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Spectral Analysis of Encrypted Chaotic Signals Using Fast Fourier Transforms and Laboratory Spectral Analyzers

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ABSTRACT

The use of acousto-optic chaos, as manifested via first-order feedback in an acousto-optic Bragg cell, in encrypting a message wave and subsequently recovering the message in the receiver using a chaotic heterodyne strategy, has been reported recently [1-3]. In examining the dynamical system analytically using computer simulation, (expected) modulated chaos waveforms are obtained within specified observation windows. Because of the relatively random nature inherent in chaos waveforms, it is essentially impossible to ascertain from the visual display of the chaotic wave whether a given message signal has in fact modulated the chaotic "carrier". In fact, it has been observed from earlier work that by appropriately controlling the chaos parameters, one may "hide" the silhouette of the message from the envelope of the modulated chaos [1]. This was found to be especially true for low-frequency chaos (in the KHz range). For chaos in the mid-RF (up to 10s of MHz) range, it is seen that the silhouette is more difficult to suppress (even though this does not affect the robustness of the encryption). To adequately determine whether modulation has in fact occurred by passing the AC signal through the sound cell bias input, one needs to examine the spectral content of the chaos wave. In this paper, we discuss the results of such spectral analyses using two different approaches, (i) fast Fourier transforms applied to the displayed waveform; and (ii) transferring the intensity-vs-time data to an Excel spreadsheet, and then applying this information to a laboratory spectrum analyzer with adequate bandwidth. The results are mutually compared and interpreted in terms of encryption and decryption properties.

Keywords: Acousto-optics, chaos, feedback, encryption, nonlinear dynamics, modulation, chaos spectrum, FFT, waveform capture, spectrum analyzer.

1. INTRODUCTION AND BACKGROUND

In recent work [1-3], it has been established that chaotic modulation and retrieval of RF signals using an A-O hybrid feedback device under first-order feedback depend strongly on the feedback gain ($\widetilde{\beta}$), the feedback time delay (TD), the amplitude (I_{inc}) of the incident light, the initial value of the intensity ($I_1(0)$) and the effective bias voltage ($\hat{\alpha}_0$). Using the chaotic properties of the hybrid acousto-optic feedback device (HAOF), Chatterjee and Al-Saedi reported that it is possible to *encrypt* relatively low-bandwidth signals (in the few KHz range) within the chaos wave, and subsequently transmit, receive, heterodyne, filter and recover the message signal from this chaotically encrypted carrier [2]. It has been shown that secure information communication for a few simple test signals applied through the acoustic bias input is possible with the HAOF device with reasonably narrow parameter (or decryption key) tolerances. Subsequently, Chatterjee and Kundur have shown that the modulation BW may be increased to the mid-RF range by appropriately shifting the chaos regime by adjusting the time delay down to the sub-μs range, with

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appropriate choice of the feedback gain in order to ensure stable, chaotic operation throughout the dynamic range of the signal [3].

While the mid-RF chaos generation and signal modulation experiments via Matlab simulation using simple time-varying signals in the RF domain have yielded satisfactory results, it has usually been found that the chaotic waveform with *and* without RF signal input does not provide any indication that modulation has taken place. True, in many cases the signal envelope manifests itself in the envelope of the chaos, but for the most part this is not consistently so. In some cases, with minor "tweaking" of one or more system parameter(s), the signal waveshape disappears from the chaos envelope.

A second critical issue deals with the specific form of modulation that is manifested in the chaos. Because of the nature of chaos itself, it is difficult to conclude definitively that the modulation resulting from adding an RF signal to the bias input of the RF source in the HAOF device is *amplitude modulation* (AM). We must note that the A-O system under investigation here is actually comprised of several levels of modulation. The first-order light itself is shifted in frequency by the acoustic frequency; hence any signal-dependent shift of the acoustic frequency would amount to an optical frequency/phase modulation. Next, we note that the detected first-order photocurrent is related to the *intensity* of the first-order light. Hence, in the absence of any modulation of the *amplitude* of the first-order light, the resulting first-order photocurrent would be a DC signal. When the feedback parameters are adjusted to where the photocurrent exhibits chaos, clearly the DC photocurrent breaks into chaotic oscillations in time (and also oscillations in the $\hat{\alpha}_0$ space, as is well known). These chaotic oscillations nominally exhibit a small amount of *self- amplitude modulation* leading to a spectral shift, as has been demonstrated [1]. When an RF signal is added to the bias input of the HAOF device, it has been established by both intuition as well as a series of simulation results that the chaos waveform evident in the photocurrent is indeed a form of chaotic amplitude modulation. This fact has been further corroborated by the recovery of the RF signal using a *heterodyne detection* strategy, which clearly applies to AM carriers.

