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Negative Index in a Lossy Chiral Metamaterial under First-Order Material Dispersion Using a Drude Model

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Abstract: Using the Drude model for complex conductivity, phase and group velocities, and indices are calculated under first-order dispersion in chiral metamaterials. Conditions are derived for negative index (NIM), and the results compared with parametric analyses.
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1. Background

Negative index in a *lossy* metamaterial has been studied for some time [1]. There are now several theoretical and experimental studies that have been reported confirming properties and applications of negative index of refraction (NIM) [2]. It is known that for waves in media with negative index, the wave vector obtains a negative sign. In a *lossy* medium, the wave vector becomes complex; its imaginary part is a result of the presence of both complex conductivity and an equivalent complex permittivity [1, 2]. In this work, we investigate the emergence of negative index in a dispersive chiral metamaterial by deriving the phase velocity via the *real* part of the wave vector and applying the Drude model for electric conductivity and the Lorentzian model for relative permittivity and permeability. Thereafter we determine the range of frequencies where phase and group velocities are in opposite directions. The overall effect of losses in NIM behavior is also explored vis-à-vis lossless propagation and also parametric analysis under loss.

2. Wave vector, phase velocity and group velocity under Drude model for conductivity

After considerable algebra, the wave vector in a lossy metamaterial can be expressed to first order as follows [3]:

$$\mathrm{Re}(\tilde{k}_{z3}) = -\omega\tilde{k}_{p0}\sqrt{\mu_0\varepsilon_0} + \omega\sqrt{\tilde{\mu}_{p0}\tilde{\varepsilon}_{p0}} + \frac{\omega\sqrt{\tilde{\mu}_{p0}\tilde{\varepsilon}_{p0}}\,\tilde{\sigma}_{p0i}}{2\tilde{\varepsilon}_{p0}\omega_0} + \omega\Omega\{-\tilde{k}'_{p0}\sqrt{\mu_0\varepsilon_0} + \frac{1}{2}\sqrt{\tilde{\mu}_{p0}\tilde{\varepsilon}_{p0}}\{\tilde{\mu}'_{pr0} + \frac{\tilde{\mu}'_{pr0}\tilde{\sigma}_{p0i}}{2\tilde{\varepsilon}_{p0}\omega_0} + \tilde{\varepsilon}'_{pro} - \frac{\tilde{\varepsilon}'_{pro}\tilde{\sigma}_{p0i}}{2\tilde{\varepsilon}_{p0}\omega_0} + \frac{\tilde{\sigma}'_{p0i}}{\omega_0\tilde{\varepsilon}_{p0}}\}\}\,. \qquad (1)$$

The phase velocity can be calculated by using $\tilde{v}_p = 1/(\mathrm{Re}(k_z)/\omega)$ and noting that $\tilde{k}_{z3}$ is complex (so that one must incorporate the real part of $\tilde{k}_{z3}$ to find $\tilde{v}_{p3}$ according to the preceding relationship [3]), we are able to calculate the phase velocity in a lossy material by using Re($\tilde{k}_{z3}$) and retaining up to the first order in Ω:

$$\tilde{v}_p = \frac{1}{\sqrt{\mu_0\varepsilon_0}\{-\tilde{k}_{p0} + \sqrt{\mu_r\varepsilon_r} + \frac{\sqrt{\mu_r\varepsilon_r}\,\tilde{\sigma}_{p0i}}{2\tilde{\varepsilon}_{p0}\omega_0} + \Omega\{-\tilde{k}'_{p0} + \frac{1}{2}\sqrt{\mu_r\varepsilon_r}\{\tilde{\mu}'_{pr0} + \frac{\tilde{\mu}'_{pr0}\tilde{\sigma}_{p0i}}{2\tilde{\varepsilon}_{p0}\omega_0} + \tilde{\varepsilon}'_{pro} - \frac{\tilde{\varepsilon}'_{pro}\tilde{\sigma}_{p0i}}{2\tilde{\varepsilon}_{p0}\omega_0} + \frac{\tilde{\sigma}'_{p0i}}{\omega_0\tilde{\varepsilon}_{p0}}\}\}\}}\,. \qquad (2)$$

