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Revisiting the Fresnel Coefficients for Uniform Plane Wave Propagation across a Nonchiral, Reciprocal and Chiral, Nonreciprocal Interface

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Revisiting the Fresnel Coefficients for Uniform Plane Wave Propagation Across a Non-chiral, Reciprocal and Chiral, Non-reciprocal Interface

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ABSTRACT

The problem of EM wave propagation in non-reciprocal chiral media has been studied by several investigators. In a recent approach, a dual-transform technique has been developed to study the problem of such propagation under paraxial and slow-envelope variation conditions. In this paper, we first outline some of the results obtained using the dual transform technique for arbitrary boundary conditions within the left boundary of a semi-infinite, non-reciprocal chiral medium for a uniform plane wave, and a fundamental Gaussian-profiled beam. Next, we explore the problem of a uniform EM wave incident at an oblique angle at an interface between a reciprocal, non-chiral medium and a non-reciprocal, chiral medium. To carry out the calculations, the appropriate Maxwell's equations are examined together with the necessary boundary conditions, and reflection and transmission coefficients are derived for both parallel and perpendicular polarizations. The results are first tested for convergence in the reciprocal, non-chiral limit, and also for their physical implications under varying interfacial conditions.

Keywords: Chiral, Reciprocity, Dual-transforms, Fresnel coefficients, Beam propagation

1. INTRODUCTION

This paper deals with a class of materials called chiral or gyrotropic media. It lies in the fields of novel, complex materials. The word chiral is derived from the Greek word, *chiro*, implying the hand. Chirality or handedness can be seen in the world around us. Sea shells, spirals, bacteria etc. show varied proportions of right or left handedness. As has been known for a long time, chiral media posses handedness, so that an electromagnetic wave propagating through it undergoes a change in polarization depending upon its initial polarization, direction of propagation, and other properties. Interestingly some work pertaining to this and other properties of chiral media was carried out as early as the late 1900s by Louis Pastuer. The mathematical model and the constitutive relations for optically active materials was first given by Drude.[1] More recently researchers from a variety of disciplines like biology and chemistry have published results which have a strong relevance to Electromagnetics and optics application.

In general the analysis of chiral media (which are generally isotropic, but may posses reciprocal or non reciprocal properties) involves highly complex mathematics, including the application of Green's functions, and a variety of special transforms. Since the basic constitutive relations in the Maxwell's equations are modified in the chiral media to include terms containing so-called chiral and reciprocity parameters (κ and χ respectively), a relatively simple method was developed by one of the current authors[a], in collaboration with Banerjee, whereby solutions for the vector electric field in an unbounded chiral medium are obtained via a dual transform technique. The technique assumes paraxiality and slowly varying envelopes, and proceeds by applying successively a 2-D (transverse) spatial Fourier transform and a Laplace transform along the propagational or the longitudinal direction.

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A straight forward application of the above technique has been shown to confirm that any arbitrary, plane polarized uniform plane wave transforms into a circular polarized wave upon propagation through the chiral medium. More recently, the work has been extended to the case of profiled plane waves, investigating in particular the intensity distribution on-axis. The axial distribution is important in that studying the axial behavior may lead realizing interesting lens like properties. For a fundamental Gaussian profiled wave it has been shown that a linearly polarized wave turns into a circular polarization and interestingly at certain so-called *achiral* distances the wave returns to its original form.

The present paper is a stepping stone towards the study of profiled beam propagation through arbitrarily shaped chiral objects which would further aid in understanding the imaging properties of a chiral object. Plane wave case has been chosen to study as any profiled beam can be decomposed into any number of plane waves. These plane waves can then be passed through a medium, and the output can then be summed up to yield the profile of the output wave. The medium can be modelled as a transfer function and once the response of that transfer function to any arbitrary plane wave is known, the output of any profiled beam can be obtained by plane wave decomposition method. The emphasis of the authors has been to make the study of the chiral media simple and avoid highly complex mathematics so that the properties of the chiral medium could be easily understood. Convergence with the achiral reciprocal media has been checked to illustrate the effect of the chirality parameter (κ) and the non-reciprocal Tellegen Parameter (χ) on the electromagnetic wave propagating through it. The study of electromagnetic properties of chiral media will help in remote sensing applications as vegetation layers,[2] pollutants etc. can be thought of as chiral media.

