7-1998

Experimental Investigations of Wavelength and Angular Errors in Holographic Gratings with Non-Bragg-matched READ Beams

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with non-Bragg-matched READ beams

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ABSTRACT

Perfect Bragg matching is generally desirable for accurate optical interconnections with holographic gratings. In reality, however, gratings may be illuminated by READ beams with non-Bragg-matched angles, or wavelengths, or both. In such cases, the scattered beams are generally misdirected, and may suffer loss of efficiency and possibly more serious errors such as crosstalk noise or missed connections. A conventional wavevector triad method of analyzing the scattered beam errors leads readily to near-Bragg estimates of the output angular misalignment. However, the READ wavevector triads appear to indicate a possible wavelength shift in the output beam even with a Bragg-matched READ wavelength, which is counter-intuitive. This paper presents some of the theoretical findings for output beam characteristics under READ misalignments, and the results of a series of experiments aimed at verifying both the output angular error as well as whether or not any unexpected wavelength shift occurs in the output beam. Based on the experimental findings, the interpretation of the misaligned READ wavevector triad is appropriately modified.

[Keywords: holographic gratings; misalignment; wavevector triad; near-Bragg scattering; interconnections]

1. INTRODUCTION

Recently there has been some work in several areas of free-space optical holographic interconnects. There has been much interest in their applications in VLSI systems, reconfigurable interconnects and photorefractive holograms with volume storage capacities [4-9]. One of the major issues in optical interconnection using holographic gratings is the sensitivity of Bragg angle mismatch caused by misalignment.

There are two types of holographic transmission gratings. The angle between the reference and object beams primarily determines the properties of holographic transmission gratings. When this angle is less then 10 degrees the fringe spacing is about the same as the recording medium thickness (emulsion). This produces a thin grating and follows the Raman-Nath approximation. When the angle is in the range of 10 degrees to 120 degrees the fringe spacing is smaller then the emulsion. This produces thick gratings and follows the Bragg law [15].

In the sections that follow, we will look at some geometrical and mathematical tools for examining transmission-type holographic gratings with unslated fringes under non-Bragg-matched READ beams. The theory is based on the classic work by Kogelnik on the analysis of holographic gratings, viz., the use of the wavevector triad model [16]. Two seemingly different but related geometrical representations of the wavevector triads will be described. Results from these analyses will be investigated experimentally to see if angular and wavelength errors occur as predicted by the wavevector triad approach. Interpretations based on the experimental findings are provided.

2. GEOMETRICAL ANALYSIS UNDER NON-BRAGG-MATCHED READ BEAMS

Holographic gratings have output deviations under non-Bragg-matched READ beam conditions. We next derive geometrical models for holographic transmission gratings which take into account general WRITE geometries, Bragg angle, READ beam wavelength and angular alignment. The effects of wavelength and angular READ beam error are quantitatively investigated for various parametric conditions.

2.1 General relations for holographic transmission gratings

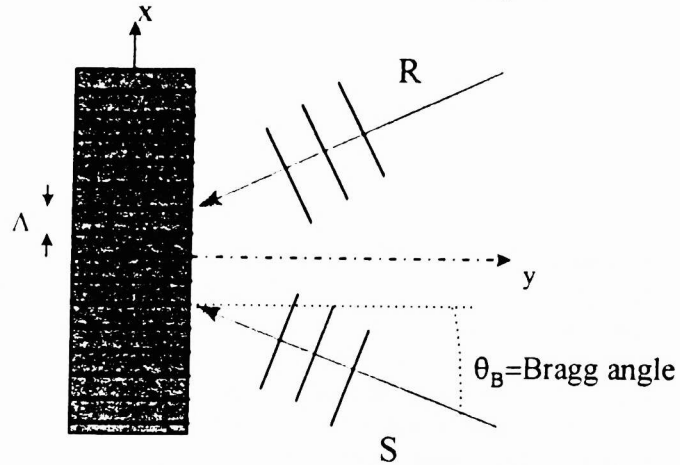The geometrical configuration for making a transmission hologram is shown in Fig.1.

Fig.1. Schematic for recording a holographic transmission grating.

