s Model?

The Black-Scholes model has been a useful tool for option pricing in the financial market. However, there are two phenomena – the leptokurtic feature and the implied volatility curve – which it fails to capture. We looked at Kou’s model which accounts for these phenomena, and it leads to an analytical solution for many option pricing problems.

Kou’s Formula:

Below is the formula derived in Kou’s article. We can see that he uses a double summation that calls several other functions.

\[ Y(\kappa; \sigma, \lambda, \rho, \eta_1; \eta_2; a, T) = \frac{e^{(\eta_1)T/2}}{\sigma \sqrt{2\pi T}} \sum_{n=1}^{\infty} \pi_n \sum_{k=1}^{\infty} \rho_{nk}(\sigma_1 \sqrt{T})^{\frac{1}{2}} \left( a - \kappa T; -\eta_1, -\frac{1}{\sigma \sqrt{T}}, -\sigma_1 \sqrt{T} \right) + \frac{e^{(\eta_1)T/2}}{\sigma \sqrt{2\pi T}} \sum_{n=1}^{\infty} \pi_n \sum_{k=1}^{\infty} \rho_{nk}(\sigma_2 \sqrt{T})^{\frac{1}{2}} \left( a - \kappa T; \eta_2, -\frac{1}{\sigma \sqrt{T}}, -\sigma_2 \sqrt{T} \right) + \pi_0 N \left( -\frac{a - \kappa T}{\sigma \sqrt{T}} \right) \]

Kou used the Upsilon equation to find the value of the option.

\[ Y(0) = S_0 Y \left( r - \beta + \frac{1}{2} \sigma^2, \sigma, \lambda, \rho, \eta_1, \eta_2; \ln \left( \frac{K}{S_0} \right), T \right) - Ke^{-rT} Y \left( r - \beta + \frac{1}{2} \sigma^2, \sigma, \lambda, \rho, \eta_1, \eta_2; \ln \left( \frac{K}{S_0} \right), T \right) \]

The \( Hh \) function can be viewed as a cumulative normal distribution function where the left tail has a polynomial growth rate and the right tail has an exponential decay.

Below are the \( Hh \) and \( I_n \) functions which are called in the Upsilon function above.

\[ Hh_n(x) = \int_x^\infty Hh_{n-1}(y) dy = \frac{1}{n!} \int_x^\infty (t-x)^n e^{-\frac{t^2}{2}} dt \]

\[ I_n(\alpha; \beta, \delta) = \int_{\alpha}^{\infty} e^{\alpha x} Hh_n(\beta x - \delta) dx \]

Density Function for Kou Model:

Below is the density function used in Kou’s model.

\[ f(x) = \begin{cases} \pi_n e^{-n(x)^2} + (1-p)\pi_n e^{0.5(x)^2} & \text{if x > 1} \\ \pi_n e^{-n(x)^2} & \text{if x > 0, x < 1} \\ \frac{\pi_n}{1-p} e^{-n(x)^2} - \frac{\pi_n}{1-p} e^{0.5(x)^2} & \text{if x < 0} \end{cases} \]

Kou’s density function (blue) is plotted against the normal curve (red). It is clear to see that the Kou’s density function accounts for discontinuity and the fat tails.