During the process of extending the chaotic signal modulation to the mid-RF (few MHz) range, and later, for the case of digitized image waveforms, it became necessary to establish whether the observed chaos waveforms under RF input signals were indeed demonstrating modulation, and specifically AM in their behavior. As mentioned, this cannot be established simply by obtaining the time waveforms of the "modulated" chaos. As a result, spectral analysis of the chaos waveform becomes necessary, whereby possible modulation characteristics might be ascertained. In section 2, we present results from Fast Fourier Transforms (FFTs) applied to finite time snapshots of the chaos waves, and compare these with (possible) equivalent AM modulated carrier waveforms with similar frequency characteristics. In section 3, we discuss an alternative spectral analysis strategy that does not require the FFT approach. Instead, the amplitude and time data of the chaos wave obtained from the Matlab simulation is stored in a separate file, and later the data is applied to the data input of a laboratory spectrum analyzer which then displays the corresponding spectral characteristics. The results using the two approaches are compared. Finally, chapter 4 presents concluding remarks and possible areas for future work.

2. CHAOS AND MODULATION IN THE MID-RF FREQUENCY RANGE

2.1 Realizing chaotic oscillations at 10 MHz or higher (from [3])

The task at this stage was to simulate and realize chaos in the HAOF system in the MHz frequency range. The main bottleneck in the previous attempts at obtaining chaos in the RF range was that for feedback delays in the μs or lower range, the corresponding computation time tended to become quite large. In the current work, it was decided to attempt running chaos simulations for sub- μs delays by using a fast laptop or tablet PC. The machine used had a 4 MHz RAM, and yielded results in a relatively short amount of time (typically several minutes). Fig.1 shows the HAOF schematic at the transmitter. An A-O optic cell operating in the Bragg regime has the first-order light detected by a photodetector (note that this process yields an output current related to the intensity of the first order light, thereby eliminating (or suppressing) the optical frequency (which is nominally shifted from the incident optical frequency by the RF/ultrasonic frequency). The detected current is then amplified (gain $\tilde{\beta}$) and returned to the bias driver input of the RF source. The effective detection and return time delay per pass is TD seconds.

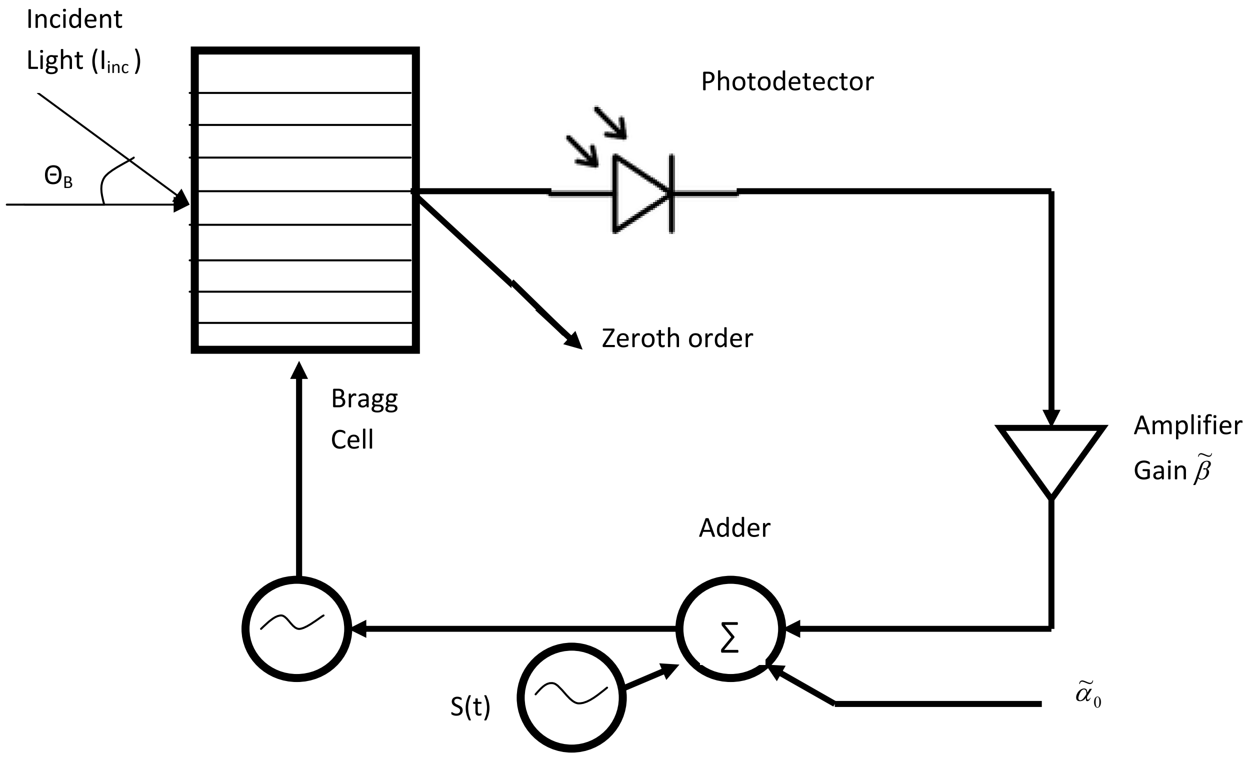
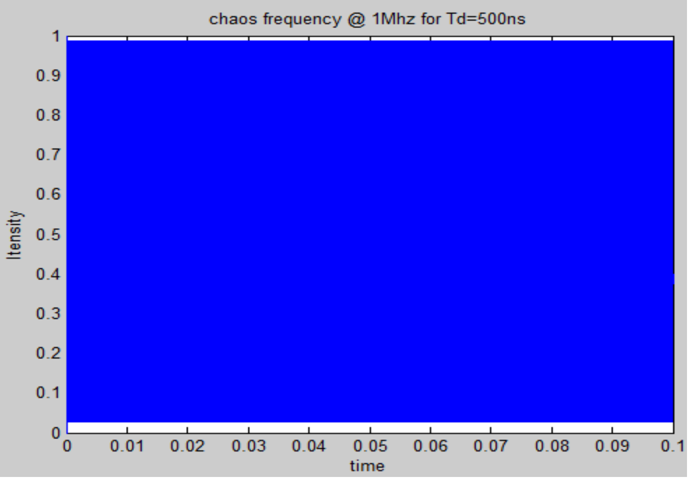