2. ANALYSIS

We assume a plane wave incidence from a non-chiral reciprocal medium on to a chiral non-reciprocal medium. Maxwell's equations along with the modified constitutive relations and boundary conditions associated with these equations are used to learn about the amplitudes of reflected and transmitted waves. The geometry used in this discussion is shown in figure 1.[3]

The two media are separated by an interface, the x-y plane at z=0. The upper half plane is the non-chiral

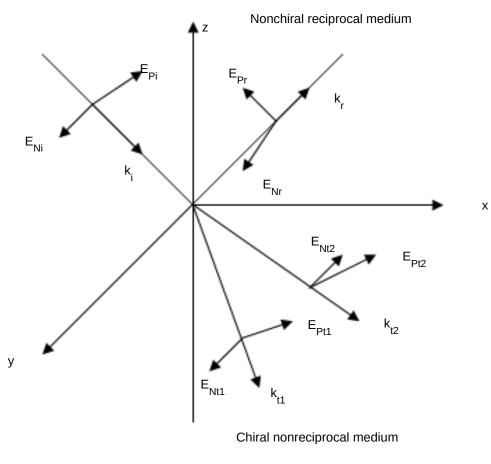

Figure 1. Geometry of the problem

reciprocal medium and the lower half plane is the chiral non-reciprocal medium. We assume that the phase of the wave varies smoothly across the boundary.

2.1. Wave Equations

In this discussion the incident and the reflected waves are assumed to propagate in the non-chiral reciprocal medium only and the transmitted wave is assumed to propagate in the chiral non-reciprocal medium. Hence we assume that the reflected part of the plane wave is reflected from the surface only and does not penetrate into the chiral non-reciprocal medium. The chiral non-reciprocal medium is described by the constitutive relations

$$\begin{aligned}\mathbf{D} &= \epsilon\mathbf{E} + \xi\mathbf{H} \\ \mathbf{B} &= \zeta\mathbf{E} + \mu\mathbf{H}\end{aligned}$$

where,

$$\begin{aligned}\xi &= (\chi - j\kappa)\sqrt{\mu_0\epsilon_0} \\ \zeta &= (\chi + j\kappa)\sqrt{\mu_0\epsilon_0} \\ \chi &= \text{Tellegen (Non-reciprocal) Parameter} \\ \kappa &= \text{Chirality Parameter}\end{aligned}$$

The chirality parameter κ is a dimensionless quantity which defines the degree of handedness of the material. In a racemic mixture where the chiral materials of both types i.e. right and left handed are equal and for non chiral materials the chirality parameter κ goes to zero. The Tellegen parameter χ describes the magnetoelectric effect. A non zero value of χ means that the material is non-reciprocal.

For the chiral medium, the Helmholtz equation can be written as

$$\nabla^2\mathbf{E} + 2\tilde{\alpha}(\nabla \times \mathbf{E}) + \tilde{k}_0^2\mathbf{E} = 0$$

where,

$$\begin{aligned}\tilde{\alpha} &= \omega\kappa\sqrt{\mu_0\epsilon_0} \\ \tilde{k}_0 &= \omega\sqrt{\mu\epsilon - \mu_0\epsilon_0(\chi^2 + \kappa^2)}\end{aligned}$$

From[4] we know that a linear polarized wave incident on a chiral, non-reciprocal media turns into a circular polarization, hence we can say that the transmitted wave consist of parallel E_{Pt} and normal E_{Nt} components which have a phase difference of $\pm\frac{m\pi}{2}$ i.e.

$$E_{Nt} = \pm jE_{Pt}$$

hence,

$$\mathbf{E}_t = E_t(\hat{\mathbf{i}}\cos\theta_t + \hat{\mathbf{k}}sin\theta_t) \pm jE_t\hat{\mathbf{j}}$$

Here the positive sign denotes right circular polarization and the negative sign indicates left circular polarization. Solving the Helmholtz equations the wave numbers in the chiral nonreciprocal medium come out to

$$\begin{aligned}k_{t1} &= -\omega\kappa\sqrt{\mu_0\epsilon_0} + \omega\sqrt{\mu\epsilon - \mu_0\epsilon_0\chi^2} \\ k_{t2} &= \omega\kappa\sqrt{\mu_0\epsilon_0} + \omega\sqrt{\mu\epsilon - \mu_0\epsilon_0\chi^2}\end{aligned}$$

Assuming that the phase varies smoothly across the boundary, the the transmitted angles are found to be

$$\begin{aligned}\theta_{t1} &= \sin^{-1}\left(\frac{k_i\sin\theta_i}{k_{t1}}\right) = \sin^{-1}\left(\frac{n_{01}\sin\theta_i}{-\kappa + \sqrt{n_{0t}^2 - \chi^2}}\right) \\ \theta_{t2} &= \sin^{-1}\left(\frac{k_i\sin\theta_i}{k_{t2}}\right) = \sin^{-1}\left(\frac{n_{0i}\sin\theta_i}{+\kappa + \sqrt{n_{0t}^2 - \chi^2}}\right)\end{aligned}$$

