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Emergence of Negative Index in a Lossy Chiral Metamaterial under First-order Material Dispersion

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Abstract: For a *lossless* dispersive chiral material, negative index (NIM) occurred only by use of parametric analysis (and not via practical models) to first-order. These findings are re-visited for the *lossy* problem and the results are compared.
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1. Introduction

It is known that counter-propagation between such standard electromagnetic (EM) entities as the Poynting vector, the propagation vector, the group velocity and the phase velocity (in paired combinations) leads to the onset of negative refractive index (NIM) behavior in the material [1]. This paper examines the possible emergence of negative refractive index in *lossy* chiral metamaterials with material dispersion up to the first order. Proceeding as in recent work [2] with lossless materials, a spectral approach is applied again leading to the derivation of EM phase and group velocities analytically, and the resulting velocities and the corresponding phase and group indices are evaluated by selecting somewhat arbitrary dispersive parameters. It has been shown that in the NIM regime, the group index (n_g) remains positive, while it is the phase index (n_p) which becomes negative [3]. For the case of lossless propagation, it is found that NIM occurs for first order under use of results derived through parametric analyses, and not for the case where the spectral approach is applied via practical models. The current work introduces material loss into the problem via dispersive conductivity. The primary aim is to investigate the effect of losses on propagation velocities, and thereby the likelihood of NIM to occur or not. The analysis is again applied to both assumed parametric behavior relative to dispersion, as well as the case of known dispersive models for permittivity, permeability, chirality and conductivity. Relative merits or the absence thereof of lossy propagation are examined.

2. EM analysis under first-order dispersion

The basic methodology consists of using Maxwell's equations, constitutive relations, spectral analyses, and plane wave solutions to eventually obtain the group, phase and energy velocities in the medium, and thereafter finding conditions for negative index [4]. The EM constitutive relations for a reciprocal chiral medium in the frequency domain are known to be:

$$\tilde{D} = \tilde{\varepsilon}\tilde{E} - j\tilde{\kappa}\sqrt{\mu_0\varepsilon_0}\,\tilde{H}, \tag{1}$$

$$\tilde{B} = j\tilde{\kappa}\sqrt{\mu_0\varepsilon_0}\,\tilde{E} + \tilde{\mu}\tilde{H}, \tag{2}$$

where $\tilde{D}$, $\tilde{B}$, $\tilde{E}$ and $\tilde{H}$ are the usual (phasor) field vectors written in terms of frequency-dependent material parameters. We introduce loss through the second Maxwell equation as follows.

$$\nabla \times \tilde{H} = \tilde{J} + j\omega\varepsilon\tilde{E}, \tag{3}$$

where the conduction current density $\tilde{J} = \sigma\tilde{E}$. Hence, from Eq. (4)

$$\nabla \times \tilde{H} = \sigma\tilde{E} + j\omega\varepsilon\tilde{E} = (\sigma + j\omega\varepsilon)\tilde{E}. \tag{4}$$

We can rewrite eq. (5) as follows

$$\nabla \times \tilde{H} = j\omega\varepsilon_c \tilde{E} , \tag{5}$$

where ε_c is the (complex) permittivity equal to $\varepsilon_c = (-j\frac{\sigma}{\omega} + \varepsilon)$, and $\varepsilon = \varepsilon_0\varepsilon_r$ represents the lossless permittivity.

Using Fourier transforms and the (chiral) constitutive relations, we obtain the following phasor fields:

$$\bar{k} \times \tilde{E}_p(\Omega) = \omega[\mathrm{j}\tilde{\kappa}_p(\Omega)\sqrt{\mu_0\varepsilon_0}\tilde{E}_p(\Omega) + \tilde{\mu}_p(\Omega)\tilde{H}_p(\Omega)], \tag{6}$$

$$\bar{k} \times \tilde{H}_p(\Omega) = \omega[\tilde{E}_p(\Omega)\tilde{\varepsilon}_p(\Omega) - j\tilde{\kappa}_p(\Omega)\sqrt{\mu_0\varepsilon_0}\tilde{H}_p(\Omega)] - j\tilde{\sigma}(\Omega)\tilde{E}_p(\Omega) , \tag{7}$$