Fig.1. Schematic diagram of A-O Bragg cell with first-order feedback and bias inputs.

We note that the bias input in this system consists of three signals- (i) a DC bias signal that establishes the operating point for the chaotic wave; (ii) a feedback signal derived from the photodetected, amplified and delayed first-order scattered optical output; and (iii) an information signal, S(t), applied to the bias input of the RF. By decreasing the time delay and increasing the value of β to 4, the chaotic oscillations can be pushed into the mid RF range where the chaos oscillates in the neighborhood of as much as 10 MHz. Figs. 2 and 3 show the chaos at 1 MHz for T_d=500ns. Fig. 3 shows the snapshot of Fig. 2 that shows the detailed oscillations. The average chaos frequency is calculated by counting the number of zero paddings and taking the reciprocal in time. Fig. 4 shows chaotic oscillations at 10 MHz for time delay of 50ns in the feedback loop. Fig 5 shows a 1.5μs snapshot of Fig.4 that shows deatiled oscillations. From the figures, it is evident that from the time scales of the plots, the chaotic oscillations are either very dense (Figs.2 and 4), or show oscillations with slow envelope variations (Figs. 3 and 5), as expected for self-AM, as discussed above. We also note here that the average chaos frequency (obtained by calculating the oscillation periods for several cycles and then averaging the result, and finally inverting the average oscillation period) is approximately equal to one-half the inverse of the feedback delay time.

At this stage, we have also used the data generated from the Matlab programs to test these results on oscilloscopes to check for their compatibility. The data generated from the Matlab programs is stored in an Excel file and connected to a arbitary signal generator. Then the output of the signal generator is connected to an oscilloscope to observe the chaos signal as a time waveform. Fig.6 shows the oscilloscope output.