The actual vectors to be used are as follows:
Incident Wave

$$\begin{aligned}\mathbf{k_i} &= k_i(\hat{\mathbf{i}}\sin\theta_i - \hat{\mathbf{k}}\cos\theta_i) \\ \mathbf{E_i} &= E_{Pi}(\hat{\mathbf{i}}\cos\theta_i + \hat{\mathbf{k}}sin\theta_i) + E_{Ni}\hat{\mathbf{j}}\end{aligned}$$

Reflected Wave

$$\begin{aligned}\mathbf{k_r} &= k_i(\hat{\mathbf{i}}\sin\theta_i + \hat{\mathbf{k}}\cos\theta_i)\\ \mathbf{E_r} &= E_{Pr}(-\hat{\mathbf{i}}\cos\theta_i + \hat{\mathbf{k}} sin\theta_i) + E_{Nr}\hat{\mathbf{j}}\end{aligned}$$

Transmitted Waves

$$\begin{aligned}\mathbf{k_{t1}} &= k_{t1}(\hat{\mathbf{i}}\sin\theta_{t1} - \hat{\mathbf{k}}\cos\theta_{t1})\\ \mathbf{E_{t1}} &= E_{t1}(\hat{\mathbf{i}}\cos\theta_{t1} + \hat{\mathbf{k}} sin\theta_{t1}) + jE_{t1}\hat{\mathbf{j}}\end{aligned}$$

$$\begin{aligned}\mathbf{k_{t2}} &= k_{t2}(\hat{\mathbf{i}}\sin\theta_{t2} - \hat{\mathbf{k}}\cos\theta_{t2})\\ \mathbf{E_{t2}} &= E_{t2}(\hat{\mathbf{i}}\cos\theta_{t2} + \hat{\mathbf{k}} sin\theta_{t2}) - jE_{t2}\hat{\mathbf{j}}\end{aligned}$$

2.2. Boundary Conditions

Assuming the μs to be the same as free space, i.e $\mu_i = \mu_r = \mu_t = \mu_0$ and replacing $\sqrt{\epsilon}$ by corresponding nonchiral refractive indices n_0. The boundary conditions associated with Maxwell's Equations are the following

a. From $\nabla \cdot \mathbf{D} = \rho$, the normal components of $\mathbf{D}$ must be continuous if there are no surface charges.
$\mathbf{D_i} = \epsilon_\mathbf{i}\mathbf{E_i}$ for the incident wave.
$\mathbf{D_r} = \epsilon_\mathbf{i}\mathbf{E_r}$ for the reflected wave.
$\mathbf{D_t} = \epsilon_\mathbf{t}\mathbf{E_t} + \xi_\mathbf{t}\mathbf{H_t}$ for the transmitted wave.

We get the boundary condition

$$[\epsilon_i(\mathbf{E_i} + \mathbf{E_r}) - \epsilon_\mathbf{t}\mathbf{E_t} + \xi_\mathbf{t}\mathbf{H_t}] \cdot \hat{\mathbf{n}} = 0$$

Evaluating the dot product we finally get

$$\begin{aligned} n_{0i}^2\sin\theta_i(E_{Pi} + E_{Pr}) &= \left[n_{0t}^2 - 2\kappa^2 - \chi^2 + j\chi\kappa + j(\chi - j\kappa)\sqrt{n_{0t}^2 - \chi^2}\right] E_{t1}\sin\theta_{t1} \\ &+ \left[n_{0t}^2 - 2\kappa^2 - \chi^2 - j\chi\kappa - j(\chi - j\kappa)\sqrt{n_{0t}^2 - \chi^2}\right] E_{t2}\sin\theta_{t2} \end{aligned} \tag{1}$$

b. From the Maxwell's Equation containing $\nabla \times \mathbf{E}$, we see that the tangential component of $\mathbf{E}$ is continuous. The boundary condition is written

$$(\mathbf{E}_i + \mathbf{E}_r + \mathbf{E}_t) \times \hat{\mathbf{n}} = 0$$

solving this boundary condition for each vector component we finally get

$$E_{Ni} + E_{Nr} = j(E_{t1} - E_{t2}) \tag{2}$$

$$(E_{Pi} - E_{Pr})\cos\theta_i = E_{t1}\cos\theta_{t1} + E_{t2}\cos\theta_{t2} \tag{3}$$

c. From $\nabla.\mathbf{B} = 0$, the normal component of $\mathbf{B}$ must be continuous. Rewriting the normal component of B in terms of E. The boundary condition is then written

$$\left[\frac{\sqrt{\mu_i\epsilon_i}}{k_i}(\mathbf{k}_i \times \mathbf{E}_i + \mathbf{k}_r \times \mathbf{E}_r) - (\omega\mathbf{k}_t \times \mathbf{E}_t)\right].\hat{\mathbf{n}} = 0$$

