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# Single-Beam Holographic Tomography creates Images in Three Dimensions

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# Single-beam holographic tomography creates images in three dimensions

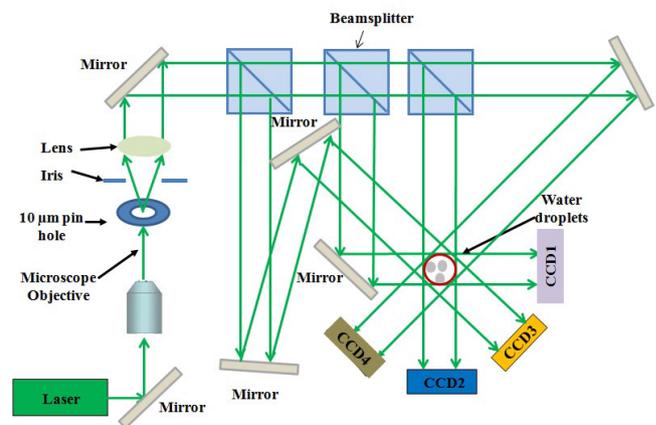
Partha Banerjee, George Nehmetallah, and Nickolai Kukhtarev

*A novel, nonintrusive technique can record and aid 3D reconstruction of the shapes of translucent objects.*

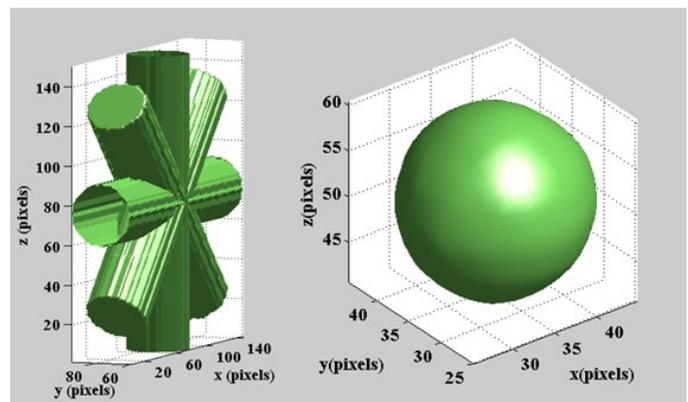
In digital holography (DH), the interference between light scattered from an object and a reference wave is recorded using a CCD camera.<sup>1-3</sup> DH has various advantages over analog holography: no film processing is needed, reconstruction is performed using numerical methods, and no further experimental setup is necessary. However, one of the disadvantages of DH is that current CCDs have a resolution of approximately 1000lines/mm, which is less than that of photographic film.

Considerable effort has been spent on determining the 2D shape and distribution of water droplets as they fall or move at high speed.<sup>4,5</sup> They are translucent, have large curvatures—like lenses—and can scatter light at very large angles. In our research, we have used DH and DH interferometry extensively to determine, for instance, model-aircraft attitudes in flight.<sup>6</sup> Use of traditional DH to determine 3D shapes<sup>7</sup> results in thousands of fringes per millimeter, which easily exceeds the resolution of CCD cameras. To solve this problem, we developed a novel technique based on single-beam holographic tomography (SHOT) to record and reconstruct the 3D shapes and distribution of water droplets and lenses.<sup>8</sup> Since the beam width spans several water droplets, light transmitted between them acts like a reference beam: it interferes with the object beam and records the holograms. Thus, single-beam ('in-line') holography reduces system complexity and can be used to determine droplet shape, although it does not provide details of the interior structure.