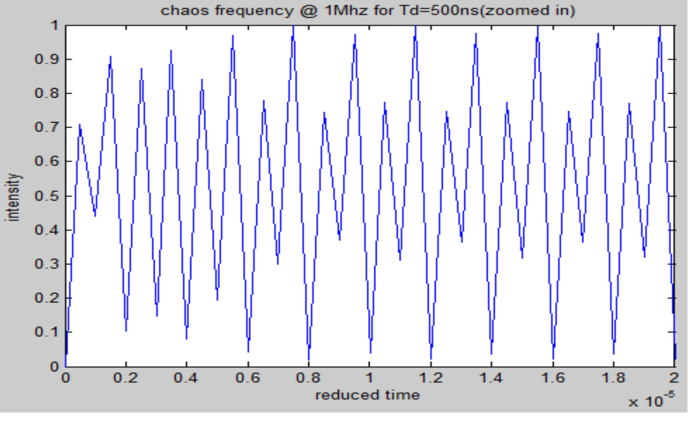
Beta=4

α_0=2

I_{1inc}=1

$I_1(0)$=0

Fig.2. Chaos Frequency of 1MHz @ T_d=500ns



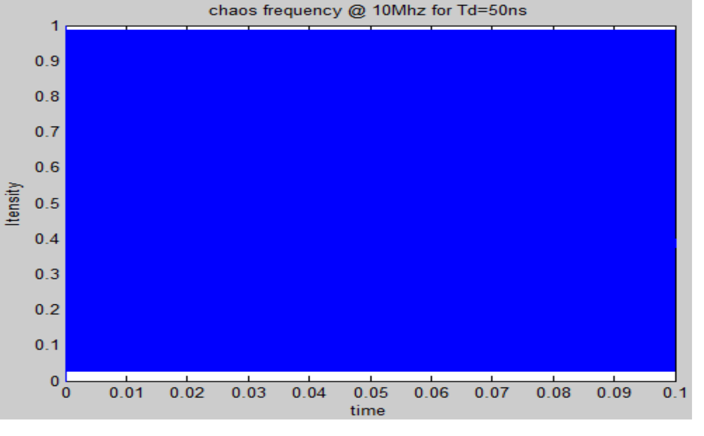
Beta=4

α_0=2

I_{1inc}=1

$I_1(0)$=0

Fig.3. 20µs snapshot chaos Frequency of 1MHz @ T_d=500ns



Beta=4

α_0=2

I_{1inc}=1

$I_1(0)$=0

Fig.4. Chaos frequency of 10 MHz @ T_d=50 ns.

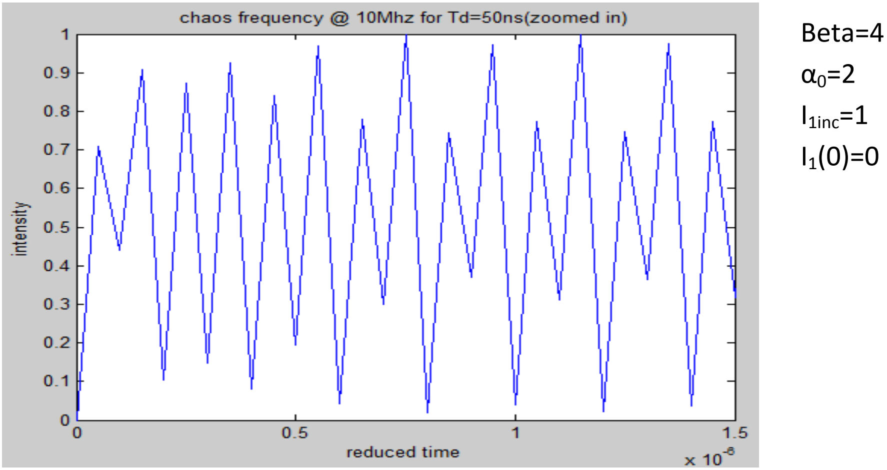


Fig.5. 1.5μs snapshot for chaos frequency of 10 MHz @ T_d=50 ns.

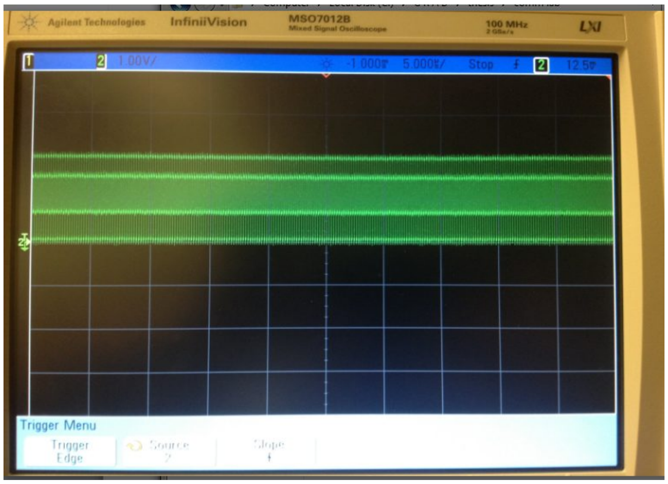


Fig.6. Chaos frequency of 10MHz @ T_d= 50 ns displayed on an oscilloscope.

2.2 Chaotic modulation for simple RF waveforms (from [3,4])

In this section, we present examples of signal modulation of a chaotic carrier for simple periodic RF waveforms such as a triangular and a rectangular pulsed waveform. The following figures show the corresponding chaotic waveforms both as Matlab output as well as oscilloscope displays obtained as described above. Two such periodic waveforms are shown in Figs.7 and 8. Note that in each case, a DC bias voltage needed to be added to the signal waveform in order that the total bias voltage would vary in a range that ensures chaotic operation of the physical system over the dynamic range of the signals. The bias voltage for these simulations was chosen as 3.5 V based on results obtained via Lyapunov exponent studies and bifurcation maps. Figs.9 and 10 show the corresponding "modulated" chaos waveforms obtained via the Matlab simulation. Note that the waveforms by themselves do not necessarily indicate simple amplitude modulation of a carrier wave by a periodic waveform. The determination of the nature of the modulation is what necessitates spectral analysis, as aimed for reporting in this paper. Figs.11 and 12 show the corresponding oscilloscope displays of the modulated chaos waveforms.

