Effect of Airfoil-Preserved Undulations on Wing Performance and Wingtip Vortex

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Effect of Airfoil-Preserved Undulations on Wing Performance and Wingtip Vortex

Honors Thesis
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December 2020
Abstract
The effect of undulation placement (leading edge, trailing edge, leading and trailing edge) on the wing performance and the wingtip vortex was investigated. Experiments were performed at the University of Dayton Low Speed Wind Tunnel (UD-LSWT) on undulated wings where the NACA 0012 airfoil cross-section is preserved along the wingspan. Sensitivity studies were done on the undulation wavelength along the span (λ/c 0.31, 0.21 and 0.15) and undulation placement (leading edge, trailing edge, and both leading and trailing edge). The leading edge undulations delayed stall until higher angles of attack, however, the maximum aerodynamic efficiency was reduced. The trailing edge undulated wing on the other hand increased the maximum aerodynamic efficiency but was not successful in stall mitigation. Wings with both leading and trailing edge undulations showed improvement in aerodynamic efficiency as well as delayed stall. The effect of the undulations on the wingtip vortex was also investigated through Particle Image Velocimetry (PIV). For the same coefficient of lift, the undulated wing cases reduced the wingtip vortex circulation by 25%. Investigations into the wingtip vortex core RMS and aerodynamic efficiency revealed a direct relationship where a higher vortex core RMS resulted in a higher aerodynamic efficiency and vice-versa.

Dedication or Acknowledgements
The author acknowledges the partial funding from Henry Luce Foundation through the Clare Boothe Luce Scholars Program for the work presented in this thesis. The author would also like to acknowledge the Ohio Space Grant Consortium (OSGC) for partially funding the primary author for performing undergraduate research. The author would also like to thank UD wind tunnel technician, Jielong Cai, and research partner, Michael Mongin, for helping with manufacturing wind tunnel models and performing experiments.
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NOMENCLATURE

$\alpha$ = Angle of attack, degrees

$\lambda$ = Wavelength, cm

$\Gamma(r)$ = Circulation as a function of radius, m2/s

$\Gamma_0$ = Total circulation, m2/s

$\mu$ = Dynamic viscosity, Pa-s

$\rho$ = Density, kg/m3

$\sigma$ = Vortex wandering amplitude, m

$\sigma_x$ = RMS wandering x component, m

$\sigma_y$ = RMS wandering y component, m

$\Omega$ = Vorticity, 1/s

$A$ = Amplitude, cm

$C_D$ = Coefficient of drag

$C_{Di}$ = Coefficient of induced drag

$C_{DP}$ = Coefficient of profile drag

$C_L$ = Coefficient of lift

$C_{L,\alpha}$ = Lift curve slope

$D$ = Drag force, N

$e$ = Span efficiency

$L$ = Lift force, N

$R$ = Strain rate, 1/s

$r$ = Vortex radius, m

$rc$ = Vortex core radius, m

$U$ = X-component of velocity, m/s

$U_{RMS}$ = RMS, m/s

$u'$ = X-component fluctuating velocity, m/s

$AR$ = Aspect ratio

$LE$ = Leading edge

$LEU$ = Leading edge undulations

$LETEU$ = Leading edge trailing edge undulations

$TE$ = Trailing edge

$TEU$ = Trailing edge undulations
I. Introduction

Even with major advancements in aviation technology, an effective reduction of induced drag has remained a velleity for many years. The lift-induced drag is responsible for more than 70% of the total drag of the aircraft during take-off and landing and about 5-15% of the total drag during cruise [1]. So far, no universal mathematical relation exists which relates the physics and properties of the wingtip vortex roll-up process, its evolution, and induced drag. Over the years, numerous methods to reduce induced drag have been conceived and implemented. Methods such as installing end plates at wingtips, winglets (most common), lift distribution tailoring (by changing the deflection of flaps on the wing), active/passive flow control methods (blowing or suction of air at the wingtip), etc. have been employed to affect the aerodynamic efficiency of the wing. Of all the methods mentioned above, the total reduction in drag has been 5-7% [1]. This reduction in drag corresponds to less fuel consumption and flight hours, leading to considerable cost reductions. If drag can be reduced even further than 5-7%, this means costs will be reduced even more drastically. Methods that have been previously implemented to improve aerodynamic efficiency leave room for improvement in terms of the influence of the devices on the wingtip vortex roll-up process. At high Reynolds numbers, the wingtip vortex core laminarizes irrespective of any external influences. This phenomenon is quantified by the Richardson number, defined in Holzapfel et al. [2], which evaluates the stratification of the vortex by evaluating the swirl velocity gradients in the vortex flow. Cotel and Breidenthal [3] and Cotel [4] determined a threshold value for the Richardson number given by a function of vortex Reynolds number. For a higher “persistence parameter,” defined by the ratio of rotational to translational speed of the vortex, the threshold value of the Richardson number was experimentally found to be $\text{Re}_{\text{vortex}}^{1/4}$. Only laminar flow is possible in the radial location of the vortex containing a Richardson number greater than this threshold value. Any perturbations will either be relaminarized or suppressed, and any diffusion in the vortex core is only possible at a molecular level [5]. If the threshold value of the Richardson number occurs at a smaller radial location from the vortex core, then most of the wingtip vortex is turbulent, yielding a poor response to flow control methods employed at the wingtip. Recent studies have shown the strength of the wingtip vortex can be significantly affected by enhancing the free shear layer interaction with the wingtip vortex [6], [7], [8]. The blockage of spanwise flow along the surface of the wing using active and passive flow control methods such as stall fences, dimples, bumps, riblets, etc. has been shown to have a strong influence on the growth and
evolution of the wingtip vortex [9], [10]. It has been shown that the airfoil-preserved undulations on the wing act as spanwise fences that prevent wingtip flow separation from spreading inboard. [11], [12]. Therefore, the undulations affect the spanwise flow over the wing, which affects the rollup and growth of the wingtip vortex. The effect of undulations on the wingtip vortex has not been investigated as most of the undulated wing studies were confined to wings with infinite aspect ratio as shown in Table 1. This study aims to characterize the effect of undulation wavelength and undulation placement on the wingtip vortex properties.

The appearance of the spanwise fences on undulated wings depends on whether the same airfoil is preserved along the wingspan. A brief background on the origins of undulations and the plethora of investigations into leading edge undulations is discussed below.

A. Leading Edge Undulations

Wings with unusual contours and undulations appear many places in nature, including the humpback whale [13], bats [14], and birds [15]. The humpback whale flipper has tubercles along the leading edge and along the trailing edge towards the tip of the flipper. The whale’s ability to perform tight maneuvers is primarily attributed to the superior performance caused by the presence of these tubercles. Bat wings are naturally contoured as the skin stretches across the bones which is comparable to how canvas stretches across the wooden ribs of a wing to provide a lifting surface. A plethora of literature exists on wings with leading edge undulations where the effect of undulation wavelength and amplitude on the airfoil performance was investigated from an experimental and/or a CFD standpoint.

Van Nierop et al. [16] investigated wing models with leading edge tubercles and found that the tubercles alter the pressure distribution, resulting in boundary layer separation behind the tubercles which ultimately leads to a delay in stall and a higher stall angle. Results from [16] indicate that the $C_L$ for the wing with the highest number of tubercles begins to deviate from the linear trend starting at $5^\circ$ and has a significantly higher stall angle when compared to the baseline. However, the maximum $C_L$ was reduced.

Shorbagy et al. [17] also performed a low-Reynolds number study on wings with leading edge tubercles and found that there was an increase in $C_L/C_D$ for wings with sinusoidal leading edge tubercles and trailing edge tubercles up to the wingtip. The results from this study for an AR 2 wing at a Reynolds number of 174,000 indicate the highest performance corresponds to a wing with a full sinusoidal leading edge and a straight trailing edge until 56% along the semi-span wing where the remainder is sinusoidal. This is the only
wing that was found to outperform the baseline between 8 and 15° angle of attack, indicating that the presence of trailing edge undulations along with leading edge undulations can increase the maximum aerodynamic efficiency while delaying stall.