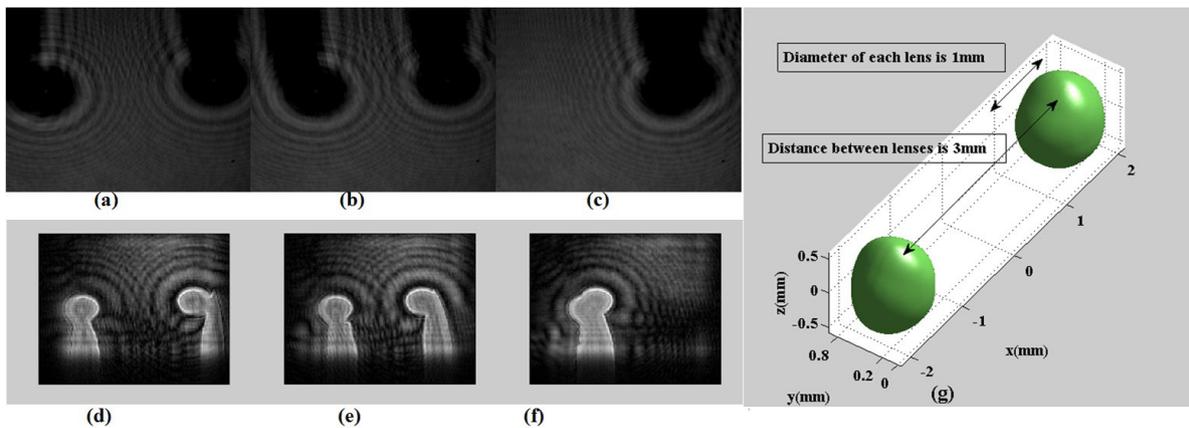
To visualize the 3D shape, multiple projections from different directions—as in tomographic imaging systems—are required. Holographic reconstruction is used to generate a 2D visualization along a certain projection, and the 3D shape is reconstructed using the SHOT multiplicative technique (SHOT-MT) or SHOT Radon-transform technique (SHOT-RTT).



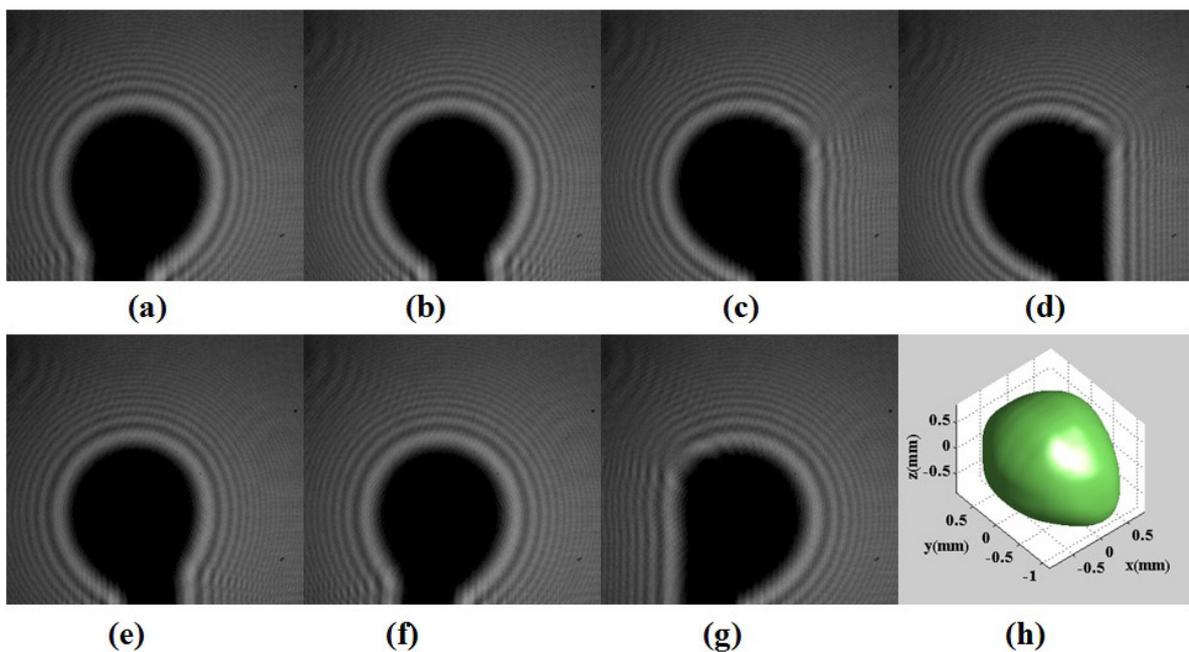
**Figure 1.** Experimental setup of our single-beam holographic-tomography (SHOT) recording scheme.



