Numerical solution of the KdV equation with periodic boundary conditions using the sinc-collocation method

Nicholas Haynes
Advisor: Dr. Muhammad Usman
Department of Mathematics

Introduction
The experimental work of Bona et. al.\(^1\) showed that waves created in a channel by pumping one end of the channel periodically eventually become periodic with the same period as the pumping.

![Waveforms](image1)

![Graphs](image2)

The theoretical work\(^2\) has shown this result for pumping with small amplitude. In this project, we use numerical techniques to demonstrate this result for pumping of arbitrary amplitude.

The Numerical Model
The KdV equation describes single waves, or solitons, propagating in a water channel. The version used here is

\[
\begin{align*}
\frac{\partial u}{\partial t} + uu_x + \beta u_{xx} + \gamma u_{xxxx} &= 0 \\
\end{align*}
\]

where \(h(t)\) is a periodic function with period \(\tau\) and arbitrary amplitude.

\[
\begin{align*}
\frac{\partial u}{\partial t} + uu_x + \beta u_{xx} + \gamma u_{xxxx} &= 0 \\
\end{align*}
\]

Numerical Results
To validate the accuracy of the method, we first demonstrate it on a test problem

\[
\begin{align*}
\frac{\partial u}{\partial t} + uu_x + u_{xxx} &= 0 \\
\end{align*}
\]

that has an exact solution. The approximated solution was found to have excellent agreement with the analytic solution, thus ensuring the accuracy of the method.

Solving the model problem produces results with a striking qualitative similarity to the observed data;

![Graphs](image3)

Indeed, when these plots are overlaid, it can be seen that the solutions become periodic with the same period as the forcing;

Stability Analysis
The method is stable if the spectral radius of the iteration matrix is less than or equal to 1.

![Graphs](image4)

For this condition to hold, it is necessary that \(1/2 \leq \theta \leq 1\) and sufficient that \(\theta = 1\).

References