where $\tilde{\varepsilon}_p$ represents the dispersive, lossless permittivity under slowly varying phasor. To get a valid set of plane-wave field solutions from the above, we assume propagation vector $\bar{k}_z$ strictly in the z-direction for simplicity [1]. A set of well-known homogenous equations for the field components are readily obtained from Eqs. (6) and (7), wherefrom by expanding the material parameters up to the first-order, four possible solutions for the wavenumber in the medium compatible with the plane wave under investigation. These solutions are:

$$\tilde{k}_z = \pm[\omega\tilde{\kappa}_p\sqrt{\mu_0\varepsilon_0} \pm \sqrt{\omega\tilde{\mu}_p(\omega\tilde{\varepsilon}_p - j\tilde{\sigma}_p)}]. \tag{8}$$

Using the relation $\tilde{v}_p = 1/(\mathrm{Re}(k_z)/\omega)$, and noting that $\tilde{k}_{z3}$ in eq.(8) is complex (so that one must incorporate the real part of $\tilde{k}_{z3}$ to find $\tilde{v}_{p3}$ according to the preceding relationship), we are able to calculate the phase velocity in a lossy material by using $\tilde{k}_{z3}$ and retaining up to the first order in Ω:

$$\tilde{v}_{p3N} = \frac{\tilde{v}_{p3}}{c} = \frac{1}{-\tilde{\kappa}_{p0} + \sqrt{\tilde{\mu}_r\tilde{\varepsilon}_r} + \left(\Omega\{-\tilde{\kappa}'_{p0} + \sqrt{\tilde{\mu}_r\tilde{\varepsilon}_r}[\frac{\tilde{\varepsilon}'_{po}}{\tilde{\varepsilon}_{p0}} + \frac{1}{4}\frac{\tilde{\varepsilon}'_{po}\,\tilde{\sigma}^2_{p0}}{\omega_0^2\tilde{\varepsilon}^3_{p0}} + \frac{1}{4}\frac{\tilde{\sigma}'_{p0}\tilde{\sigma}_{p0}}{\omega_0^2\tilde{\varepsilon}^2_{p0}}]\}\right)}. \tag{9}$$

Using the phase index definition $n_{p3} = c/\tilde{v}_{p3}$, one obtains:

$$n_{p3} = \left\{-\tilde{\kappa}_{p0} + \sqrt{\tilde{\mu}_r\tilde{\varepsilon}_r} + \left(\Omega\{-\tilde{\kappa}'_{p0} + \sqrt{\tilde{\mu}_r\tilde{\varepsilon}_r}[\frac{\tilde{\varepsilon}'_{po}}{\tilde{\varepsilon}_{p0}} + \frac{1}{4}\frac{\tilde{\varepsilon}'_{po}\,\tilde{\sigma}^2_{p0}}{\omega_0^2\tilde{\varepsilon}^3_{p0}} + \frac{1}{4}\frac{\tilde{\sigma}'_{p0}\tilde{\sigma}_{p0}}{\omega_0^2\tilde{\varepsilon}^2_{p0}}]\}\right)\right\}. \tag{10}$$

3. Numerical results

In what follows, we present numerical results for the phase velocity in lossy and lossless materials. We first examine the frequency-dependent (normalized) phase velocity and phase index in a lossy material based on eqs. (10) and (11). The results are plotted in Fig. 1(a) and (b). This is followed by a similar graphical examination of the (normalized) phase velocity and phase index in *lossless* material as shown in Fig. 1(c) and (d). The plots in Fig. 1 are useful for the purpose of examining the effect of material loss in the emergence or absence of NIM in the absence or presence of loss to first order. The plot of $\tilde{v}_{p3N}$ versus frequency is carried out by assuming $\mu_r = \varepsilon_r = 4$, $\tilde{k}_{p0}$ = 5, $\tilde{k}'_{p0} = 0.1\frac{\tilde{\varepsilon}'_{po}}{\varepsilon_0}$, and $\tilde{\varepsilon}'_{po}$ =3.13*10^{-18}, $\tilde{\sigma}'_{p0}$ =2*10^4, and $\tilde{\sigma}_{p0}$ = 10^{-3}. In Fig. 1(a) the phase velocity transitions from negative to positive around the resonance and the resonance frequency in case of lossy material is equal to 5.1*10^5 rad/sec. Likewise, the phase velocity in lossless material is also transitions from negative to positive around the resonance as seen from Fig. 1(c) and also in this case the resonance frequency is 3.14*10^6 rad/sec. These transitions are further indicated in the index plots of Figs. 1(b) and 1(d). Incidentally, an earlier examination of the lossless problem to first order using both $\tilde{v}_{p3N}$ and $\tilde{v}_{g3N}$ in the parametric analysis indicated that NIM occurs for the parameters as chosen; however, NIM does not occur for the lossless problem in first order when a specific practical model is applied [4]. For the lossy case, as discussed here, the phase index again exhibits a transition from negative to positive at a lower frequency. It is likely this is also indicative of NIM behavior; this may be verified further by examining the corresponding lossy group velocity. Specifically, we find that the phase index n_{p3} in the lossy material is negative in the range of from 0 to $5.1*10^5$ rad/sec and n_{p3} in the lossless material is negative within the frequency range of approximately 0 up to about $3.14*10^6$ rad/sec. Emergence of NIM in the lossy material to first order requires further examination which is currently being pursued.