Evaluating the dot product yields

$$n_{0i}(E_{Ni} + E_{Nr})\sin\theta_i = -j\left(\sqrt{n_{0t}^2 - \chi^2} - \kappa\right)E_{t1}\sin\theta_{t1} + j\left(\sqrt{n_{0t}^2 - \chi^2} + \kappa\right)E_{t2}\sin\theta_{t2} \tag{4}$$

d. The tangential component of **H** is continuous if there are no surface currents. The boundary condition is then written

$$\left[\frac{1}{k_i}\sqrt{\frac{\epsilon_i}{\mu_i}}(\mathbf{k}_i \times \mathbf{E}_i + \mathbf{k}_r \times \mathbf{E}_r) - \frac{1}{\mu_t}\left(\frac{\sqrt{\mu_t \epsilon_t - \mu_0 \epsilon_0 (\chi^2 + \kappa^2)}}{\tilde{k}_0}\mathbf{k}_t \times \mathbf{E}_t - \zeta_t \mathbf{E}_t\right)\right] \times \hat{\mathbf{n}} = 0$$

Equating each vector component,

$$n_{0i}\left(E_{Pi} + E_{Pr}\right) = \left[\sqrt{n_{0t}^2 - \chi^2} - 2\kappa + j\chi\right] E_{t1} + \left[\sqrt{n_{0t}^2 - \chi^2} + 2\kappa - j\chi\right] E_{t2} \tag{5}$$

$$n_{0i}(E_{Ni} - E_{Nr})\cos\theta_i = \left[\sqrt{n_{0t}^2 - \chi^2} - 2\kappa + j\chi\right] jE_{t1}\cos\theta_{t1} + \left[\sqrt{n_{0t}^2 - \chi^2} + 2\kappa - j\chi\right] jE_{t2}\cos\theta_{t2}$$

3. FRESNEL COEFFICIENTS

3.1. Reflection and Transmission coefficients

Here we define the interface as a system which can be written in terms of the following matrices. The transmission coefficient matrix

$$\begin{pmatrix} E_{t1} \\ E_{t2} \end{pmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{pmatrix} E_{Ni} \\ E_{Pi} \end{pmatrix} \tag{6}$$

and the reflection coefficient matrix

$$\begin{pmatrix} E_{Nr} \\ E_{Pr} \end{pmatrix} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} \begin{pmatrix} E_{Ni} \\ E_{Pi} \end{pmatrix} \tag{7}$$

Where the entries of the matrices are,

$$t_{11} = \frac{-2jn_{0i}\cos\theta_i(n_{0i}\cos\theta_{t2} + n_+\cos\theta_i)}{n_{0i}(n_+ + n_-)(\cos\theta_{t1}\cos\theta_{t2} + \cos^2\theta_i) + \cos\theta_i(\cos\theta_{t1} + \cos\theta_{t2})(n_{0i}^2 + n_+ n_-)} \tag{8}$$

$$t_{12} = \frac{2n_{0i}\cos\theta_i(n_+\cos\theta_{t2} + n_{0i}\cos\theta_i)}{n_{0i}(n_+ + n_-)(\cos\theta_{t1}\cos\theta_{t2} + \cos^2\theta_i) + \cos\theta_i(\cos\theta_{t1} + \cos\theta_{t2})(n_{0i}^2 + n_+ n_-)} \tag{9}$$

$$t_{21} = \frac{2jn_{0i}\cos\theta_i(n_{0i}\cos\theta_{t1} + n_-\cos\theta_i)}{n_{0i}(n_+ + n_-)(\cos\theta_{t1}\cos\theta_{t2} + \cos^2\theta_i) + \cos\theta_i(\cos\theta_{t1} + \cos\theta_{t2})(n_{0i}^2 + n_+ n_-)} \tag{10}$$

$$t_{22} = \frac{2n_{0i}\cos\theta_i(n_-\cos\theta_{t1} + n_{0i}\cos\theta_i)}{n_{0i}(n_+ + n_-)(\cos\theta_{t1}\cos\theta_{t2} + \cos^2\theta_i) + \cos\theta_i(\cos\theta_{t1} + \cos\theta_{t2})(n_{0i}^2 + n_+ n_-)} \tag{11}$$

$$r_{11} = \frac{n_{0i}(n_+ + n_-)(\cos^2\theta_i - \cos\theta_{t1}\cos\theta_{t2}) + \cos\theta_i(\cos\theta_{t1} + \cos\theta_{t2})(n_{0i}^2 - n_+ n_-)}{n_{0i}(n_+ + n_-)(\cos\theta_{t1}\cos\theta_{t2} + \cos^2\theta_i) + \cos\theta_i(\cos\theta_{t1} + \cos\theta_{t2})(n_{0i}^2 + n_+ n_-)} \tag{12}$$

