A Transfer Function Based Frequency Model for Propagation of a Chaos Wave through Modified von Karman Turbulence under Various Chaos and Turbulence Conditions

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Abstract: Complex phasor fields for electromagnetic wave propagation through von Karman turbulence and acousto-optic RF chaos are derived *en route* to the effective transfer function between chaos and narrow turbulence. Results are tested for several turbulence and chaos conditions.

1. Mathematical background

Turbulence has a large influence on electromagnetic (EM) wave propagation. Turbulence effects arise from atmospheric refractive index fluctuations that cause both large and small variations of EM wave propagation [1]. This work analyzes the overall propagation problem within the framework of a linear system and thereby develops an equivalent *transfer function* model derived from cross- and self-spectral densities using both chaos and turbulence, as will be shown.

1.1 Modified von Karman Spectrum (MVKS)

The MVKS power spectral density for turbulence incorporating both the inner scale (ℓ_0) and outer scale L_0 is given by [2]:

$$\Phi_n(k) = 0.033\ C_n^2\ \frac{\exp\left(-\frac{k^2}{k_m^2}\right)}{(k^2+k_0^2)^{\frac{11}{6}}} \quad , \ 0 \leq k \ll \infty \tag{1}$$

where C_n^2 is the medium structure parameter, $k_m = {}^{5.92}/_{l_o}$ is an equivalent wavenumber related to the inner scale,
$k_0 = {}^{2\pi}/_{L_o}$ is a wavenumber related to the outer scale, and k is the unbounded (non-turbulent) wavenumber in the medium. In the above equation, Φ_n(k) represents the so-called power spectral density (PSD) of the refractive index of the medium.

1.2 Acousto-Optic (A-O) chaos

The first-order detected intensity of the standard Bragg cell transmitter with feedback follows the nonlinear dynamical equation [3,4]:

$$I_1(t) = I_{inc}\sin^2\left[0.5\left(\tilde{\alpha}_0(t) + \tilde{\beta}(t - T_D)\right)\right], \tag{2}$$

where $\tilde{\alpha}_0$ is the peak optical phase delay through the medium, $\tilde{\beta}$ is the effective feedback gain, I_{inc} is the incident intensity, and T_D is the feedback time delay including photo detector conversion delay. Under certain values of the four parameters mentioned above, the system may be driven through a series of dynamical regimes, namely, monostable, bistable, multistable, and chaos. In this paper, we generate a scalar chaos field $E_C(t)$ at different pre-selected chaos frequencies (given approximately by f_{ch}=1/(2*T_D)).

1.3 Turbulence time waveform $E_T(t)$ derived from spatial model

In order to generate the turbulence time waveform (needed for developing the aforementioned transfer function), we propagate a uniform plane EM wave over a distance (L) using the MVKS model whereby a narrow phase screen is placed half way between the object and image planes. For each sampling time interval (ΔT), the on-axis field at (0,z) (E(0,z)) is evaluated. We repeat this process for pre-selected intervals ΔT in order to generate the on-axis transmitted

field *as a function of time*. The turbulence frequency (f_T) is determined by taking the reciprocal of the repetition interval ΔT.

1.4 The cross-correlation and cross-power spectral density functions

For two independent random processes (C and T for chaos and turbulence respectively), the cross-correlation $R_{CT}(\tau)$ and cross-power spectral density $S_{CT}(f)$ functions may be expressed as [5,6]:

$$R_{CT}(\tau) = E(E_C(t)E_T(t+\tau)), \tag{3a}$$

and

$$S_{CT}(f) = \mathfrak{F}\{R_{CT}(\tau)\}, \tag{3b}$$

where $\mathfrak{F}$ denotes Fourier transform.

The effective transfer function between the chaos and the turbulence using the time models described above may be found using the following formula [6]:

$$H_T(f) = \frac{S_{CT}(f)}{S_T(f)}. \tag{4}$$

2. Results for specific cases of turbulence

In this section, we will examine the time waveforms for both chaos and turbulence and the resultant transfer function by propagating a uniform plane wave through a narrow phase screen characterizing the random behavior of the turbulence (weak and strong). The parameters used in this simulation are chosen as: inner scale ℓ_0 =1mm ; outer scale L_0 = 1km; phase screen grid size and resolution 500mmx500mm and 513x513 pixels respectively; the propagation distance L = 5km. The chaos time delay T_D=0.5μs (f_{ch}=1MHz) and the turbulence interval ΔT=0.01s (f_T=100Hz).