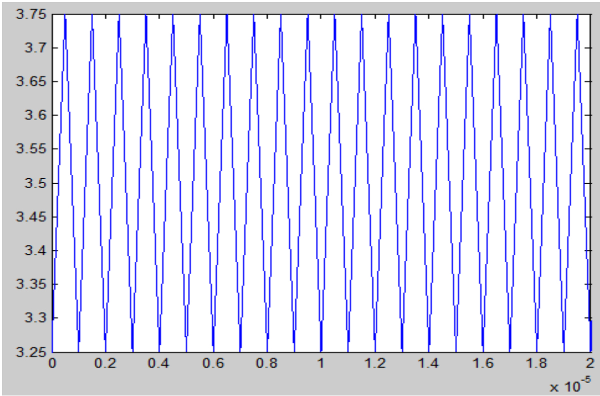


Fig.7. Periodic triangular wave with DC bias.

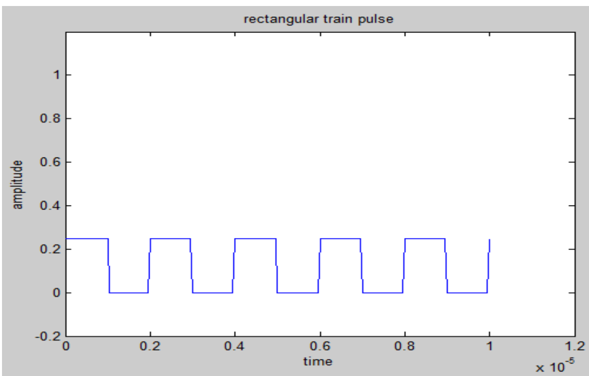
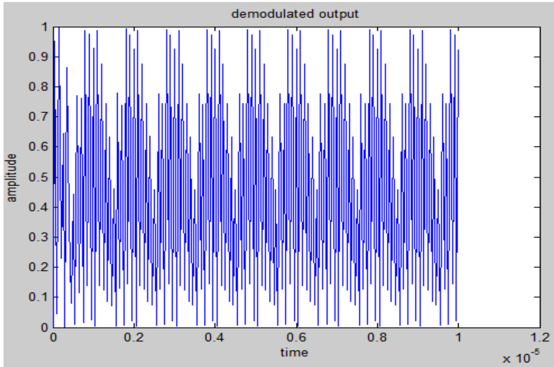
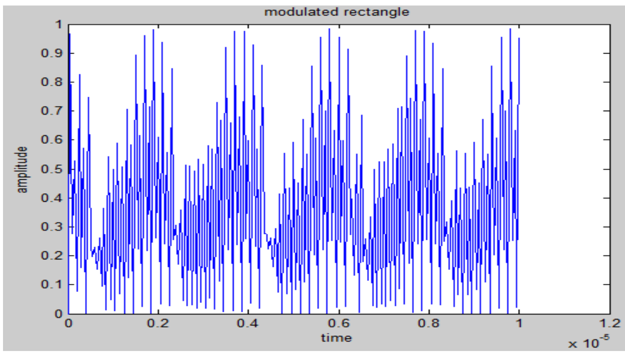


Fig.8. Periodic rectangular wave shown without DC bias.



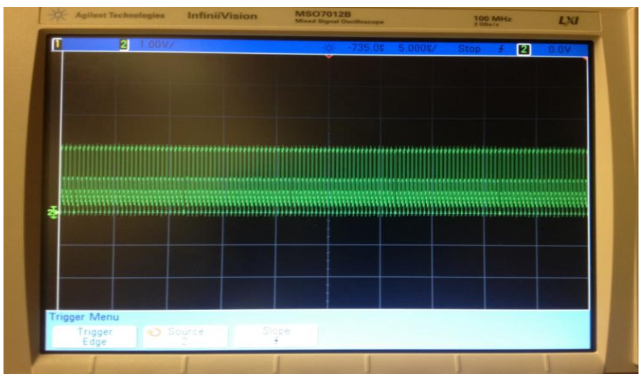
Beta=4
$Alpha_0$=3.5
Td=50ns
I_{inc}=1
$I_1(0)$=0

Fig.9. Modulated chaos wave for triangular pulse input.