Farouk et al. [18] carried out a computational study with spherical protrusions on the leading edge of a NACA 0012 wing and found that while the protrusions help delay stall, the magnitude of $C_L$ was compromised across all angles of attack. Hansen et. al [19] investigated the changes in wing performance with leading edge tubercles of varying wavelength and amplitude for distinct airfoil profiles and found that the tubercle parameters do not result in a significant impact in the pre-stall angle of attack, but there is a very noticeable impact in the post-stall range. Hansen et al. [20] also found that flow separation occurs further aft of the airfoil in the tubercle peak when compared to the tubercle trough, and that maximum suction occurs at the tubercle peak cross-section as well. However, only marginal improvement in aerodynamic efficiency was found with the tubercle airfoil when compared to the baseline unmodified wing at 10° angle of attack.

As this section evinces, vast amounts of literature exist on the effect of leading edge tubercles on wing performance [16], [17], [18], [19], [21], and stall delay [23], [24] and has been considered for applications such as wind turbines [25] and propellers [26]. In applications such as these, it is important to identify the types of airfoils used in the undulated wings for design purposes. Some leading edge tubercles studies involve serrated-type or undulated leading edges where the airfoil cross-section is destroyed unintentionally. In other cases, the airfoils are religiously maintained across the undulations. ‘Airfoil-preserved’ undulations can be defined as undulations where the thickness of the airfoil is varied along with the chord in creating the undulations to preserve the airfoil’s thickness-to-chord ratio. If the airfoil is not preserved, the thickness of the airfoils at peaks and troughs will be the same. Therefore, the thickness-to-chord ratio of airfoil sections along the span will also not be the same. Figure 1 shows the differences between airfoil-preserved leading edge undulations and non-airfoil-preserved leading edge undulations. Studies involving leading edge protuberances also inherently do not maintain the airfoil section throughout.
Fig 1 Schematic denoting the differences between a) airfoil-preserved undulations and b) undulations without airfoil preservation.

Table 1 below shows some of the investigations into leading edge undulations with and without airfoil preservations. As seen in Table 1, there is a mixture of airfoil-preserved undulations and non-airfoil-preserved undulations investigations.

The effects of preserving the airfoil along the span are not well understood. Aftab and Ahmed [43] performed investigations on both spherical leading edge undulations and airfoil-preserved leading edge undulations and found that both airfoils performed similarly, and the reduction in maximum aerodynamic efficiency when compared to the baseline was significant. Given the variety of undulations in the literature, the airfoil-preserved undulations were chosen as the type of undulations for the current study to generate the wavy pattern on the wing surface which is hypothesized to affect the spanwise flow, thereby affecting the rollup and growth of the wingtip vortex. As seen in Table 1, a variety of airfoils with and without leading edge undulations have also been investigated. The NACA 63-021 and NACA 0021 was investigated more frequently due to their similarity to the cross-section of the humpback whale flipper. NACA 0012 airfoil was chosen for this study as the performance characteristics of this airfoil are well researched and understood.

It is also important to note that most of the literature mentioned in this section and in Table 1 focus on leading edge undulations as a means to delay stall. However, a genetic optimization study conducted by Lohry et al. [44] on undulated wings found that the optimized geometry (in terms of maximum aerodynamic efficiency) is a wing with undulations not along the leading edge, but along the trailing edge. Wings with trailing edge undulations seems to provide superior performance in improving aerodynamic efficiency when compared to wings with leading edge undulations. Our hypothesis states that the wings with trailing edge undulations may not be successful in mitigating stall but may improve aerodynamic performance. Our
hypothesis also states that improving aerodynamic efficiency and delaying stall can be achieved by a combination of both leading and trailing edge undulations.

**Table 1 List of references with and without airfoil-preserved leading edge undulations**

<table>
<thead>
<tr>
<th>Airfoil Preserved</th>
<th>Airfoil Not Preserved</th>
<th>Airfoil AR</th>
<th>Type of Investigation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miklosovic et al. [27]</td>
<td>NACA 0020</td>
<td>4.45</td>
<td>Experimental</td>
</tr>
<tr>
<td>Miklosovic et al. [28]</td>
<td>NACA 0020</td>
<td>4.45</td>
<td>Experimental</td>
</tr>
<tr>
<td>Johari et al. [29]</td>
<td>NACA 63 - 021</td>
<td>∞</td>
<td>Experimental</td>
</tr>
<tr>
<td>Watts and Fish [31]</td>
<td>NACA 63 - 021</td>
<td>∞</td>
<td>CFD</td>
</tr>
<tr>
<td>Hansen et al. [19]</td>
<td>NACA 0020 NACA 0021 NACA 63 - 021</td>
<td>∞</td>
<td>Experimental</td>
</tr>
<tr>
<td>Van Nierop et al. [16]</td>
<td>NACA 0018</td>
<td>∞</td>
<td>Low Order Model</td>
</tr>
<tr>
<td>Dropkin et al. [32]</td>
<td>NACA 63 - 021</td>
<td>∞</td>
<td>Experimental/CFD</td>
</tr>
<tr>
<td>Favier at al.[33]</td>
<td>NACA 0020</td>
<td>∞</td>
<td>CFD</td>
</tr>
<tr>
<td>Guerrerio and Sousa [34]</td>
<td>NASA LS(1)-0417</td>
<td>1, 1.5</td>
<td>Experimental</td>
</tr>
<tr>
<td>Hansen et al. [35]</td>
<td>NACA 0021</td>
<td>∞</td>
<td>Experiment/CFD</td>
</tr>
<tr>
<td>Skillen et al. [36]</td>
<td>NACA 0021</td>
<td>∞</td>
<td>CFD</td>
</tr>
<tr>
<td>Weber et al. [37]</td>
<td>NACA 0020</td>
<td>~ 4</td>
<td>CFD</td>
</tr>
<tr>
<td>Shorbagy et al. [17]</td>
<td>NACA 0021</td>
<td>2, 7</td>
<td>Experimental</td>
</tr>
<tr>
<td>Zhang et al. [38]</td>
<td>NACA 63 - 021</td>
<td>∞</td>
<td>Experimental</td>
</tr>
<tr>
<td>Rostamzadeh et al. [39]</td>
<td>NACA 0021</td>
<td>∞</td>
<td>Experiment/CFD</td>
</tr>
<tr>
<td>Cai et al. [40]</td>
<td>NACA 63 - 021</td>
<td>∞</td>
<td>Experiment/CFD</td>
</tr>
<tr>
<td>Torro and Kim [41]</td>
<td>NACA 0021</td>
<td>∞</td>
<td>CFD</td>
</tr>
<tr>
<td>Talboys et al. [42]</td>
<td>NACA 0021</td>
<td>∞</td>
<td>Experimental</td>
</tr>
<tr>
<td>Farouk [18]</td>
<td>NACA 0012</td>
<td>∞</td>
<td>CFD</td>
</tr>
<tr>
<td>Aftab and Ahmed [43]</td>
<td>NACA 4415</td>
<td>∞</td>
<td>CFD</td>
</tr>
</tbody>
</table>

The main focus of this present study is to investigate the changes in aerodynamic performance of wings with leading edge, trailing edge and leading and trailing edge undulations, the effect of the undulation
placement on the wingtip vortex, and the balance of lift induced and profile drag.