The S (scattered or object beam) and the R (reference beam) form the angle $2\theta_{mw}$ in the medium of the holographic material. Λ is the grating space, and λ_{rw} and λ_{mw} are the WRITE input wavelengths outside the medium, θ_{iw} and θ_{mw} are the WRITE input angles outside the medium, n is the average refractive index of the holographic material, and n_1 is the peak refractive index modulation. Wavelength selectivity is the change in wavelength $\Delta\lambda$ of the READ beam from the WRITE beam wavelength λ_{mw} such that the diffraction efficiency goes to zero. If a READ wavelength is such that λ_{mr} is equal to $\lambda_{mw} + \Delta\lambda$, and there is angular alignment, then the wavelength selectivity for a grating for which the outside medium is air can be approximated as:

$$\frac{\Delta\lambda}{\lambda_{mw}} \cong \frac{1-n}{n} \pm \frac{\lambda_{mw} \cot\theta_{mw}}{2nd \sin\theta_{mw}} \sqrt{1 - \left(\frac{n_1 d}{n\lambda_{mw} \cos\theta_{mw}}\right)^2} . \qquad (1)$$

We can also find the angular selectivity which is the $\Delta\theta$ READ beam deviation from the WRITE Bragg angle that gives zero diffraction efficiency:

$$\Delta\theta \cong \pm \frac{\lambda_{mw}}{2nd \sin\theta_{mw}} \sqrt{1 - \left(\frac{nd_1}{n\lambda_{mw} \cos\theta_{mw}}\right)^2} - (1-n)\tan\theta_{mw} . \qquad (2)$$

The READ wavelength and angular deviations are assumed to be well within the limits given by these selectivities.
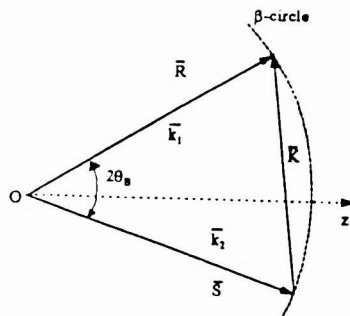
2.2 Wavevector triads

Fig.2. Wavevector triad showing WRITE and READ beams in a transmission grating.

In the above figure a wavevector representation of two beams inside a holographic material is shown. R denotes the reference beam and S denotes the object or scattered beam. The resulting (K) vector is the grating vector inside the medium. This wavevector shows the recording of a holographic transmission grating. We are assuming the vector-Floquet closure of the k-vectors in accordance with S+K=R for the wavevector triad. In thick transmission gratings the WRITE beams can be incident at any angle on the medium, but we will consider only symmetrical angles on the holographic medium. The wavelength of the WRITE beams must be the same. Therefore in the wavevector triad representation, the WRITE wave vectors are of equal length. Since we are assuming a thick holographic grating the wavevector triad shows the ideal Bragg condition. This means that there is a high efficiency first order scattering.

2.3 READ wavelength misalignment

In optoelectronic interconnections situations may arise that cause misalignment of the READ beam. These situations could be a result of geometrical constraints or instability of light sources. If a READ wavelength deviates from λ_{mw} by $\Delta\lambda$, it will cause the diffracted light to deviate by $\Delta\theta_{m,out}$. In our analysis we make certain assumptions, which include:

    (1) the READ beam is a monochromatic plane wave.
    (2) the angular errors are within 10% of the Bragg angle, and hence in the near-Bragg regime.
    (3) the small wavelength deviation $\Delta\lambda$ satisfies near-Bragg regime.
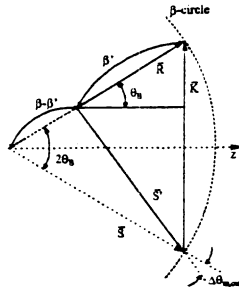

Fig.3. Wavevector triads showing output angular misalignment (after [17]).