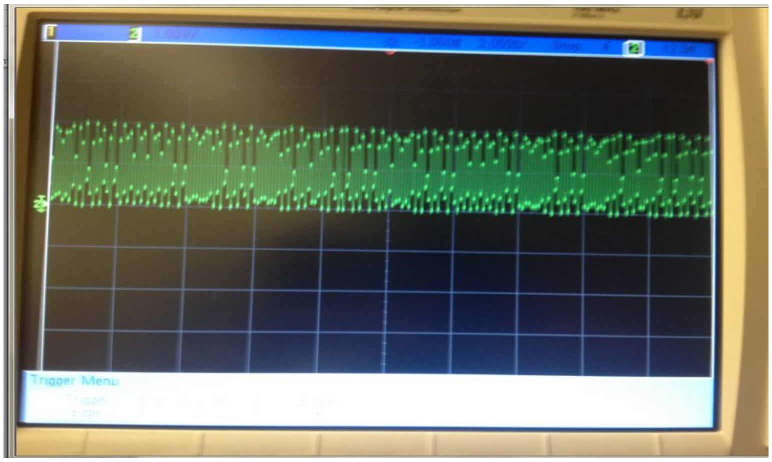
Beta=4
$Alpha_0$=3.5
Td=50ns
I_{inc}=1
$I_1(0)$=0

Fig.10. Modulated chaos wave for rectangular pulse input.



Beta=4
$Alpha_0$=3.5
Td=50ns
I_{inc}=1
$I_1(0)$=0

Fig.11. Oscilloscope display of modulated chaos wave for triangular input.



Beta=4
$Alpha_0$=3.5
Td=50ns
I_{inc}=1
$I_1(0)$=0

Fig.12. Oscilloscope display of modulated chaos wave for rectangular pulse input.

3. SPECTRAL STUDIES OF CHAOS AND MODULATION

3.1 Introduction

The frequency characteristics of any signal are important for the study the frequency spectrum of the signal, and thereby ascertain the corresponding signal behavior in the time domain. Since chaos is made up of random oscillations, study of its frequency characteristics becomes vital to know its frequency components. In this section, we discuss the frequency characteristics of an unmodulated chaos wave and a message-encrypted chaos wave. This is done by calculating and plotting the basic Fast Fourier Transform (FFT) of the signals. All the FFT plots have been carried out in MATLAB using software simulation.

In our work, in order to check for the reliability of the results obtained from MATLAB, we have also tried to study these frequency characteristics on hardware RF equipment such as signal generators, oscilloscopes and spectrum analyzers. For this approach, we first executed the software simulations in MATLAB. Then the results from the simulations are collected and stored in an Excel file in the form of voltages versus time. These values are stored column-wise and the Excel sheet is saved as .csv file. This Excel data is then stored in a hardware device such as a pen drive which is then connected to an arbitrary signal generator that generates the arbitrary signal (chaos wave and message-encrypted chaos in this case). The signal generated from the signal generator may then be connected to an oscilloscope to view the signal, or to a spectrum analyzer to view its frequency spectrum.

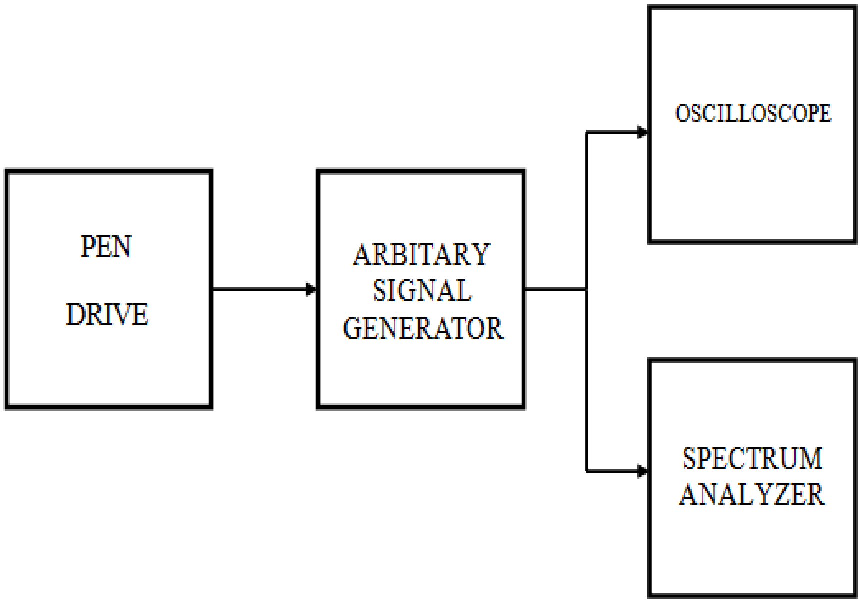
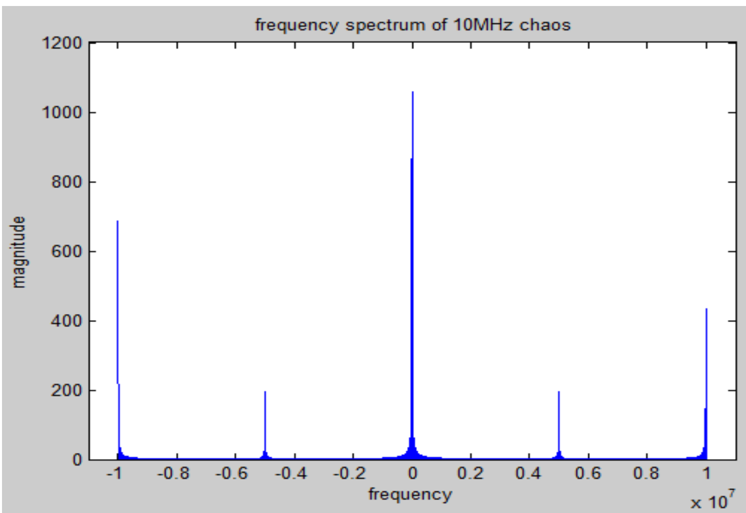