$$r_{12} = \frac{-2jn_{0i}\cos\theta_i(n_-\cos\theta_{t1} - n_+\cos\theta_{t2})}{n_{0i}(n_+ + n_-)(\cos\theta_{t1}\cos\theta_{t2} + \cos^2\theta_i) + \cos\theta_i(\cos\theta_{t1} + \cos\theta_{t2})(n_{0i}^2 + n_+ n_-)} \tag{13}$$

$$r_{21} = \frac{2jn_{0i}\cos\theta_i(n_+\cos\theta_{t1} - n_-\cos\theta_{t2})}{n_{0i}(n_+ + n_-)(\cos\theta_{t1}\cos\theta_{t2} + \cos^2\theta_i) + \cos\theta_i(\cos\theta_{t1} + \cos\theta_{t2})(n_{0i}^2 + n_+ n_-)} \tag{14}$$

$$r_{22} = \frac{n_{0i}(n_+ + n_-)(\cos^2\theta_i - \cos\theta_{t1}\cos\theta_{t2}) + \cos\theta_i(\cos\theta_{t1} + \cos\theta_{t2})(n_{0i}^2 - n_+ n_-)}{n_{0i}(n_+ + n_-)(\cos\theta_{t1}\cos\theta_{t2} + \cos^2\theta_i) + \cos\theta_i(\cos\theta_{t1} + \cos\theta_{t2})(n_{0i}^2 + n_+ n_-)} \tag{15}$$

where,

$$n_+ = \sqrt{n_{0t}^2 - \chi^2} + (2\kappa - j\chi)$$
$$n_- = \sqrt{n_{0t}^2 - \chi^2} - (2\kappa - j\chi)$$

4. CONCLUSION

These coefficients are complex as opposed to the non-chiral non-chiral case. The complex nature of these equations is attributed to the complex terms in ζ and ξ. If we set κ and χ to zero in the above equations, they revert back to well established Fresnel coefficients in the non chiral medium. The magnitude and the phase of these coefficients have been plotted in the Figures section for both non-chiral reciprocal and chiral non-reciprocal case. In each case rarer to denser medium propagation is taken. The denser to rarer case can be found in[5] and.[6] We can see by comparing the corresponding plots that the change in magnitude of the coefficients is rather slight, which is attributed to the change in refractive index of the medium brought about by χ and κ. However there is a greater change in the phase behavior mainly because of the complex nature of these equations.

The amplitudes of the transmission coefficient for a non chiral reciprocal medium appear one half of the known values in the graphs, this is because they are partial transmission coefficients on adding the two graphs we get the total transmission coefficient. However for the chiral nonreciprocal medium they are not merely half the total value but they depend on the values of parameters χ and κ. Chiral medium of specific value of κ is known to favor a certain left circular polarization over right circular polarization or vice versa i.e. the velocity of an RCP wave may be greater than or less than LCP wave.

It is not possible to test experimentally the amplitude reflection and transmitted coefficients. The parameter that can be measured experimentally is the energy. The energy coefficients are not merely squares of the amplitude components. Work on the graphs of Reflectivity and Transmissivity is still underway and inferences are yet to be drawn from these graphs.

5. FIGURES

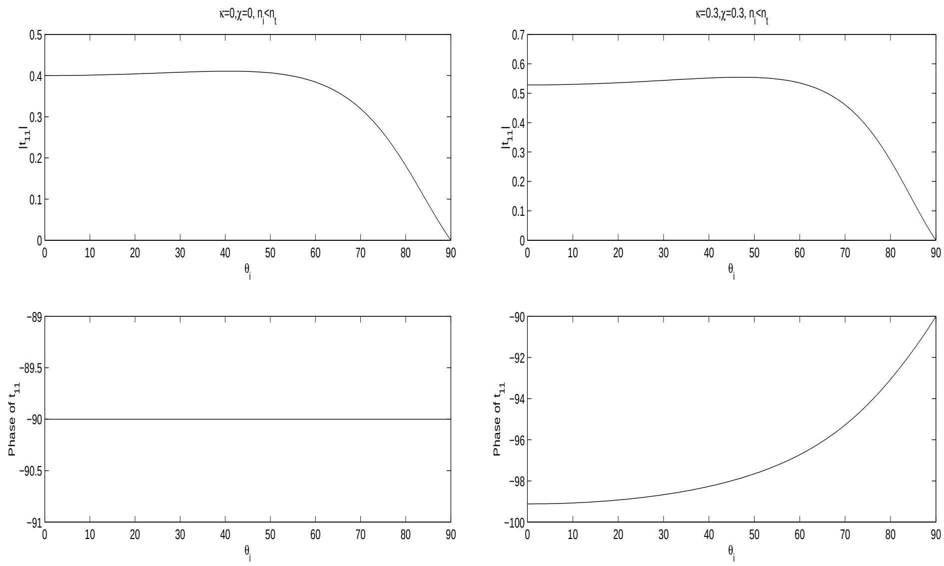

Figure 2. Magnitude and phase of amplitude transmission coefficient t_{11} are plotted for various values of κ and χ