Using the phase index definition $n_{p3} = c/\tilde{v}_{p3}$, one obtains:

$$n_{p3} = c[\sqrt{\mu_0\varepsilon_0}\{-\tilde{k}_{p0} + \sqrt{\mu_r\varepsilon_r} + \frac{\sqrt{\mu_r\varepsilon_r}\,\tilde{\sigma}_{p0i}}{2\tilde{\varepsilon}_{p0}\omega_0} + \Omega\{-\tilde{k}'_{p0} + \frac{1}{2}\sqrt{\mu_r\varepsilon_r}\{\tilde{\mu}'_{pr0} + \frac{\tilde{\mu}'_{pr0}\tilde{\sigma}_{p0i}}{2\tilde{\varepsilon}_{p0}\omega_0} + \tilde{\varepsilon}'_{pro} - \frac{\tilde{\varepsilon}'_{pro}\tilde{\sigma}_{p0i}}{2\tilde{\varepsilon}_{p0}\omega_0} + \frac{\tilde{\sigma}'_{p0i}}{\omega_0\tilde{\varepsilon}_{p0}}\}\}\}]\,. \qquad (3)$$

Similarly, we calculate the group velocity by using $\tilde{v}_g = 1/\frac{\partial\,\mathrm{Re}(k_{z3})}{\partial\Omega}$, so that

$$v_{g3} = \frac{1}{\sqrt{\mu_0\varepsilon_0}[-\tilde{k}_{p0} + 4 + \frac{\tilde{\sigma}_{p0i}}{2\varepsilon_0\omega_0} - \omega_0\tilde{k}'_{p0} + 2\omega_0\tilde{\mu}'_{pr0} + \frac{\tilde{\mu}'_{pr0}\tilde{\sigma}_{p0i}}{4\varepsilon_0} + 2\omega_0\tilde{\varepsilon}'_{pro} - \frac{\tilde{\varepsilon}'_{pro}\tilde{\sigma}_{p0i}}{4\varepsilon_0} + \frac{\tilde{\sigma}'_{p0i}}{4\varepsilon_0} + \Omega\{-2\tilde{k}'_{p0} + 4\tilde{\mu}'_{pr0} + \frac{\tilde{\mu}'_{pr0}\tilde{\sigma}_{p0i}}{2\varepsilon_0\omega_0} + 4\tilde{\varepsilon}'_{pro} - \frac{\tilde{\varepsilon}'_{pro}\tilde{\sigma}_{p0i}}{2\varepsilon_0\omega_0} + \frac{\tilde{\sigma}'_{p0i}}{2\omega_0\varepsilon_0}\}]}\,. \qquad (4)$$

Next, we find the group index by dividing the free-space speed of light by the group velocity as follows

$$n_{g3} = c\{\sqrt{\mu_0\varepsilon_0}[-\tilde{k}_{p0} + 4 + \frac{\tilde{\sigma}_{p0i}}{2\varepsilon_0\omega_0} - \omega_0\tilde{k}'_{p0} + 2\omega_0\tilde{\mu}'_{pr0} + \frac{\tilde{\mu}'_{pr0}\tilde{\sigma}_{p0i}}{4\varepsilon_0} + 2\omega_0\tilde{\varepsilon}'_{pro} - \frac{\tilde{\varepsilon}'_{pro}\tilde{\sigma}_{p0i}}{4\varepsilon_0} + \frac{\tilde{\sigma}'_{p0i}}{4\varepsilon_0}$$
$$+\Omega\{-2\tilde{k}'_{p0} + 4\tilde{\mu}'_{pr0} + \frac{\tilde{\mu}'_{pr0}\tilde{\sigma}_{p0i}}{2\varepsilon_0\omega_0} + 4\tilde{\varepsilon}'_{pro} - \frac{\tilde{\varepsilon}'_{pro}\tilde{\sigma}_{p0i}}{2\varepsilon_0\omega_0} + \frac{\tilde{\sigma}'_{p0i}}{2\omega_0\varepsilon_0}\}]\} . \quad (5)$$

3. Numerical results and discussion

To track the behavior of a lossy chiral material in terms of the phase and group velocities calculated earlier, we have assumed the Drude model for electric conductivity and the Lorentzian model for relative permittivity and permeability. We first examine the frequency-dependent (normalized) phase velocity and phase index in a lossy material based on eqs. (2) and (4). The plots of $\tilde{v}_{p3N}$ and $\tilde{v}_{g3N}$ versus frequency is carried out by using $\mu_r = \varepsilon_r = 4$,