Fig.13. Hardware waveform analysis setup using Matlab chaos data.

Fig.13 depicts the procedure discussed above about the hardware setup of performing the spectral studies of the chaos waveform. The oscilloscope outputs corresponding to periodic triangular and rectangular wave responses of the HAOF chaotic device were shown earlier.

3.2 Experiments and Results

Using the procedures described, spectral studies have been carried out using both MATLAB software and a laboratory spectrum analyzer. The following results show the outputs using both approaches. Figs. 14 and 15 show the software simulation and spectrum analyzer outputs for a 10 MHz chaos wave without any RF input.



Beta=4
$Alpha_0$=3.5
I_{inc}=1
$I_1(1)$=0
Td=50ns

Fig.14. FFT plot of a snapshot of the 10 MHz chaos (as in Fig.4).

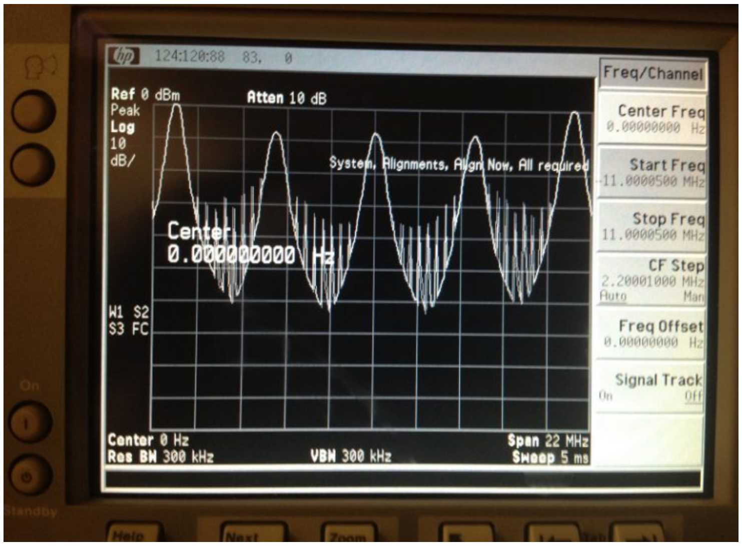


Fig.15. Spectrum analyzer display of the chaos wave of Fig.4.

As seen from the above, the spectra consist of large DC components (arising from the DC bias applied to the RF input), along with strong sidebands at about 10 MHz and subharmonics at 5 MHz. Comparing the two plots, we observe essentially the same characteristics. We note that the spectrum analyzer output show some additional fine spectral lines, which are likely artifacts resulting from the discrete data points used in generating the spectrum. This requires further investigation. Figs.16 and 17 show the simulation and spectrum analyzer outputs for a 2.5 MHz triangular wave encrypted on a 10 MHz chaos. From both figures, we find once again that the simulation result is in complete agreement with that of the spectrum analyzer output.

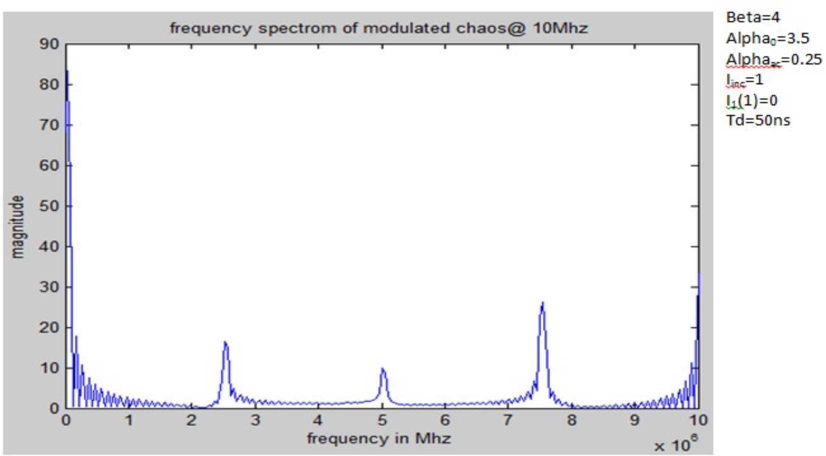


Fig.16. Simulation output for 10MHz chaos with 2.5 MHz periodic triangular input.