II. Experimental Setup

A. Wind Tunnel

All experiments were conducted at the University of Dayton Low Speed Wind Tunnel (UD-LSWT) in the open jet configuration. The UD-LSWT has a 16:1 contraction ratio, 6 anti-turbulence screens and 4 interchangeable 76.2cm x 76.2cm x 243.8cm (30” x 30” x 96”) test sections. The test section is convertible from a closed-jet configuration to an open-jet configuration with the freestream range of 6.7 m/s (20 ft/s) to 40 m/s (140 ft/s) at a freestream turbulence intensity below 0.1% at 15 m/s measured by hot-wire anemometer. The contraction feeds into a pressure/air sealed room where the test section is located. The effective length of the test section in the open jet configuration is 182 cm (72”). A 137 cm x 137 cm (44” x 44”) collector collects the expanded air on its return to the diffuser. The velocity variation for a given RPM of the wind tunnel fan is found using a Pitot tube connected to an TSI T600 Micromanometer.

B. Test Model Design and Fabrication

Nine different airfoil-preserved undulated wing cases (3 with trailing edge undulations, 3 with leading edge undulations and 3 with both leading and trailing edge undulations) were considered for the sensitivity study. The three cases for each undulation placement have different numbers of undulations on the wing surface (6, 9 and 12) which corresponds to $\lambda/c$ values of 0.31, 0.21 and 0.15 respectively. The value of the chord $c$ used in normalizing the wavelength is 12.7 cm. The different cases are depicted in Fig 2. The baseline wing has a chord length of 12.7 cm and a wingspan of 25.4 cm resulting in a semi-span AR of 2 (full span AR 4). The undulated wings have a mean chord of 11.43 cm (4.5 in) and an effective full-span AR of 4.5. The mean chord used in calculating the aerodynamic coefficients of the undulated wings was found by averaging the maximum chord and minimum chord. Table 2 depicts the wavelength, amplitude, surface area, and planform measurements for each undulated wing. A NACA 0012 airfoil was maintained throughout the span for all wing cases. The planform area was kept constant for all undulated wing at 285.35 cm² and the baseline NACA 0012 wing planform area was 325.48 cm².
Table 2 Dimensions for undulated wings

<table>
<thead>
<tr>
<th>Case</th>
<th>A, cm</th>
<th>λ, cm</th>
<th>Surface Area, cm²</th>
<th>Planform Area, cm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>LE λ/c 0.31</td>
<td>2.54</td>
<td>3.91</td>
<td>313.06</td>
<td></td>
</tr>
<tr>
<td>TE λ/c 0.31</td>
<td>2.54</td>
<td>3.91</td>
<td>308.23</td>
<td></td>
</tr>
<tr>
<td>LETE λ/c 0.31</td>
<td>0.635 (LE)</td>
<td>1.905 (TE)</td>
<td>3.91</td>
<td>308.02</td>
</tr>
<tr>
<td>LE λ/c 0.21</td>
<td>2.54</td>
<td>2.61</td>
<td>319.95</td>
<td></td>
</tr>
<tr>
<td>TE λ/c 0.21</td>
<td>2.54</td>
<td>2.61</td>
<td>311.16</td>
<td></td>
</tr>
<tr>
<td>LETE λ/c 0.21</td>
<td>0.635 (LE)</td>
<td>1.905 (TE)</td>
<td>2.61</td>
<td>310.71</td>
</tr>
<tr>
<td>LE λ/c 0.15</td>
<td>2.54</td>
<td>1.96</td>
<td>328.60</td>
<td></td>
</tr>
<tr>
<td>TE λ/c 0.15</td>
<td>2.54</td>
<td>1.96</td>
<td>315.17</td>
<td></td>
</tr>
<tr>
<td>LETE λ/c 0.15</td>
<td>0.635 (LE)</td>
<td>1.905 (TE)</td>
<td>1.96</td>
<td>314.32</td>
</tr>
</tbody>
</table>

All the test models shown in Fig 2 were generated in SolidWorks. To generate the airfoil-preserved undulations, two NACA 0012 airfoil profiles with different chord lengths were used. The larger airfoil had a chord of 12.70 cm while the smaller airfoil had a chord length 10.16 cm. The profiles were placed half a wavelength from each other and were lofted. In the case of wings with trailing edge undulations (TEU), the leading edge of both the smaller and larger airfoil profiles were kept at a constant location. For the wing with leading edge undulations (LEU), the trailing edge of both airfoils were kept at a constant location. The wings with leading edge and trailing edge undulations (LETEU) were modeled by placing the aerodynamic center of both airfoil profiles at the quarter-chord of the entire wing. The mean quarter-chord of the airfoils on TEU
wings is at 22.5% of the main wing chord and for the LEU wings, it is at 27.5% of the main wing chord. The loft command allows the airfoil cross-section to be preserved in all wings. The loft was then mirrored to produce the desired number of undulations. All wings were 3D printed using the University of Dayton Gorilla Maker 3D printer.

C. Force-Based Experiment

The ATI Gamma Sensor was used to measure the normal force (N) and axial force (A) on the wing which were transformed into lift and drag forces which were later converted to lift and drag coefficients, $C_L$ and $C_D$. The range and resolution for each direction of the sensor is shown in Table 3.

<table>
<thead>
<tr>
<th>Table 3 Specifications of ATI Gamma Sensor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_x, N$</td>
</tr>
<tr>
<td>Range</td>
</tr>
<tr>
<td>Resolution</td>
</tr>
</tbody>
</table>

The sensor was located underneath the splitter plate along with the Griffin motion rotary stage which was used to control the angle of attack. The rotary stage was controlled using Galil motion software. A schematic of force-based test setup is shown in Fig 3.

Table 4 shows the test matrix for the force-based experiments. All undulated wings and the baseline NACA 0012 wing were tested at a freestream velocity of 35 m/s which corresponds to a Reynolds number of approximately 282,000. The experiments were conducted at an angle of attack range of $-17^\circ$ to $17^\circ$ in $1^\circ$ increments.
increments. Two trials of each case were conducted to ensure repeatability.

Table 4 Test Matrix for Force-Based Experiments

<table>
<thead>
<tr>
<th>Case</th>
<th>Number of Undulations</th>
<th>Size, m x m</th>
<th>Freestream speed, m/s</th>
<th>AOA Range, °</th>
<th>AOA Increment, °</th>
<th>Reynolds Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>LE λ/c 0.31</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TE λ/c 0.31</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LETE λ/c 0.31</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LE λ/c 0.21</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TE λ/c 0.21</td>
<td>9</td>
<td>0.127 x</td>
<td>35</td>
<td>-17 to 17</td>
<td>1</td>
<td>281,587</td>
</tr>
<tr>
<td>LETE λ/c 0.21</td>
<td></td>
<td>0.254</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LE λ/c 0.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TE λ/c 0.15</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LETE λ/c 0.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BASELINE</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>312,874</td>
</tr>
</tbody>
</table>

D. PIV Experiment

Cross-stream Particle Image Velocimetry (PIV) was conducted to analyze how the undulations affected the wingtip vortex compared to the straight NACA 0012 wing (Fig 4).

![Fig 4 Schematic of PIV experiments and equipment used in UD-LSWT](Image)

In the force-based experiments, it was determined that the number of undulations do not have a significant impact on the aerodynamic performance of the wing. Therefore, the wings that were considered in the PIV experiment were chosen with the intent to represent each of the undulation placements (TEU, LEU, and
The wings were chosen based on which showed the best aerodynamic performance in the force-based experiments out of each undulation placement. The wings that were chosen to be compared to the baseline case in the PIV experiment were LE $\lambda/c$ 0.15, TE $\lambda/c$ 0.31, and LETE $\lambda/c$ 0.15. The test matrix is shown in Table 5.