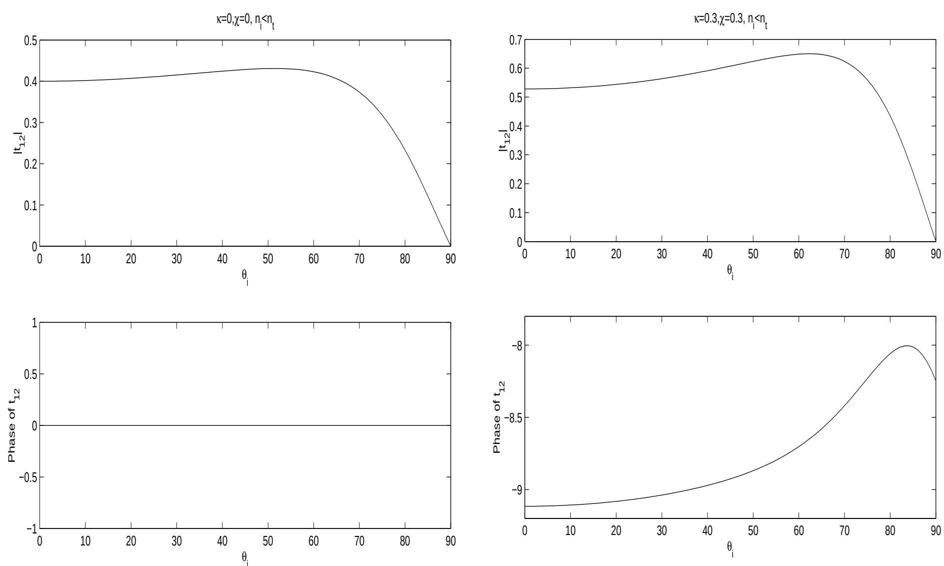

Figure 3. Magnitude and phase of amplitude transmission coefficient t_{12} are plotted for various values of κ and χ

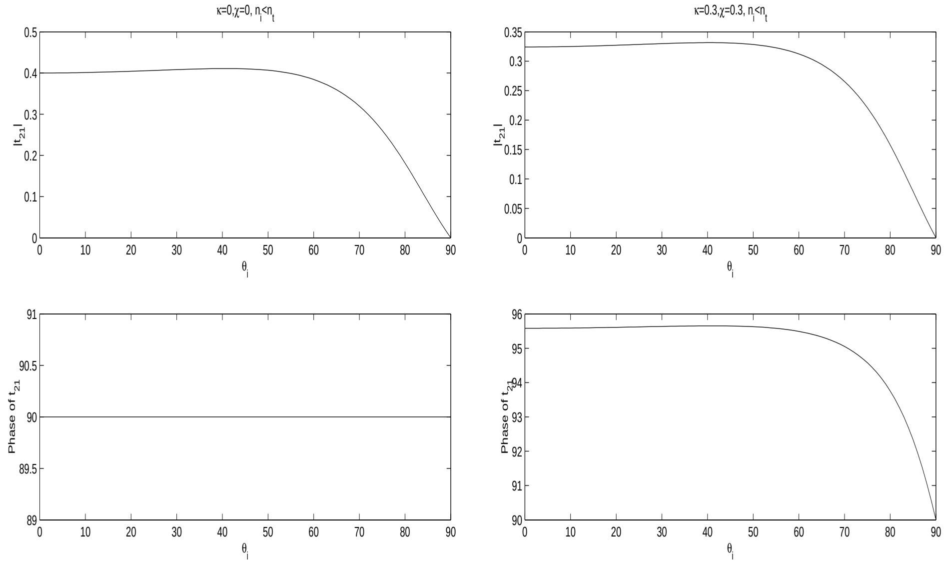

Figure 4. Magnitude and phase of amplitude transmission coefficient t_{21} are plotted for various values of κ and χ