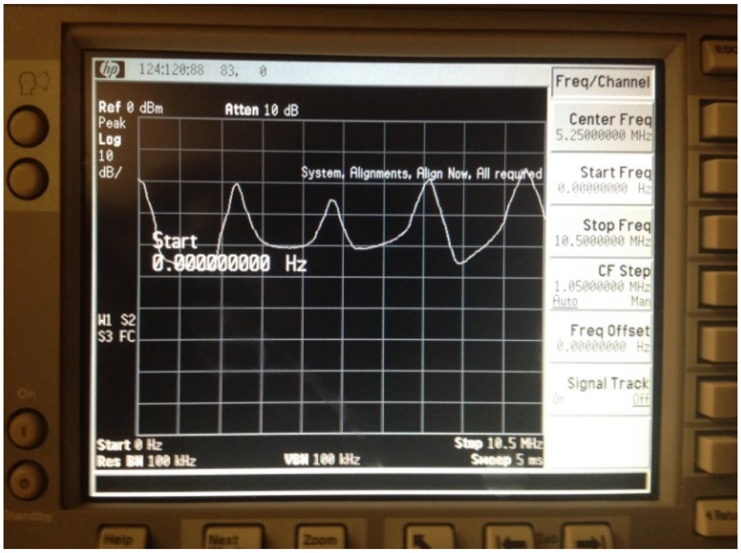


Fig.17. Spectrum analyzer display of the modulated chaos wave of Fig.9.

From the above figures, it is clear that the chaos wave under periodic triangular bias drive shows strong spectral components at DC and around the chaos frequency of 10 MHz, but also additonal sidebands separated by 2.5 MHz, the fundamental frequency of the periodic wave. Not shown along with the above waveform is a spectral (Fourier) plot of a 10 MHz carrier wave amplitude modulated by a 2.5 MHz triangular signal wave. The result corresponds closely to the spectrum shown in Fig.16. This affirms that the presence of an RF input at the bias input of the RF driver does indeed effectively amplitude modulate an equivalent chaotic carrier. Thus, spectral analysis serves as a useful tool in the understanding of the effect of specific waveforms applied to the HAOF nonlinear system on the overall behavior of the system in the chaotic or non-chaotic regimes.

4. CONCLUDING REMARKS

In this paper, we have discussed the emergence of chaotic weves arising from a nonlinear feedback system (the HAOF device) both in the presence of RF bias-drive signals and without. The need for determining the spectral characteristics of the resulting chaotic waves in order to ascertain whether the input signal has afected the chaos "carrier" in some manner (as in amplitude modulation) has been highlighted. It has been shown that the frequency characteristics of the chaos waveforms may be determined in three ways. The first is to transfer the time data of the chaotic output obtained from Matlab to an oscilloscope via an Excel datasheet. The second is to use an FFT routine via Matlab to directly display the spectra corresponding to a finite time snapshot of the waveform. The third approach is to use the Excel datasheet, stored in a pen drive or other suitable USB storage device, and then use the data to generate the oscilloscope waveform that may then be used as an input to a laboratory spectrum analyzer to directly display the spectra. The results have been shown to be generally compatible and in agreement with general expectations.

Further work is needed to examine in greater detail the finer spectral lines seen in the spectrum analyzer outputs, as well as the subharmonics in the unmodulated chaos waveforms. This work will find potential use for better understanding of chaos encryption and decryption of image and video waveforms which are expected to be more complex. The work in this area is currently in progress.

REFERENCES

1. M.R. Chatterjee and M. Al-Saedi, "Examination of chaotic signal encryption and recovery for secure communication using hybrid acousto-optic feedback," Opt. Eng. 50, 055002 (May 02, 2011).

2. M. Al-Saedi and M.R. Chatterjee, "Examination of the nonlinear dynamics of a chaotic acousto-optic Bragg modulator with feedback under signal encryption and decryption," Opt. Eng., 51(1), 018003 (January 01, 2012).

3. M.R. Chatterjee and A. Kundur, "Information encryption and retrieval in mid-RF range using acousto-optic chaos," Proc. SPIE 8406, 840608 (2012).

4. A. Kundur, MSEE Thesis, University of Dayton, Dayton, Ohio (2012).