**Table 5 Test Matrix for Cross-Stream PIV Experiment**

<table>
<thead>
<tr>
<th>Case</th>
<th>Number of Undulations</th>
<th>Size, m x m</th>
<th>Freestream speed, m/s</th>
<th>AOA Range, °</th>
<th>AOA Increment, °</th>
<th>Reynolds Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>LE $\lambda/c$ 0.15</td>
<td>12</td>
<td>0.127 x 0.254</td>
<td>25</td>
<td>2 to 10</td>
<td>2</td>
<td>201,133</td>
</tr>
<tr>
<td>TE $\lambda/c$ 0.31</td>
<td>6</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LETE $\lambda/c$ 0.15</td>
<td>12</td>
<td>0.254</td>
<td></td>
<td></td>
<td></td>
<td>223,481</td>
</tr>
<tr>
<td>BASELINE</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The PIV experiment was conducted using a Vicount smoke seeder with glycerin oil, a 200 mJ/pulse Nd:YAG frequency doubled laser (Quantel Twins CFR 300) and an Imperx B2021 camera with a 200 mm f/2.8 lens. A plano-convex and plano-concave lens were used in series to open the laser beam into a sheet. The laser sheet was placed 3 chord lengths behind the wing. The laser and camera were triggered simultaneously using a Quantum composer pulse generator. In each test case, 500 image pairs were obtained at a sampling rate of 15 Hz and were processed using ISSI Digital Particle Image Velocimetry (DPIV) software. Two iterations were performed in DPIV processing with 64-pixel interrogation windows in the first iteration and 32-pixel interrogation windows in the second iteration. Table 4 shows the test matrix for the cross-stream PIV experiment. The cross-stream PIV experiment was conducted at a Reynolds number of 201,000. This Reynolds number is lower than the Reynolds number observed for the force-based experiments in Table 4. This is because the freestream speed for the PIV experiment was chosen to be 25 m/s rather than 35 m/s due to wingtip vortex wandering being more prevalent at higher Reynolds numbers. At a lower Reynolds number, the changes in the vortex wandering between the wings can be quantified accurately. The camera was located more than 10 chord lengths downstream from the trailing edge of the wing to reduce the effects of the camera disrupting the flow. The time delay between the laser pulses is a function of the size of the field of view and the velocity in the wingtip vortex. The boundary of the wingtip vortex core (where the maximum azimuthal velocity is the highest) was chosen to calculate the time delay as the variation of velocity across a wingtip vortex is sinusoidal. As the maximum azimuthal velocity of the wingtip vortex increases with angle of attack,
it was necessary to change the time delay between the laser pulses at each angle of attack to obtain 8 to 10-pixel particle displacement at the wingtip vortex core boundary.

III. Force-Based Experiment Results

A. Coefficient of Lift

The coefficient of lift results from the force-based experiments are shown in Fig 5. The lift curve slope of all the wing cases are compared with the ideal lift curve slope predicted by the Helmbold Equation shown in Eq. (1):

\[ C_{L,a} = \frac{\pi AR}{1 + \sqrt{1 + \left(\frac{AR}{2}\right)^2}} \]  

(1)

It can be seen in Fig 5 that the lift coefficient for all the cases remains linear at lower angles of attack and agrees extremely well with theoretical predictions for AR 4 and AR 4.5. Major differences in the lift coefficients are only seen at higher angles of attack. It can also be seen that the undulations did not alter the linearity of the lift curve slope until stall. Any differences at lower angles of attack between the undulated cases falls within the standard deviation represented by the error bars. The differences between the cases are only statistically valid at higher angles of attack. The errors bars shown in Fig 5 indicate the 95th percentile confidence interval.

1. Sensitivity on Undulation Placement

Wings with LEU are illustrated in Fig 5. The lift coefficient for the LEU wings is comparatively lower than all other cases, especially at higher angles of attack. However, no discernible stall angle was observed, and the lift coefficient continues to increase with the increase in angle of attack and does not reach its maximum value within the angle of attack range considered. Similar post-stall behavior in the lift coefficient was documented in literature investigating the wings with leading-edge undulations [16], [17], [19]. At 17° the LEU case \( C_L \) exceeds that of the TEU case. The differences between the LEU wing \( C_L \) and the baseline are shown in Fig 5b in terms of percent difference between the observed values for the undulated wing in question and the baseline NACA 0012 wing. The percent difference was calculated at each angle of attack using Eq. (2). Similar calculations were performed for \( C_D \) and \( C_L/C_D \) shown in Fig 8b and Fig 12b respectively. As the magnitude of \( C_L \) and \( C_D \) are lower at lower angles of attack, even small differences
between the coefficients will result in a higher percent difference. However, it should be noted that the percent difference is with respect to the lower magnitude baseline lift coefficient. As the magnitude of $C_L$ and $C_D$ increase with the increase in angle of attack, the percent difference becomes more significant since it is calculated with respect to a higher magnitude baseline lift coefficient.

$$\text{% Difference} = \frac{C_{L\text{undulated}} - C_{L\text{baseline}}}{C_{L\text{baseline}}} \times 100\%$$

Up to $8^\circ$, the LEU wing lift coefficient is similar to the baseline but after $8^\circ$, the LEU wing lift coefficient decreases significantly on an average of 10%, the highest decrease being 17% at $16^\circ$. However, at $17^\circ$, the wing starts to perform better than the baseline as the LEU wing $C_L$ continues to increase while the baseline wing $C_L$ decreases post-stall.

Fig 5 Variation of $C_L$ a) comparison of all cases and b) percent difference from baseline NACA 0012 wing.
The lift coefficient of leading edge undulated wings shown in Fig 5 is compared with the results from Hansen et al. [35] (infinite AR), Miklosovic et al. [28] (AR 4.45), Shorbagy et al. [17] (AR 2) and Guerreiro and Sousa [34] (AR 1.5) in Fig 6. The airfoils used in these references are mentioned in Table 1. Even though different airfoils were used in these investigations, the results from present study compare well with the results from Miklosovic et al. [28] who investigated the humpback whale flipper model of AR 4.45. In all of these cases shown in Fig 6, stall was effectively mitigated at higher angles of attack. Except for the results from Hansen et al. [35], the different undulated wing cases show similar performance where the lift curves coalesce with each other at lower angles of attack similar to the results from the current study. The comparison with the results seen in the literature validate the data obtained in the current study.

TEU wings are represented by red curves in Fig 5. The TEU wings exhibited greater $C_L$ magnitude when compared to other undulated wings, even outperforming the baseline until the stall angle. Unlike the LEU wings, the TEU wings show an abrupt stall at 14°. Fig 5b illustrates that the TEU wings produce more lift until 14°, outperforming the baseline by an average of 10%. However, at angles of attack greater than 15°, the performance of the TEU wings is similar to the baseline.

LETEU wings are illustrated by yellow curves in Fig 5. The $C_L$ trends for these cases mirror that of both the LEU wing and TEU wing with no discernible stall angle and higher $C_L$ than the baseline wing at low angles of attack. The LETEU wings follow very closely with the TEU wings up until the wings begin to
stall at 12° at which point $C_L$ begins to deviate. Although $C_L$ is lower for LETEU wings as compared to the TEU wings from the region of 12° to 15°, $C_L$ continues to increase and shows superior performance at 17°. The better overall performance of the LETEU wings is also documented in similar studies where the undulations in leading and trailing edges are investigated, especially at the wingtips [17]. This result indicates that a combined LE and TE undulations help mitigate stall without sacrificing the $C_L$.

2. Sensitivity on Undulation Wavelength

The LEU wing cases in Fig 5b show that from 3 to 15°, the LE $\lambda/c$ 0.31 case exhibits a higher $C_L$ at higher angles of attack. At lower angles of attack, there are no significant differences between LEU wings with varying undulation wavelength. For the TEU and LETEU wings however, the differences in the $C_L$ are scattered at lower angles of attack and from 9°, $C_L$ becomes independent of undulation wavelength. From these observations, it appears there are no strong correlations between wing performance and undulation wavelength along the wingspan, especially prior to stall.

