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Abstract: A transfer function formalism is applied to track propagation of modulated chaos waves through modified von Karman phase turbulence; the demodulated signal is examined vis-à-vis performance relative to turbulence strength in comparison with non-chaotic propagation.

1. Background

Electromagnetic wave (EM) propagation through discrete random phase screen(s) and an extended random medium has been studied for many years [1]. This work is motivated by research involving generation and encryption of acousto-optic (AO) chaos, and examining propagation of such chaotic waves through phase turbulence. In ongoing work [2], a transfer function model has been derived using cross-correlation and cross-power spectral density functions using both *unmodulated* chaos and turbulence. Characteristics of information-encrypted chaos are examined in the current paper.

2. Thin phase screen generation

Shown in Fig.1 is a schematic representing arbitrary profiled beam propagation through a narrowly turbulent region of space which is expressed as a thin random phase screen developed from the modified von Karman (MVKS) phase turbulence model. The phase screen is derived using a randomly generated transverse phase distribution given by:

$$\varphi_{ij} = Re\left\{IFFT\left((a+jb)\sqrt{\Phi_p \Delta k_x \Delta k_y}\right)\right\},$$

where *IFFT* represents the inverse *fast Fourier transform* operation, Δk_x, Δk_y is the incremental spatial frequencies, Φ_p is the power spectral density (given below) evaluated in the transverse plane, and (a and b) are random numbers generated in order to appropriately mimic the random noise-like characteristics of the von Karman phase. Correspondingly, the MVKS model in the spatial domain is expressed as:

$$\Phi_p(k) = 0.23 r_0^{-\frac{5}{3}} \exp\left(-k^2/k_m^2\right)/(k^2+k_0^2)^{11/6} \ ,$$

where r_0 is Fried parameter, , $k_m = 5.92/\ell_0$ is an equivalent wavenumber related to the inner scale l_0, $k_0 = 2\pi/L_0$ is a wavenumber related to the outer scale L_0, and k is the unbounded wavenumber in the medium. L is the beam propagation distance and Δz is the distance increment between alternate propagation segments in the split-step analysis. Details of the phase screen generation and it use via the split-step method are discussed in ref.[3,4]

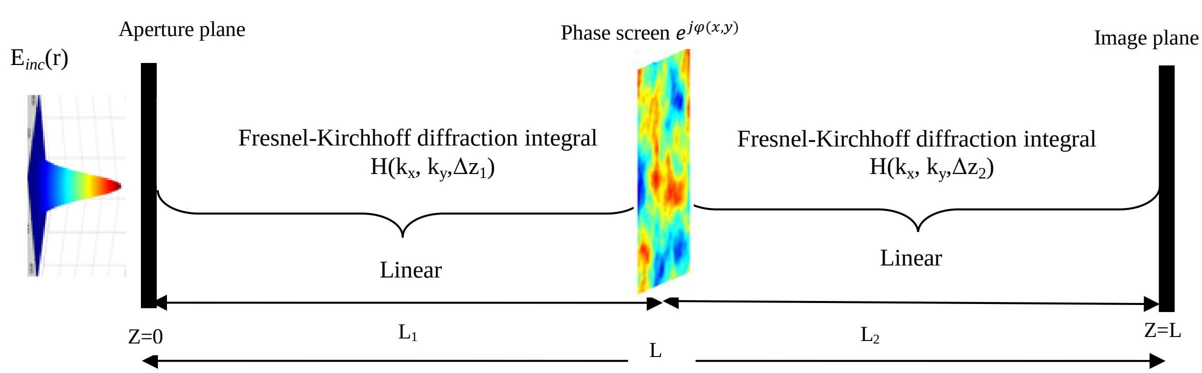

Fig. 1. Schematic illustration of propagation through random phase screen.