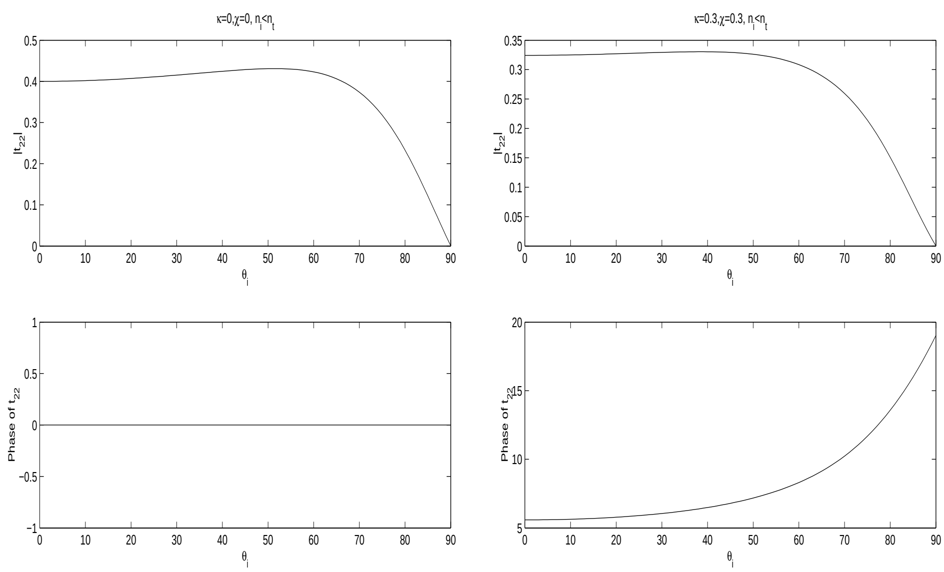

Figure 5. Magnitude and phase of amplitude transmission coefficient t_{22} are plotted for various values of κ and χ

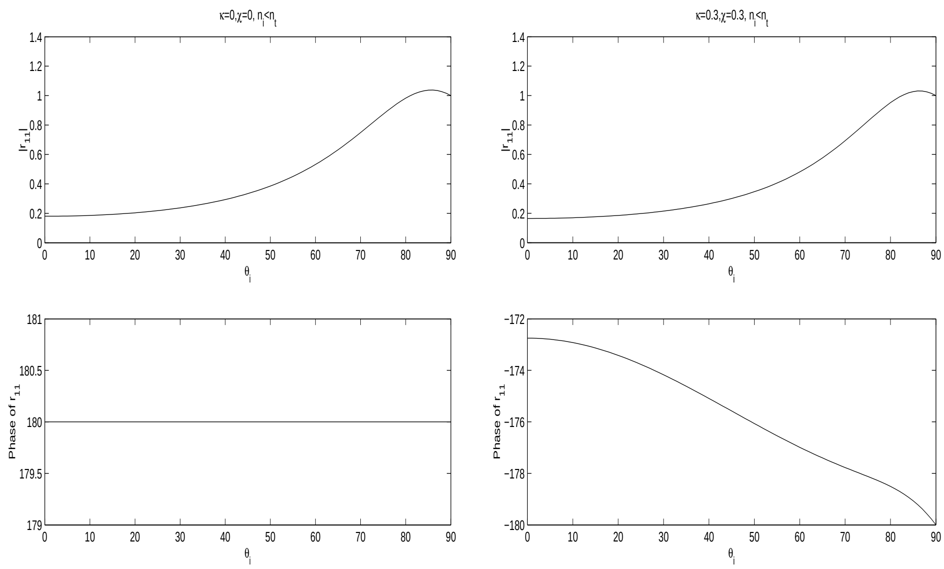

Figure 6. Magnitude and phase of amplitude reflection coefficient r_{11} are plotted for various values of κ and χ