3. Variation in Lift Curve Slope

The lift curve slope of each case was calculated using the linear regions of Fig 5a and is represented in Fig 7. The experimental baseline NACA 0012 lift curve slope of 0.0648 deg$^{-1}$ is shown as a green line in Fig 7 for better comparison. The LEU wing lift curve slope is similar to the baseline and the TEU and LETEU wing cases show an increment in lift curve slope by 5%.

![Fig 7 Variation of lift curve slope with undulation placement and number of undulations](image)

This slight increment in lift curve slope could be attributed to the slight increase in the effective aspect ratio of the undulated wings. A different trend is observed for the TEU wings where the lift curve...
slope decreases from wavelengths of 3.91 cm ($\lambda/c \ 0.31$) to 2.61 cm ($\lambda/c \ 0.21$) and then increases from 2.61 cm ($\lambda/c \ 0.21$) to 1.96 cm ($\lambda/c \ 0.15$) while the other cases continually decrease in the lift curve slope with decreasing undulation wavelength (greater number of undulations). Despite the trends, the change in lift curve slope magnitude does not exceed 0.0015 deg$^{-1}$ within any undulation placement group, which suggests that the undulation wavelength does not have a significant impact on the wing performance. However, in terms of the undulation placement, similar trends are observed in Fig 7 as those observed in Fig 5 where the TEU wings show the highest $C_L$ and lift curve slope, the LEU wings perform least favorably in both $C_L$ and lift curve slope, and the LETEU wings perform better than LEU wings but not as well as TEU wings.

B. Coefficient of Drag

The coefficient of drag results from the force-based experiments are shown in Fig 8a. The variation in $C_D$ with angle of attack for each undulated wing is compared to the baseline NACA 0012 wing in terms of percent difference in Fig 8b.

1. Sensitivity on Undulation Placement

Similar to lift coefficient variation, there are no significant differences between the baseline and the other undulation cases until 8°. However, after 8°, significant deviations from the baseline case are observed. The drag coefficient of the LEU wings is significantly greater than the baseline at higher angles of attack as observed in Fig 8b. However, as observed in the lift coefficient variation, there is no abrupt increase in coefficient of drag due to stall which is beneficial for maneuverability. On average, the LEU wing increases drag by 30% when compared to the baseline at angles of attack lower than 15°. At 17°, the percent difference in $C_D$ decreases, a trend observed in all the undulated wing cases.

The TEU wings showed an overall favorable drag performance as compared to the baseline and the LEU wings, especially in the range of 8° to 14° as observed in Fig 8b. In this range, the TE $\lambda/c \ 0.15$ wing achieved a reduction in $C_D$ of approximately 10% as compared to the NACA 0012 wing. However, similar to the lift coefficient variation, the onset of stall is marked by a sudden increase in the drag coefficient. Soon after the stall angle, there is an abrupt reduction in $C_D$ at 17° as observed in Fig 8b.
Similar to the lift coefficient case, the LETEU wings perform better at lower angles of attack with comparatively lower $C_D$ and also shows post-stall benefits at higher angles of attack. Unlike the TEU wings, $C_D$ increases gradually near the stall angle. As shown in Fig 8b, the percent difference from the baseline of all LETEU wing cases are favorable from the range of 8° to 11°, where the percent difference is similar to that of the TEU wings.

2. Sensitivity on Undulation Wavelength

While the lift coefficient did not show any significant differences as a function of undulation wavelength, noticeable differences are seen in the drag coefficient, as seen in Fig 8b. The LEU wing cases show high scatter in the percentage difference in the drag coefficient from the baseline for different numbers of undulations, especially at lower angles of attack. The scatter in the $C_D$ deviation from the baseline is lower for the LETEU cases. However, no discernable trends in undulation wavelength are observed between the different undulation placement cases.

3. Induced Drag and Profile Drag

The lift coefficient variation for different cases shown in Fig 5a was used to determine the coefficient of induced drag using Eq. (3) below

$$C_{D_i} = \frac{C_L^2}{\pi \, e_v \, AR}$$

where $e_v$ is the span efficiency that considers viscous effects. Spedding and McArthur [45] proposed an expression for span efficiency equation as
\[ e_v = (1 + \delta + \kappa \pi \text{AR})^{-1} \]  

(4)

where \( \delta \) is the induced factor of drag [46] which is a function of Fourier coefficients that describe the distribution of lift on the wing. It can also be considered as a variable that measures the departure of the elliptic loading distribution. By inspection, \( e_v \) contains both the effect of AR and the approximate shape of the lift-drag polar in the form of fitting constant \( \kappa \), which was found to be 0.05. In 1926, Glauert [47] derived an expression for the induced factor drag and lift as

\[ 1 + \delta = \sum_{n=1}^{N} n \left( \frac{A_n}{A_1} \right)^2 \]  

(5)

where \( A_n \) is the Fourier coefficients which describe the lift distribution on a wing.

**Fig 9 a)** Variation of induced factor of lift (\( \tau \)) and induced factor of drag (\( \delta \)) with respect to AR/\( a_0 \)  
**b)** Variation of \( \delta \) with respect to \( \tau \).

Because the lift distribution on a contoured wing is unknown and may be uneven due to the presence of surface undulations, the value of \( \delta \) was determined by correlating it with the corresponding experimentally obtained induced factor of lift value \( \tau \). The induced factor of lift first appeared in Prandtl’s derivation of lifting line theory to predict the lift curve slope of a finite wing as a function of the 2D lift curve slope and aspect ratio given in Eq. (6).

\[
\frac{dC_L}{d\alpha} = \frac{a_0}{1 + \frac{a_0}{\pi \text{AR}} (1 + \tau)}
\]

(6)

where \( a_0 \) is the 2D lift curve slope which is equivalent to \( 2\pi \) rad\(^{-1}\). Using the lift curve slope values from Fig
6 for different wing cases, the value of $\tau$ was determined using Eq. (6). The corresponding value of $\delta$ for a given value of $\tau$ was determined from Fig 9a. The relationship between $\tau$ and $\delta$ shown in Fig 9 was developed by Glauert [47].

Table 6 shows the values of $\tau$, $\delta$ and $e_v$ for each wing case. The value of $\delta$ for a given wing case was then substituted in Eq. (3) and Eq. (4) to determine the variation of induced drag. The induced drag coefficient is then subtracted from the total drag coefficient obtained from the experiment to determine the profile drag coefficient. The variation of induced and profile drag coefficient is shown in Fig 10a and Fig 10b, respectively. Due to lower lift production, the LEU wing cases resulted in a lower induced drag coefficient when compared to the other undulated wings while remaining comparable to the baselines until the onset of stall. The TEU and LETEU cases show an increase in $C_{D_i}$ prior to the onset of stall which is also to be expected as these cases showed an increase in $C_L$ compared to the baseline and LEU wings. These differences can be seen more clearly at higher angles of attack as there are not significant deviations between any cases at lower angles of attack.