$\tilde{\varepsilon}_{pr0} = 1 + \frac{\omega_p^2}{\omega_c^2}$, $\tilde{\varepsilon}'_{pr0} = \frac{2\omega_0\omega_p^2}{\omega_c^4}$, $\tilde{\mu}_{pr0} = 1 + \frac{\omega_m^2}{\omega_c^2}$, $\tilde{\mu}'_{pr0} = \frac{2\omega_0\omega_m^2}{\omega_c^4}$, $\tilde{\kappa}_{pro} = \frac{\alpha_c\omega_0}{\omega_c^2}$, $\tilde{\kappa}'_{pro} = \frac{\alpha_c}{\omega_c^2}$, $\alpha_c = 0.1\omega_c$,
$\tilde{\sigma}_{por} = 2.66*10^{-44}$,

$\tilde{\sigma}_{poi}$ =1.33*10^{-18}, $\tilde{\sigma}'_{por}$ = 1.33*10^{-12} and $\tilde{\sigma}'_{poi}$ =1.33*10^{-18} , and $\tilde{\sigma}_{p0}$ = 1.33*10^{-12}, where τ is the relaxation time for the material. In Fig. 1(a) the phase velocity transitions from negative to positive around the resonance and the resonant frequency in case is equal to 2.43*10^{7} rad/sec. Likewise, the group velocity transitions from positive to negative around the resonance as seen from Fig. 1(c) and also in this case the resonant frequency is 3.41*10^{9} rad/sec. This is followed by a similar graphical examination of the (normalized) phase index and group index as shown in Fig. 1(b) and (d). The plots in Fig. 1(d) are useful for the purpose of examining the NIM region, in this plot we can see that n_{g3} starts to be negative around the frequency 3.393*10^{9} rad/sec. Correspondingly, n_{p3} is positive in this region. This contradicts the requirement for NIM that the phase index should be negative and the group positive. The comparison between Fig. 1(b) and (d) shows that the material does not exhibit NIM for any positive frequency with the chosen parameters. Incidentally, NIM does not occur for the *lossless* problem in first order when a specific practical model is applied [4]. Examining the physical problem with further parameter choices (including conductive losses) in order to achieve NIM is currently being pursued, and will be reported on in due course.

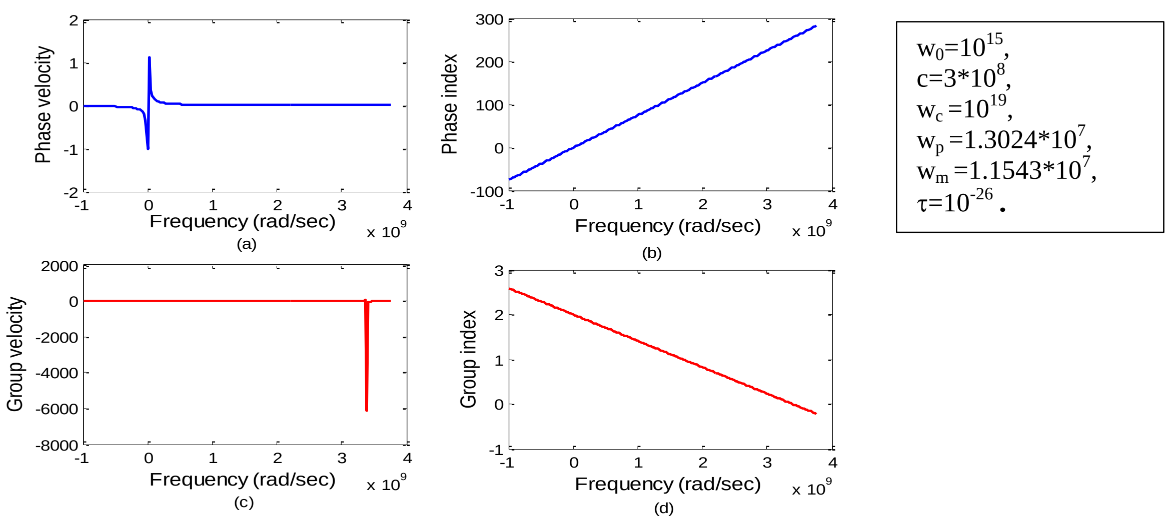

Fig. 1. (a) Phase velocity in lossy material under first-order; (b) phase index in lossy material under first- order; (c) group velocity in lossy material under first-order; (d) group index in lossy material under-first-order.

4. References

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