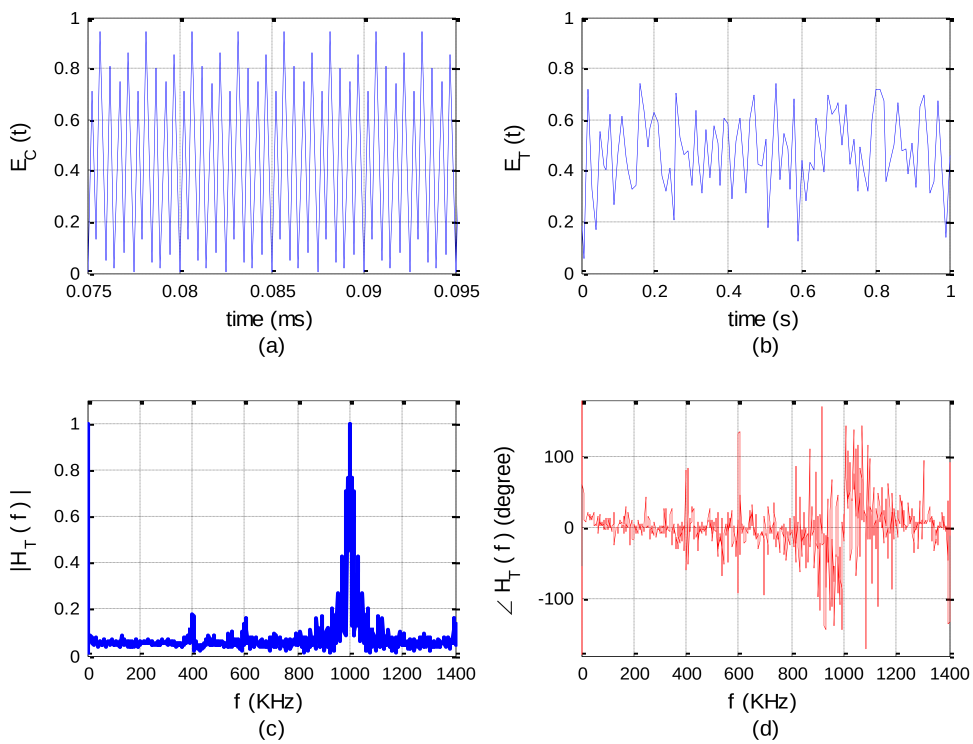

Fig. 1. Propagation of EM waves through weak turbulence (C_n^2=1.0713x10^{-18} $m^{-2/3}$), (a) chaos time waveform with frequency f_{ch}=1MHz, (b) turbulence time waveform with frequency (f_T=100Hz), (c) transfer function magnitude, and (d) phase of the transfer function.

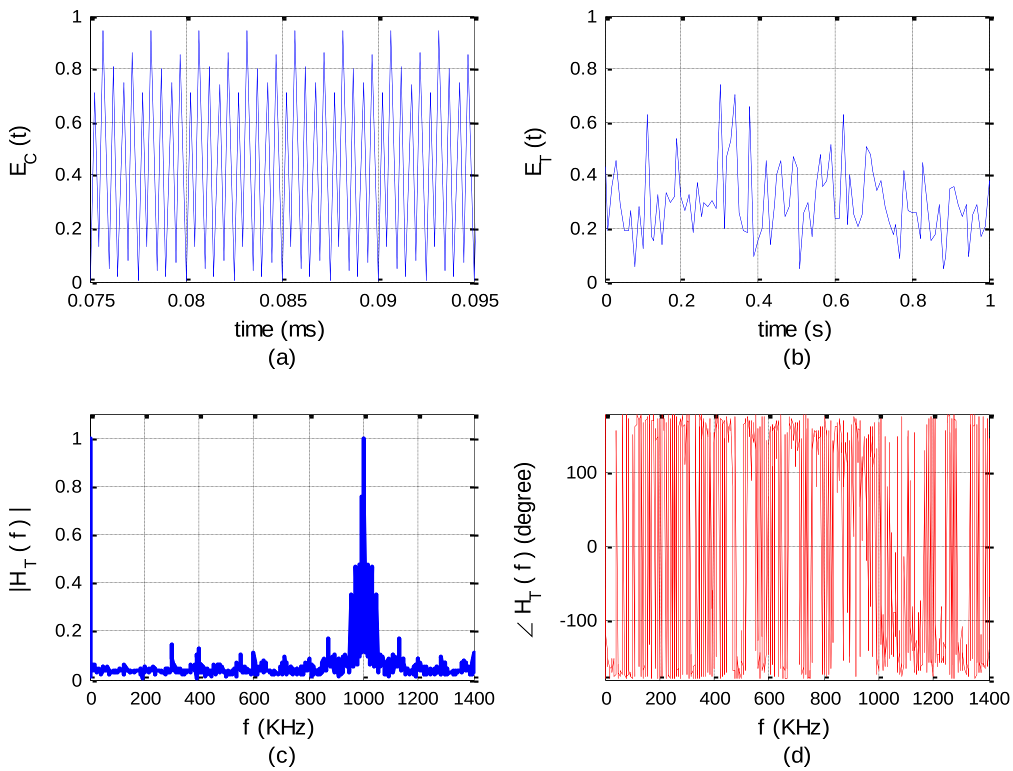

Fig. 2. Propagation of EM waves through strong turbulence (c_n^2=1.0713x10^{-13} $m^{-2/3}$), (a) chaos time waveform with frequency f_{ch}=1MHz, (b) turbulence time waveform with frequency (f_T=100Hz), (c) transfer function magnitude, and (d) phase of the transfer function.

3. Discussion of results and ongoing work

From the figures shown (Figs. 1 and 2), the average amplitude of the turbulence time waveform appears to move downwards for stronger turbulence (Fig.1b and Fig. 2b). The transfer function magnitude in the two cases are virtually unchanged, with the maximum magnitude around the chaos frequency ($f_{ch}=1MHz$) as illustrated in Fig.1c and Fig. 2c. It may be noted that the magnitude spectrum contains a strong component nearer to the average turbulence frequency; however, in the neighborhood of the chaos wave, it is the latter which is predominant. Regarding the phase angle of the transfer function, we can observe that phase fluctuations occur near the chaos frequency in the case of weak turbulence (as in Fig.1d), while in the case of strong turbulence, the (rapid) phase fluctuations spread out along the frequency axis (as in Fig. 2d). Overall, this corroborates the parallel finding that propagating EM waves are generally distorted in the presence of strong phase turbulence [7]. In ongoing work, we will examine more general cross-correlation ($R_{CT}(\tau)$) and cross-power spectral density $S_{CT}(f)$ functions for different chaos and turbulence conditions (including different chaos parameters; turbulence interval and variable turbulence strength).

4. References

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