Table 6 Values of $\tau$, $\delta$ and $e_v$ for all wing cases tested

<table>
<thead>
<tr>
<th>Case</th>
<th>Number of Undulations</th>
<th>Induced Factor of Lift, $\tau$</th>
<th>Induced Factor of Drag, $\delta$</th>
<th>Span Efficiency, $e_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LE $\lambda/c$ 0.31</td>
<td>6</td>
<td>0.164</td>
<td>0.045</td>
<td>0.569</td>
</tr>
<tr>
<td>TE $\lambda/c$ 0.31</td>
<td>6</td>
<td>0.160</td>
<td>0.043</td>
<td>0.570</td>
</tr>
<tr>
<td>LETE $\lambda/c$ 0.31</td>
<td>6</td>
<td>0.186</td>
<td>0.056</td>
<td>0.566</td>
</tr>
<tr>
<td>LE $\lambda/c$ 0.21</td>
<td>9</td>
<td>0.105</td>
<td>0.021</td>
<td>0.577</td>
</tr>
<tr>
<td>TE $\lambda/c$ 0.21</td>
<td>9</td>
<td>0.108</td>
<td>0.022</td>
<td>0.577</td>
</tr>
<tr>
<td>LETE $\lambda/c$ 0.21</td>
<td>9</td>
<td>0.091</td>
<td>0.016</td>
<td>0.579</td>
</tr>
<tr>
<td>LE $\lambda/c$ 0.15</td>
<td>12</td>
<td>0.114</td>
<td>0.025</td>
<td>0.576</td>
</tr>
<tr>
<td>TE $\lambda/c$ 0.15</td>
<td>12</td>
<td>0.113</td>
<td>0.024</td>
<td>0.576</td>
</tr>
<tr>
<td>LETE $\lambda/c$ 0.15</td>
<td>12</td>
<td>0.123</td>
<td>0.027</td>
<td>0.575</td>
</tr>
<tr>
<td>BASELINE</td>
<td>0</td>
<td>0.124</td>
<td>0.035</td>
<td>0.601</td>
</tr>
</tbody>
</table>

The profile drag coefficient provides key insight into the way the contours are affecting the profile and pressure drag of the wing. At lower angle of attack cases, the profile drag stays relatively constant for all cases. At an angle of attack of 7°, the LEU cases show significant increase in the profile drag coefficient whereas the TEU wing cases show almost constant profile drag coefficient, similar to the baseline case until stall. As usual, the LETEU wing cases fall within the range between the LEU and TEU cases. Since slightly higher $C_L$ was observed for the TEU wings and LETEU wings as compared to the baseline, the $C_{D_{i}}$ values
also show the same trend. Similarly, the LEU wings show lower $C_D$ compared to the baseline at higher angles of attack since the LEU wings produce less lift than the baseline at higher angles of attack until stall.

The net $C_D$ of the wing is given by,

$$C_D = C_{D_0} + kC_L^2$$  \hspace{1cm} (7)

where $C_{D_0}$ is the zero lift drag coefficient and $k = 1/\pi eAR$. Both $C_{D_0}$ and $k$ can be obtained graphically by plotting the drag polar and $C_L^2$ as a function of $C_D$ respectively. The drag polar for the different undulated wing cases is shown in Fig 11a. The trends once again indicate there are no significant variations as a function of undulation wavelength, however, there are significant variations as a function of undulation placement.

The drag polar of the TEU case follows very close to that of the baseline wing. For a given magnitude of $C_D$, the TEU cases show an increased $C_L$ magnitude when compared to the baseline. The LEU wings, on the other hand, start to generate higher $C_D$ for a given magnitude of $C_L$ with increase in angle of attack. The LEU drag polar trend starts to deviate from the baseline at a $C_L$ magnitude of 0.5, beyond which the wing produces significantly higher drag coefficient than the lift coefficient. This point is denoted as the “inflection point” in Fig 11a. Keeping consistent with the trends seen in the $C_L$ and $C_D$ plots above, the LETEU wing cases fall in between the LEU and TEU wing cases. The addition of TEU along with LEU increases the “inflection point” to a higher $C_L$ magnitude of 0.7. The drag polar trends once again indicate that a combination of LEU and TEU is beneficial in delaying stall without sacrificing $C_L$.

![Fig 10 Variation of a) induced drag and b) profile drag with angle of attack](image-url)
The $C_{D0}$ obtained from the drag polar curves for the different wing cases are listed in Table 7. The $C_{D0}$ values for the LEU cases are comparatively higher than the $C_{D0}$ values for the TEU and LETEU cases. The LETEU case once again falls in between LEU and TEU cases. Since $C_{D0}$ indicates the zero-lift drag coefficient, it is comprised of the profile drag of the wing. The drag polar trends indicate that placing the undulations along the LE increases the profile drag of the wing when compared to placing the undulations along the TE.

The drag due to lift is quantified by the second term on the RHS of Eq. (7) which is a function of wing aspect ratio and span efficiency. The factor $k$ can be graphically obtained by finding the slope of the $C_D$ vs $C_L^2$ curve since Eq. (7) is essentially an equation of a line where the slope is $k$. The variation of $C_D$ vs $C_L^2$ is shown in Fig 11b. Since the magnitude of $C_L$ is squared, any slight differences between the undulated cases will be magnified which allows the formation of significant conclusions. An ideal $C_D$ vs $C_L^2$ curve will remain linear until stall. However, the presence of undulations on the wing makes the trends in Fig 11b far from ideal. Both the TEU and baseline cases follow similar trends in Fig 11b with the total drag coefficient being lower in magnitude for a given $C_L^2$ value when compared to the baseline. The presence of stall can clearly be seen where the magnitude of $C_D$ increases significantly with decrease in $C_L^2$. This trend is absent in the LEU and in the LETEU cases. The linearity of the slope starts to deviate in the LEU case at a $C_L^2$ value of 0.3 and the slope increases significantly thereafter and becomes non-linear. However, the magnitude of $C_D$ does not decrease with increase in $C_D$. The deviation from linearity occurs at a $C_L^2$ value of 0.56 for the LETEU case and shows similar trend at higher $C_L^2$ values as the LEU case. The trends once again indicate

**Fig 11 a) Drag polar b) Variation of $C_D$ vs $C_L^2$ for different undulated wing cases**
that wings with both LEU and TEU delay stall while maintaining $C_L$. The average slope of the linear region of $C_D$ vs $C_L^2$ variation for the different undulated wing cases shown in Fig 11b was determined to be 0.12 using linear regression. The average $R^2$ value of the regression was 0.98. The range of $C_L^2$ values used to calculate the slope in each undulation location cases are annotated in Fig 11b using solid lines. The corresponding span efficiency $e$ calculated was found to be 0.6 which closely matches with the span efficiency calculated by Eq. (4).

C. Aerodynamic Efficiency

The aerodynamic efficiency quantified by the ratio of $C_L$ and $C_D$ for the different wing cases are shown in Fig 12a along with the percentage differences in the efficiency between the undulated wing cases and the baseline shown in Fig 12b.

1. Sensitivity on Undulation Placement

As expected, the LEU wings show the least favorable aerodynamic efficiency magnitude and variation with respect to angle of attack when compared to other wing cases. The LEU wings exhibited a lower aerodynamic efficiency than the baseline wing at all angles of attack with the exception of the LE $\lambda/c_{0.15}$ wing from 4° to 8°. The $C_L/C_D$ ratio reaches its peak from 6° to 7° for the LEU wings and then decreases with increase in angle of attack falling significantly less than the baseline wing efficiency. As shown in Fig 12b, the LEU wings show a significant reduction in aerodynamic efficiency beyond 7°.

The TEU wings on the other hand showed an overall favorable aerodynamic efficiency compared to the baseline wing with the exception of the TE $\lambda/c_{0.15}$ case which shows a trend similar to that of the baseline wing. These variations were to be expected as the TEU wings produced higher lift and lower drag than the baseline for most of the angle of attack range considered. Fig 12b shows that the TE $\lambda/c_{0.31}$ case achieved an almost 20% increase in aerodynamic efficiency as compared to the baseline wing and did so consistently from 6° to 14°.

All LETEU wings showed a higher $C_L/C_D$ than the baseline from the range of 4° to almost 12°, and then decrease in a trend similar to that of the LEU wings when the wings reach stall. Fig 12b indicates that the LETE $\lambda/c_{0.15}$ wing achieved an increase in aerodynamic efficiency of almost 20% from approximately 6° to 8°.