In the ideal case the READ beam satisfies the exact Bragg condition, which is when the READ beam has the same angle and wavelength as the WRITE beam. In the real world the READ beam can have a $\Delta\lambda$, which will cause an error $\Delta\theta_{m,out}$ in the output beam direction. This $\Delta\theta_{m,out}$ can be shown after some algebra to be:

$$\Delta\theta_{m,out} = \tan^{-1}\left(\frac{K - \beta'\sin\theta_B}{\beta'\cos\theta_B}\right) - \theta_B = -\left(\tan^{-1}\left[\left(1 + 2\frac{\Delta\lambda}{\lambda_{mw}}\right)\tan\theta_B\right] - \theta_B\right). \quad (3)$$

2.4 READ angular misalignment

An optical interconnection system may suffer from misalignment of the light source and the holographic grating. In this section we will present the result of a quantitative analysis using wave vector triads to find the angular error in the output beam. In Fig.4(a and b), we see positive and negative misalignment angles $\Delta\theta_{m,in}$ relative to the Bragg angle.
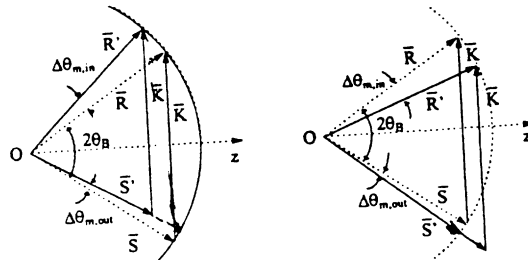

Fig.4. (a) Wavevector diagram showing output angular misalignment for READ counterclockwise angular deviation; (b) same as (a), for clockwise angular deviation (after [17]).

Clockwise angles are assumed to be the negative and counterclockwise angles are assumed to be positive direction. From the wavevector triad it is clear that a positive $\Delta\theta_{m,in}$ results in a positive $\Delta\theta_{m,out}$ and a negative $\Delta\theta_{m,in}$ results in a negative $\Delta\theta_{m,out}$. Using geometrical considerations, the relationship between $\Delta\theta_{m,in}$ and $\Delta\theta_{m,out}$ is found to be:

$$\Delta\theta_{m,out} = \theta_B - \cot^{-1}\left(\frac{\cos\left(\theta_B + \Delta\theta_{m,in}\right)}{2\sin\theta_B - \sin\left(\theta_b + \Delta\theta_{m,in}\right)}\right), \qquad (4)$$

assuming that $\Delta\theta_{m,in}$ is well within the angular selectivity, implying near-Bragg operation.

The $\Delta\theta_{m,in}$ in the wavevector triad also seem to result in a change in the length of S' vector. Therefore there seems to be a change in *wavelength* of the output beam. Using geometrical relationships, the S' is found to be:

$$S' = S\left(\cos\Delta\theta_{m,out} \pm \sqrt{\left(\cos^2\Delta\theta_{m,out} - \left(2\cos\Delta\theta_{m,in} - 1\right)\right)}\right). \qquad (5)$$

The wavelength can be found by using the relationship:

$$S' = \frac{2\pi}{\lambda_{out}}n. \qquad (6)$$

The above implies that a change in the input READ angle could result in a change in the output wavelength in addition to the change in the output angle, which is entirely counter-intuitive.

2.5 Alternative wavevector triad method

The $\Delta\theta_{m,in}$ can also be due to a rotation of the grating vector K instead of the READ beam rotation. This is shown by the wavevector triad S''+K=R in Fig.5. In this wavevector triad the assumptions and rules of vector triads remain the same. We see from this wavevector triad that there is again a change in the output beam angle.
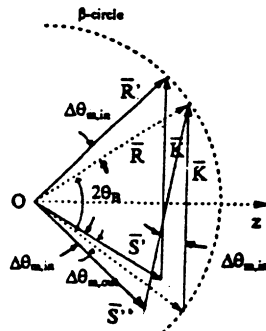

Fig.5. Alternative wavevector triad model.

The direction of S'' appears to be different from S', but this is because the vector S'' is relative to grating rotation. To bring it back to the coordinates of the READ beam, S'' must be rotated by $\Delta\theta_{m,in}$ in the positive direction. Once this transform is made we see that the direction of the S'' aligns with S', which is the direction from the previous wavevector triad method.

3. EXPERIMENTAL INVESTIGATION OF POSSIBLE WAVELENGTH SHIFT

As indicated, an output wavelength error or deviation under READ angular error seems to be counter intuitive. With a beam propagating in a linear medium (holographic grating) from air and exiting back to air, such a wavelength shift should not occur? We next attempt to resolve this question experimentally.