**Figure 2.** Schematic showing the principle of how the beams are used to generate a 3D image using the SHOT-multiplicative reconstruction technique.



**Figure 3.** (a)–(c) Three holograms of two 1mm-diameter spherical ball lenses. (d)–(f) Reconstruction along the projection angles  $\theta = 0, 45$ , and  $90^\circ$ . (g) Final 3D shape.



**Figure 4.** (a)–(g) Series of holograms of a large water droplet hanging from one side of a dripper, along the orientations  $\theta = 0, 45, 90, 135, 180, 225$ , and  $270^\circ$ , respectively. (h) 3D reconstruction.

In SHOT-MT, digital holograms  $h_j(x, y)$  are recorded that correspond to each orientation  $\theta_j = [0 : j\pi/M : \pi]$ ,  $j = 1, 2, \dots, M$  with respect to the  $y$  axis, i.e., the axis perpendicular to the plane of the diagram in Figure 1. Subsequently,  $h_j(x, y)$  are numerically reconstructed and the intensities  $I_j$  are computed on multiple planes around the point at a distance  $d$ , corresponding to the middle of the test volume where the droplet is located. The numerical reconstruction involves using the discretized form of the Fresnel diffraction formula.<sup>1-3,7,8</sup> After some coordinate

transformations, the 3D shape and distribution of this droplet can be reconstructed by multiplying the multiple reconstructed intensities  $I = \prod_1^M I_j$  (see Figure 2).

In SHOT-RTT, a 3D matrix of all numerically reconstructed holograms from different angles is formed and the inverse 3D Radon transform is calculated by computing the inverse 2D Radon transform of each slice using the Fourier slice

*Continued on next page*

theorem.<sup>8</sup> The Radon transform is an integral transform used in tomography, where an image is created from the scattering data associated with cross-sectional scans of an object. Morphological image-processing techniques are then applied to the inverted 3D matrix to get the final 3D shape.

Both SHOT-MT and SHOT-RTT can be extended from visualization of a single to multiple droplets, such as in a cloud or rain. They enable visualization of the 3D shape of the droplet, as well as estimation of the radii of curvature around each point on the 3D shape. In addition, both methods allow accurate quantification of the number density of the droplets and their separation.

We provide two illustrative examples of SHOT and a reconstruction using SHOT-MT. Figure 3(a)–(c) shows three holograms of two ball lenses glued onto the ends of syringe needles. Figure 3(d)–(f) shows their reconstruction along three projection angles  $\theta = 0, 45,$  and  $90^\circ$  using the SHOT-MT technique. Figure 3(g) shows the 3D picture. Note that the needles attached to hold the lenses result in some deformation, which will be absent in regular experiments. Figure 4(a)–(g) shows a series of holograms of a large droplet hanging from one side of a syringe, along the orientations  $\theta_j = 0, 45, 90, 135, 180, 225,$  and  $270^\circ$ , respectively. Figure 4(h) shows the 3D picture. Again, deformation where the syringe is attached to the droplet is apparent.

In summary, we have described a nonintrusive technique for recording and 3D reconstruction of the shapes of lenslets and water droplets using SHOT-MT and SHOT-RTT. The main advantage of SHOT-MT is that it usually requires fewer projections than SHOT-RTT and, hence, fewer CCD cameras. In addition, the SHOT-MT algorithm is straightforward. However, its main disadvantage is that the reconstruction program needs more memory for computation because of the 3D matrix rotations involved. Work toward techniques for multiplexing images on the same CCD and development of more user-friendly and faster software is currently in progress.

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Georges Nehmetallah received his PhD from the University of Dayton in 2006. To date, he has published over 30 refereed journal papers and conference proceedings. He is a research associate, investigating digital holography, metamaterial applications to sensors technology, and nanoparticle dispersed liquid-crystal cell fabrication and characterization.

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