2. Sensitivity on Undulation Wavelength
While the TE $\lambda/c$ 0.31 case performs the best out of the TEU cases, the LE $\lambda/c$ 0.15 and LETE $\lambda/c$ 0.15 cases perform best out of the LEU and LETEU cases. This shows that there is no strong correlation between aerodynamic efficiency and the number of undulations.

**Fig 12 Variation of $C_l/ C_D$ a) comparison of all cases and b) percent difference from NACA 0012 wing.** The maximum deviation in the aerodynamic efficiency between the number of undulations for any particular case is on the order of 10% with greater deviations seen in both the LEU and TEU cases as compared to the LETEU cases. This could be attributed to the scatter in the $C_D$ measurements at lower angles of attack. All the LETEU wings showed a higher $C_l/C_D$ than the baseline from the range of 4° to almost 12°, and then decrease in a trend similar to that of the LEU wings at stall. Fig 12b indicates that the LETE $\lambda/c$ 0.15 wing achieved an increase in aerodynamic efficiency of almost 20% from approximately 6° to 8°. To analyze the way both leading edge and trailing edge undulations affect the wingtip vortex, high performing cases in each undulation placement category: the LETE $\lambda/c$ 0.15 case, TE $\lambda/c$ 0.15 case and LE $\lambda/c$ 0.15 case were investigated through PIV. The results from the force-based testing are summarized in Table 7.
IV. Wingtip Vortex PIV Results

A. Vortex Wandering Corrections

The oscillations of the wingtip vortex in the cross-stream plane, referred to as vortex wandering, if not corrected for, biases the wingtip vortex measurements causing the vortex to have a large radius with lower vorticity and circulation than it does. The vortex wandering was quantified by tracking the vortex center across the PIV image using Q-criterion, a well-established method to find vortex center in the flow.

The equation for Q-criterion is

$$Q = \frac{1}{2} \left( \|\Omega\|_2^2 - \|S\|_2^2 \right)$$

(8)

Where $\Omega_{ij}$ is the rotation tensor given by

$$\Omega_{ij} = \frac{1}{2} \begin{bmatrix} 0 & \left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) \\ \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) & 0 \end{bmatrix}$$

(9)

and $S_{ij}$ is the strain tensor given by

$$S_{ij} = \frac{1}{2} \begin{bmatrix} 2 \frac{\partial u}{\partial x} & \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \\ \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & 2 \frac{\partial v}{\partial y} \end{bmatrix}$$

(10)

An example Q-criterion contour is shown in Fig 13 where a higher Q magnitude is observed in the vortex center indicating a larger vorticity magnitude and lower strain magnitude in the vortex center.
The peak Q-locations in each image pair were found and shifted to the center of the field of view. Once the vortex centers were determined, the RMS wandering amplitude $\sigma$ given by Eq. (11) was quantified

$$\sigma = \sqrt{\sigma_x^2 + \sigma_y^2}$$

where $\sigma_x$ is the RMS wandering amplitude component in the horizontal x-direction and $\sigma_y$ is the RMS wandering amplitude component in the vertical y-direction. On average the $\sigma$ in each case was determined to be around 0.014 meters.

**B. Wingtip Vortex Azimuthal Velocity**

In order to quantify the effect of undulations on the wingtip vortex growth and evolution, the wingtip vortex normalized azimuthal velocity profiles of the baseline and the undulations wings are compared against the well-known Batchelor’s wingtip vortex model [48] given by

$$\frac{U}{U_{\text{Max}}} = \left(1 + \frac{1}{2\alpha_L}\right) \frac{1}{\eta} \left(1 - \exp(-\alpha_L \eta^2)\right)$$

where $U_{\text{Max}}$ is the maximum velocity in the X direction, $\alpha_L$ is Lambs constant: 1.256, $\eta$ is $r/r_c$ where $r$ is the radial location in the wingtip vortex and $r_c$ is wingtip vortex core radius determined by the radial location where maximum azimuthal velocity occurs.
For most cases shown in Fig 14, it can be seen that the velocity distribution follows the Batchelor’s model trend very well at $r/r_c$ greater than 1. However, inside the wingtip vortex core, deviations from the Batchelor’s model can be observed, especially at lower angles of attack. This is due to the lower tangential velocity of the wingtip vortex at lower angle of attack cases as well as a strong interaction with the free shear layer wake. This interaction is well documented in [49] and [50]. The undulated wings also show similar behavior as the baseline but do not show higher deviations from the model at lower angles of attack ($2^\circ$) as observed in the baseline case.

C. Wingtip Vortex Vorticity

The normalized z-vorticity of the wingtip vortex shown in Fig. 15 was determined numerically from the $\Omega_{z,1}$ component for the rotation tensor in Eq. (9) using the central difference technique. It can be seen that the magnitude of the vorticity increases with an increase in angle of attack for all cases considered. A quick visual investigation of the contours reveals no significant variations between the vorticity contours with the exception of LE $\lambda/c$ 0.15 case. The wingtip vortex was found to be smaller at each angle of attack when compared to the other cases. This indirectly indicates a reduction in lift as observed in the lift coefficient plot shown in the previous section.
D. Wingtip Vortex Circulation

After determining the vortex center through Q-criterion, the wingtip vortex circulation was determined as a function of wingtip vortex radius. This wingtip vortex circulation as a function of vortex radius obtained from the experimental data is then compared to the ideal Lamb-Oseen vortex model [51] described by Eq. (13)

\[
\Gamma(r) = \Gamma_0 \left( 1 - \exp \left( -\frac{r^2}{r_c^2} \right) \right)
\]  

(13)

The \( \Gamma_0 \) in Eq. (13) was changed until a good fit (a good \( R^2 \) value) was achieved between the model and the experimental data. Once the \( R^2 \) value reached its maximum, the corresponding \( \Gamma_0 \) value was designated as the value of circulation for the wingtip vortex for each case at each angle of attack. This method of determining wingtip vortex circulation was developed by Stevens [52] and has been employed in Corkery et al. [53] and Stevens and Babinsky [54]. Fig. shows an example of experimental data compared to the Lamb-Oseen vortex to find wingtip vortex circulation. The Lamb-Oseen model is fitted with the experimental data for the LE \( \lambda/c \) 0.15 wing at 10° angle of attack by finding the \( \Gamma_0 \) which results in the highest \( R^2 \) value. In this

Fig 15 Normalized vorticity from 2° to 10° angles of attack for baseline and undulated wings
scenario, a value of 0.6 m$^2$/s resulted in the highest $R^2$ value of 0.999. The same method was used to obtain the circulation for all wings from 2° to 10° angles of attack. The average $R^2$ value for all cases was 0.985.

Fig 16 a) Variation of circulation as a function of vortex radius b) Determining total circulation of wingtip vortex using the Lamb-Oseen vortex model

The variation of normalized circulation with angle of attack for each case is shown in 17a. For all cases, circulation increases with increase in angle of attack as expected. The undulated wings show similar variation compared to the baseline at 2° and 4° before beginning to trend below the baseline from 6 to 10°. This shows that for a given angle of attack, the undulated wings produce a lower wingtip vortex circulation, and therefore weaker wingtip vortex than the baseline case. The TEU case shows a higher wingtip vortex circulation for a given angle of attack compared to the LEU and LETEU cases. The variation of normalized circulation with $C_{D,i}$ obtained from experimental data is shown in Fig b. The undulated wings trend below the baseline case as $C_{D,i}$ increases. For a given coefficient of induced drag, the undulated wings produce lower wingtip vortex circulation than the baseline case, with the LETEU cases showing the lowest circulation. This trend is mirrored in the variation of circulation with $C_L$ shown in 17c. Again, the undulated wings trend below the baseline.
Fig 17 Variation of circulation as a function of a) angle of attack, b) coefficient of induced drag, c) coefficient of lift, and d) coefficient of drag

For the same value of $C_L$, the undulated wings produce lower wingtip vortex circulation than the NACA 0012 wing with as high as almost 33% difference in the LETEU case at a $C_L$ of about 0.64. This shows that the undulated wings are producing the same $C_L$ with a lower wingtip vortex circulation. This violates the Kutta-Joukowski theorem of linear variation between the $C_L$ and circulation. However, it should be noted that the Kutta-Joukowski theorem was derived under the assumptions of inviscid and irrotational flow. It is possible that the undulations increased the viscous interactions of the wingtip vortex when compared to the baseline leading to a lower wingtip vortex circulation value for the lift coefficient condition. The change in normalized circulation with $C_D$ is also shown in 17d. The undulated wings and NACA 0012 wing trend together with the exception of the LEU case. Given a certain circulation value, the LEU case produces a higher $C_D$ than the NACA 0012 wing, TEU case, and LETEU case. This trend is in excellent agreement with the increase in $C_D$ that is observed for the LEU case in the force-based data (Fig 8).