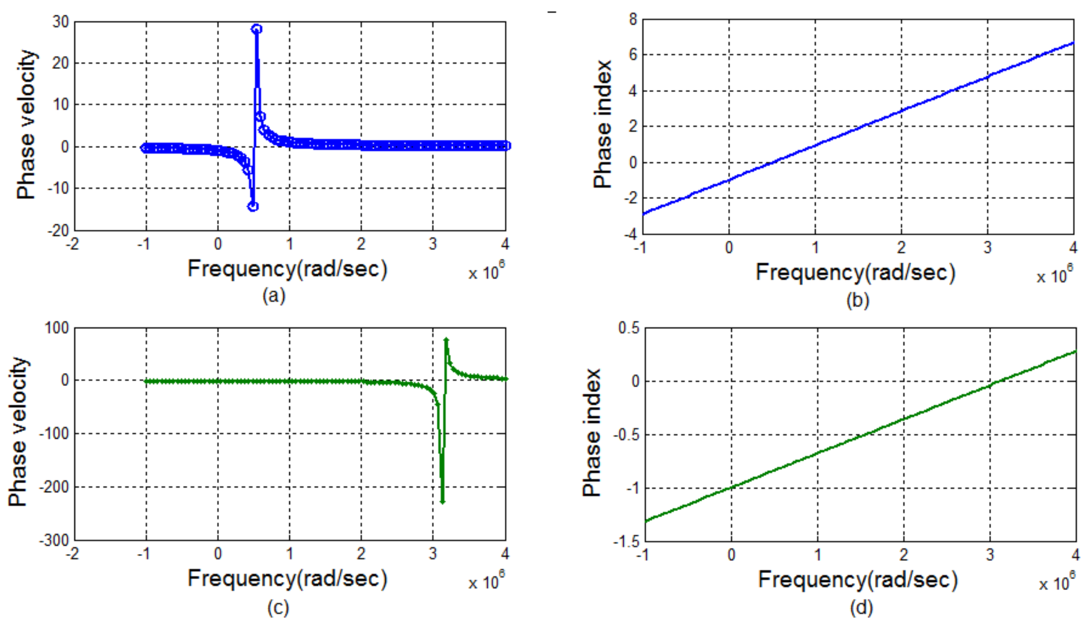

Fig. 1. (a) Phase velocity in lossy material under first-order; (b) phase index in lossy material under first- order; (c) phase velocity in lossless under first-order; (d) phase index in losslesss material under-first-order.

4. Conclusion

Starting from the phasor approach and a slowly varying envelope approximation (SVEA) combined with Maxwell's equations and the constitutive relations in a reciprocal chiral material, expressions for electromagnetic phase velocity and index for a *lossy* chiral material have been derived assuming first-order material dispersion. Overall it is established from those analyses that n_{p3} in a *lossy* material is negative within the range of frequency from 0 to $5.1*10^5$ rad/sec which is less than the corresponding range in the lossless material. This result does not automatically establish NIM behavior. To examine the NIM characteristics under loss, group velocity and index measures need to be derived, and thereafter compared with those for the phase velocity and index. An important follow-up will be to apply practical dispersive models to the *lossy* problem in order to see if the presence of losses fundamentally changes the observed NIM characteristics for the lossless dispersive problem under first- or second-order dispersion. These analyses are currently in progress, and shall be reported upon in the future.

5. References

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