3. Cross-correlation and cross-power spectral density functions applied to encrypted chaos propagation

In [2], the system transfer function $H_T(f)$ was derived using an *unmodulated* chaos wave $E_C(t)$ with specific values of the average chaos and effective turbulence frequencies. In this paper, a *modulated* chaos wave as the EM input field is incorporated where the power spectral density of the chaos wave ($S_{Cm}(f)$) is calculated via an auto-correlation function ($R_{Cm}(\tau)$). Defining a corresponding transfer function $H_T(f)$ based on the spectral densities we compute (a) the cross-power spectral density ((S_{TCm}(f))); and (b) the cross-correlation function ($R_{TCm}(\tau)$) between the *modulated* chaos and turbulence using the following relations on an equivalent *linear systems* basis:

$$S_{Tcm}(f) = H_T(f) S_{cm}(f) \,, \qquad (3)$$

$$R_{Tcm}(\tau) = F^{-1}\{S_{Tcm}(f)\} \;. \qquad (4)$$

4. Results and discussion for specific cases of chaos and turbulence

In this section, we will examine the propagation of a *modulated* chaos wave with average frequency f_{ch} (=1MHz) through weak turbulence with effective frequency f_T (=100Hz). The parameters used in this simulation are chosen as: inner scale ℓ_0 =1mm; outer scale L_0 = 1km; phase screen grid size and resolution 500mmx500mm and 513x513 pixels respectively; the propagation distance L = 5km.

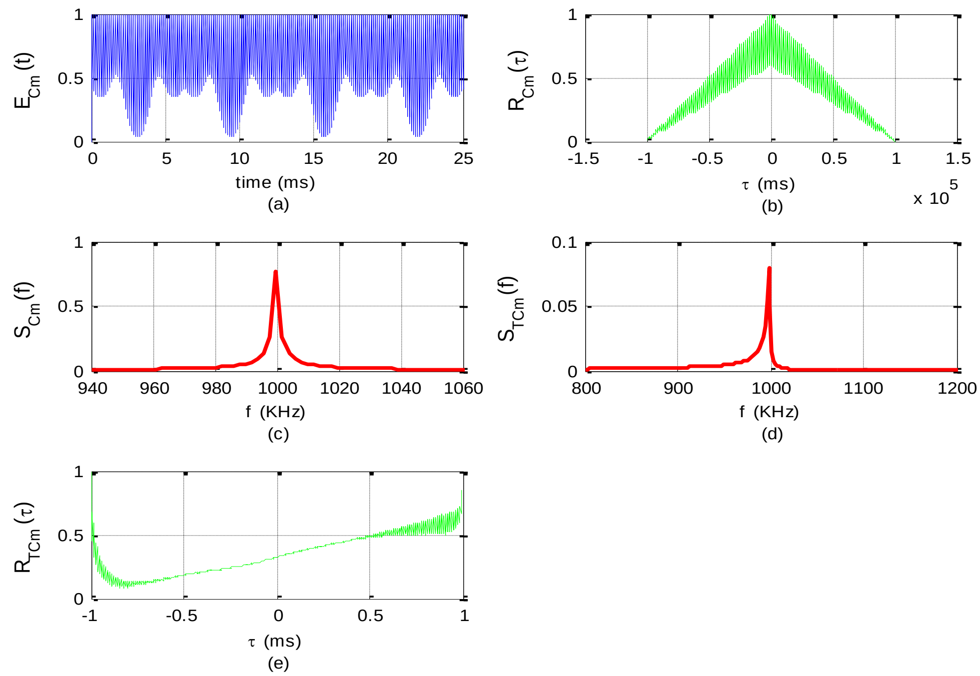

Fig. 2. Propagation of modulated chaos waves through weak turbulence (C_n^2=1.0713x10^{-18} $m^{-2/3}$) (a) modulated chaos time waveform with frequency f_{ch}=1MHz; (b) chaos auto-correlation function; (c) chaos power spectral density; (d) cross-power spectral density; and (e) the cross-correlation function between turbulence and chaos.

From Fig.2(e), by definition $R_{TCm}(\tau)$) is the product of the expectation value of E_T(t) and that of E_{Cm}(t) (assuming that chaos and turbulence are statistically independent). In our ongoing work, the goal is to extract the *modulated* chaos field E*cm*(t) from $R_{Tcm}(\tau)$, such that we may apply a standard heterodyne scheme to recover the originally transmitted signal E_C(t). The ultimate intent is to examine the survivability of the message wave encrypted in chaos through moderate to extreme turbulence.

5. References

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[2] M.R. Chatterjee and F.H.A. Mohamed, "A Transfer Function Based Frequency Model for Propagation of a Chaos Wave through Modified

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