In order to understand the lower circulation magnitude for the same $C_L$ condition in the undulated
wing cases, the variation of wingtip vortex circulation is compared with the wingtip vortex Reynolds number defined by Eq. (14).

\[ Re_{\text{Vortex}} = \frac{\rho V_Z r_c}{\mu} \]  

(14)

where \( V_Z \) is the wingtip vortex azimuthal velocity, \( r_c \) is the wingtip vortex core radius and \( \mu \) is dynamic viscosity. The vortex Reynolds number indicates the ratio of the vortex inertia to the viscous forces and was calculated for each wing case for each angle of attack. The circulation of the wingtip vortex for all the cases when plotted against the vortex Reynolds number shows a slight increase in the circulation for a given vortex Reynolds number for the LEU and LETEU cases when compared to the baseline at higher angles of attack (Fig 18). It can be observed in Fig 18 that the curves for all cases collapse at lower angles of attack. Beginning at a Reynolds number of 100, the TEU case deviates from the other cases where a higher vortex circulation is achieved at a given vortex Reynolds number. Figures 17b-c showed that for a given circulation, a lower \( C_{D,i} \) and a higher \( C_L \) are achieved by the TEU case. This observation combined with the observation from Fig 18 suggests a relationship may exist between the vortex Reynolds number and \( C_L \), namely that the vortex Reynolds number for the TEU case would be lower than other wings producing the same \( C_L \). This means that the TEU case may either reduce the vortex inertia or increase the viscous forces. This relationship requires further investigation.

![Fig 18 Variation of wingtip vortex Circulation with wingtip vortex Reynolds number](image_url)

Fig 18 Variation of wingtip vortex circulation with wingtip vortex Reynolds number

Ever since the design of the first jumbo jet, the mean and fluctuating quantities of the wingtip vortex has been studied in relation to steady state aerodynamic forces (lift and drag force on an aircraft) in an effort to mitigate the strength of wingtip vortices [55]. The rollup of wingtip vortex, although being a complex process, is directly influenced by the aerodynamics around the wing which is also responsible for the
aerodynamic forces. Direct correlations between the Reynolds stress and RMS of the wingtip vortex and the drag force have been documented in detail in [56]. From a turbulent standpoint, the fluctuations derive energy from the mean flow which is responsible for the aerodynamic forces observed on the wings. Therefore, the connection between the mean forces and the fluctuations provide a greater insight into the effect of undulations on the wingtip vortex. Hence, the relationship between the wingtip vortex RMS and the steady state coefficient of lift and drag is explored in the section below along with the vortex Reynolds number. The decrement in the wingtip vortex Reynolds number is accompanied by the increase in the wingtip vortex RMS of the velocity fluctuations. The RMS of the velocity is calculated by

\[ U_{\text{RMS}} = \sqrt{u'^2} \]  

(15)

where \( u' \) is the fluctuating velocity about the x-axis. The variation of the freestream normalized peak \( U_{\text{RMS}} \) with coefficient of lift, the vortex Reynolds number and the coefficient of drag is shown in Fig 19a, Fig 19b, and Fig 19c, respectively. It is readily observed that the TEU wing case has the highest RMS until a certain lift coefficient of value of 0.4. As the \( C_L \) increases, the RMS in the baseline case increases.

\[ \text{Fig 19 Variation of the normalized peak } U_{\text{RMS}} \text{ as a function of } a) \ C_L \ b) \text{ vortex Reynolds number and } c) \ C_D \]
The contour plots of the $U_{RMS}$ around the $C_L$ value of 0.4 is shown in Fig 19a. for both the TEU wing case and the baseline. The contour plots indicate the trend observed in the peak $U_{RMS}$. The TEU case has the highest RMS in the core of the vortex when compared to the baseline for the same value of $C_L$. Similar trends are also observed in the variation of peak RMS with the vortex Reynolds number. The TEU wing case has the higher RMS for a given vortex Reynolds number indicating the increased turbulence level in the vortex when compared to the other cases. There seem to be a correlation between the RMS and the aerodynamic performance of the wing. At lower angles of attack, as indicated in the lift, drag and aerodynamic coefficient plots, the TEU wing case delivered a superior performance when compared to the baseline. And correspondingly, the RMS in the vortex core for the TEU wing case is also higher than the other cases for a given $C_L$ and vortex Reynolds number. This correlation is still observed on the other end of the spectrum. The LEU wing case delivered poor performance overall at lower angles of attack as shown in the lift coefficient, drag coefficient and aerodynamic performance plots. Coincidentally, the wingtip vortex core RMS is significantly lower at any given $C_L$ condition greater than 0.4 and a given vortex Reynolds number greater than 100. Similar trends are also seen in the peak RMS vs $C_D$ where the TEU wing shows a greater RMS at a given coefficient of drag condition when compared to all the other cases. Coincidentally, the LEU wing case shows lower RMS when compared at a given $C_D$ condition. All these correlations indicate that the vortex RMS is a good indicator of the aerodynamic performance of the wing.

V. Conclusions

Wings with airfoil preserved undulations were investigated to determine their effect on the aerodynamic performance and on the balance of induced and profile drag of the wing.

- The force-based aerodynamic coefficients show clear differences in aerodynamic performance between the TEU, LEU, and LETEU cases even at lower angles of attack. The TEU wing cases perform better than the LEU and LETEU wing cases, especially at lower angles of attack. At higher angles of attack, the LEU and LETEU wing cases hint the post-stall benefits observed in the previous literature. It was also observed that the zero-lift drag coefficient and induced drag coefficient has fundamentally changed between the undulated wing cases.

- The lift coefficient seems to be independent of the undulation wavelength for the TEU, LEU and
LETEU wing cases. However, subtle changes were observed between the undulation wavelengths under study in drag coefficient and in the aerodynamic performance. However, no discernible trends were observed.

- The top performing candidates of each category: TE $\lambda/c$ 0.31, LE $\lambda/c$ 0.15, and LETE $\lambda/c$ 0.15 cases were chosen for PIV investigations, to provide insight into the way undulations affect the balance of induced and profile drag of the wing. The TEU wing case provided superior aerodynamic efficiency (higher lift coefficient but lower induced drag) and showed an increased circulation for a similar vortex Reynolds number when compared to the other cases. The reduction in the wingtip vortex Reynolds number is compensated by the increased RMS in the wingtip vortex core. This relationship is also observed in the LEU wing case which showed poor aerodynamic performance at low angles of attack and resulted in a lower RMS in the wingtip vortex core. This relationship indicate that the vortex RMS has an inverse relationship with the aerodynamic performance and the wingtip vortex RMS plays a key role in affecting the balance of induced and profile drag of the wing.
VI. References


[34] Favier, Julien, Alfredo Pinelli, and Ugo Piomelli. "Control of the separated flow around an airfoil using a