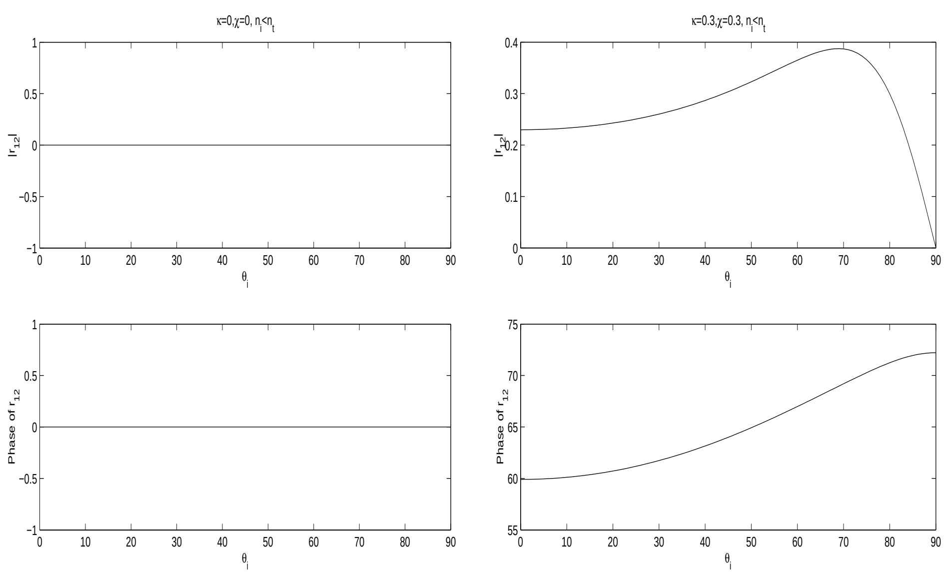

Figure 7. Magnitude and phase of amplitude reflection coefficient r_{12} are plotted for various values of κ and χ

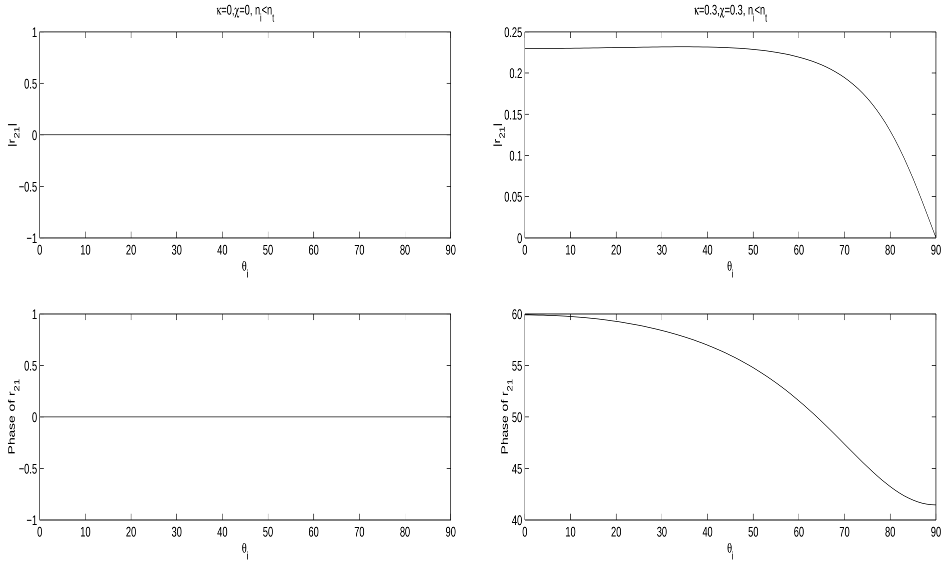

Figure 8. Magnitude and phase of amplitude reflection coefficient r_{21} are plotted for various values of κ and χ

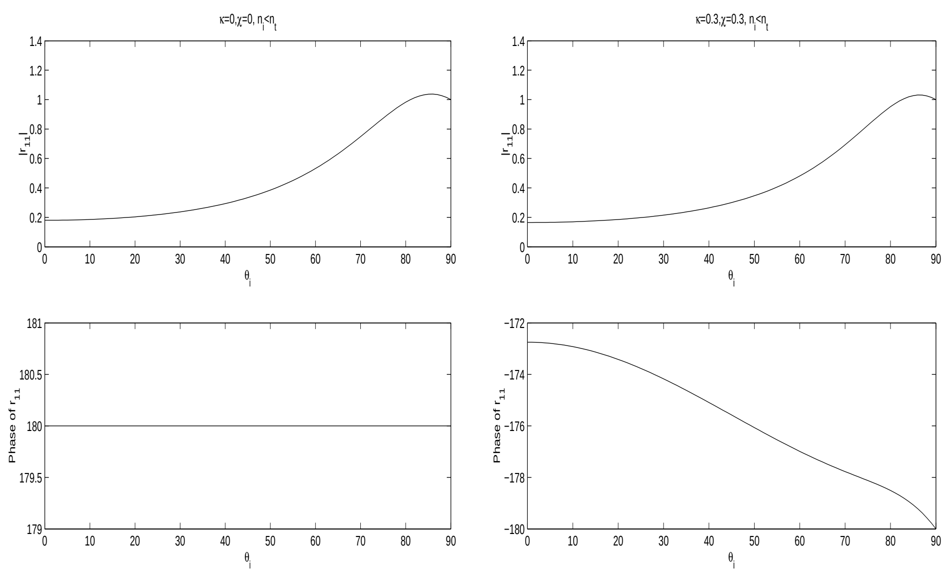

Figure 9. Magnitude and phase of amplitude reflection coefficient r_{22} are plotted for various values of κ and χ

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