3.1 Experiment to measure wavelength shift

To carry out the experiment, first a holographic transmission grating is recorded with a Bragg angle of 25° inside the holographic medium. This translates to 39° outside the film. Once the holographic transmission grating was recorded and developed, an experiment to measure the wavelength shift was designed. A few attempts initially to carry out the measurement with a dispersive prism and with a narrow slit were discarded due to insufficient resolution. Ultimately, acceptable results were obtained using an optical spectrometer. The experimental setup is shown in Fig.6.
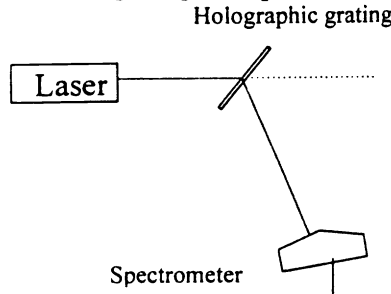
Holographic grating

Fig.6. Setup for wavelength measurement using spectrometer.

The optical spectrometer used had a resolution of 4nm. Two beams were directed into the optical spectrometer. The first was the 632.8 nm HeNe wave and the second was the output beam of the grating with a READ angular error of 1.5°. The experiment was repeated several times, but no wavelength shift was observed. Therefore, there appears to be no wavelength shift (at least not within the 4nm resolution of this setup) in the output beam of a non-Bragg-matched READ beam. From this observation, we conclude that we must change the interpretation of the wavevector triad. In the corrected wavevector triad, the output beam vector must have a length equal to the READ beam vector. Therefore one solution or interpretation is to project the output beam vector back to the β circle. This is shown in Fig.7. A possible explanation for the apparent change in the resulting K-vector is that the process involved is second order for the K-vector rotation.
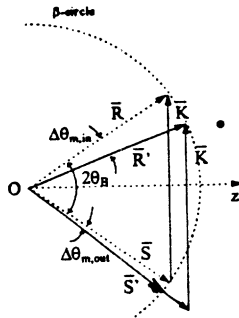

Fig.7. Wavevector triad showing projected (scattered) wavevector.

3.2 Experiment to measure output angular error

We saw earlier how the wavevector triad method predicts an angular error for a non-Bragg READ beam. An experiment was designed to measure this output angular error. The first experiment was with a 10° Bragg angle holographic transmission grating.
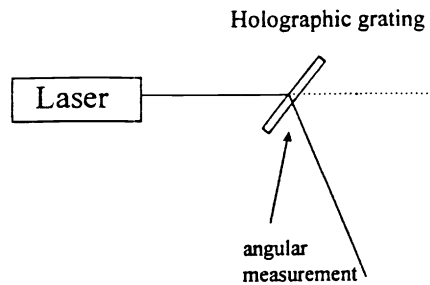
Holographic grating

Fig.8. Experimental setup for output angular error measurement.

The 10° Bragg angle holographic transmission grating was first read at the exact Bragg angle and then the grating was rotated at .5 degrees up to 1°. This is the limit of angular error necessary to be within 10% of the Bragg angle. The results of this data are compared with eq.(4) and shown in Fig.9.
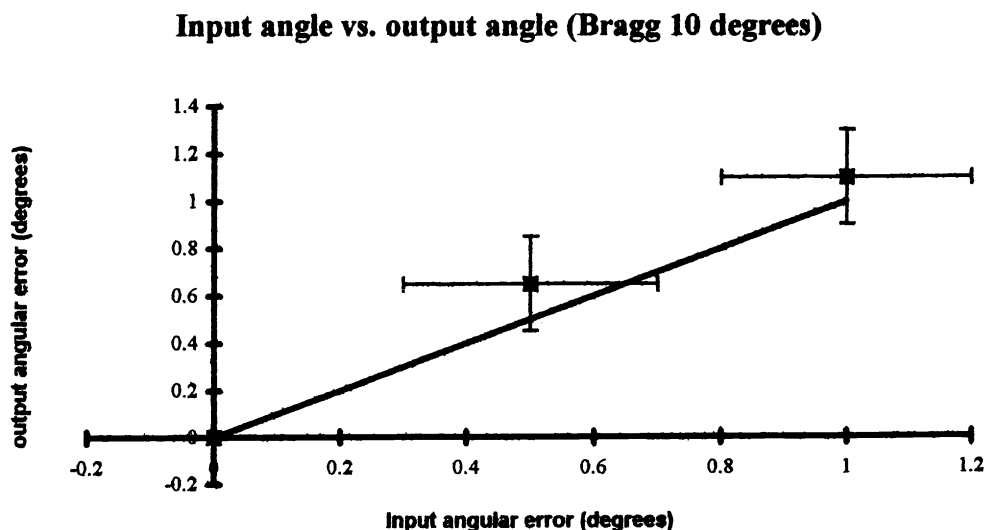
Input angle vs. output angle (Bragg 10 degrees)

