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Generalized Multi-Latin Squares
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ABSTRACT
The research explores properties of generalized multi-latin squares and proposes ways to construct them. A \( (n, t, m, p, g) \)-generalized multi-latin square is an array consisting of \( n \) rows and \( n \) columns, where each cell is filled with \( m \) symbols from a collection consisting of \( t \) different symbols, any symbol appears in each row and in each column \( p \) times, and any pair of different symbols occur together \( g \) times. Understanding trivial examples, the properties, and the mathematical relationships behind the problem reveals multiple examples and a systematic way to build generalized multi-latin squares.

RELATED DESIGNS
\( \text{BIBD} \) is a pair \((V, B)\) where \( V \) is a collection of \( v \) symbols and \( B \) is a collection of \( k \)-subsets of \( V \) (blocks) such that each element of \( V \) is contained in exactly \( r \) blocks and any \( 2 \)-subset of \( V \) is contained in exactly \( \lambda \) blocks. \text{Parallel classes} partition the set of blocks so that a symbol appears once in each class. \( \text{RBIBD} \) is a \( \text{BIBD}(v, k, \lambda) \) whose blocks can be partitioned into parallel classes.

CONSTRUCTION METHOD
Let \( c \) be the number of parallel classes and \( \ell \) be the number of block in each parallel class from the \( \text{RBIBD}(v, k, \lambda) \).
1. Build \( c \times \ell \times \ell \) square permuting the blocks each row
2. Arrange the \( \ell \times \ell \) squares into a \( c \times c \) square
3. To extend the generalized multi-latin square, expand the \( c \times c \) square by a factor of \( c \) to use every parallel class the same number of times.
4. Let \( N \times N \) be the size of the square filled with \( \ell \times \ell \) squares.
   Therefore, \( N^2 = c \cdot s \) for some natural number \( s \).
   The result is a \((N \cdot \ell, v, k, N, s \cdot \ell)\)-generalized multi-latin square.

GENERALIZED MULTI-LATIN SQUARES
\( \text{RBIBD}(9, 3, 1) \)

Symbols: \( V = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \)
Block size: \( k = 3 \)
Number of block in each class:
\( \ell = v/k = 9/3 = 3 \)
Number of blocks:
\( \binom{v}{2} / \binom{k}{2} = \binom{9}{2} / \binom{3}{2} = 12 \)
Number of Parallel classes:
\( c = \left[ \binom{9}{2} / \binom{3}{2} \right] / 3 = 4 \)

\( \text{RBIBD}(51, 3, 1) \)

\((425, 51, 3, 25, 425)\)-generalized multi-latin square
\((595, 51, 3, 35, 833)\)-generalized multi-latin square

REFERENCES

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FUTURE RESEARCH
& APPLICATIONS
- To classify and characterize the existence of generalized multi-latin squares for any given parameters.
- Experiment variable design