Fig.9. Output vs. input angular error for $\theta_B = 10°$.

In the figure, the data is shown by the squares and the solid line is from eq.(4). The error bars in the graph are because of the fact that the angular measurement could only be made within an error of $\pm.25°$. The experimental agreement to the wavevector triad model seems to be good. The data is within the error bars. However, even though the data looks reasonable, it is not too convincing since there are only two sets of data points. Since the experiment has limitations on the accuracy of the angular measurement, the only way to increase the number of data points is to increase the Bragg angle of the grating.

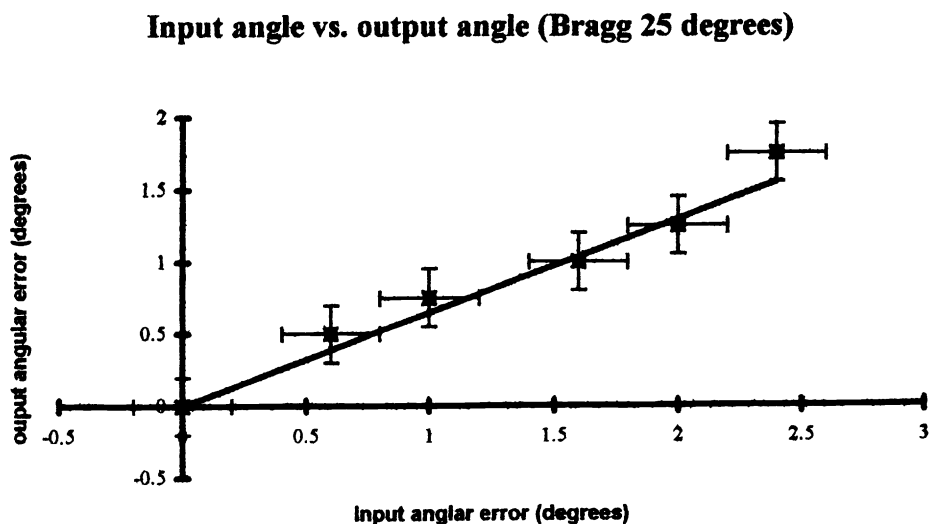### 3.3 Experiment with larger Bragg angle

Input angle vs. output angle (Bragg 25 degrees)

Fig.10. Output vs. input angular error for $\theta_B = 25°$.

The input angular error experiment with $\theta_B = 25°$ has more data points. The data is within the error bars and in agreement with the quantitative analysis of the wavevector triad model. This experiment shows a more convincing agreement with theory because of the number of data points that are within the error bars. We therefore conclude that the angular error analysis of the wavevector triad has been "tested" by experimental measurements, and found to be in good agreement with the observed data.

4. CONCLUDING REMARKS

In conclusion, experiments based on the wavevector triad model were performed. The experimental results were compared to those predicted by the wavevector triad model. The first experiment was to determine if there was a wavelength shift in the output beam of a non-Bragg-matched READ beam. The experiment resulted in no observable wavelength shift. Therefore a new interpretation of the wavevector triad was presented. An experiment for the measurement of output angular errors for non-Bragg-matched READ beams was also conducted. A very straightforward experiment was made using measurements with planar, two-beam holographic gratings read out under several off-Bragg READ beam angles. There was good experimental agreement with the wavevector triad model.

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