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# Nonlinear dynamics of Bragg-domain acousto-optic hybrid feedback for first-order scattering of profiled optical beams

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## Abstract:

Acousto-optic Bragg diffraction with profiled input beams is numerically examined under open loop via a transfer function formalism. Thereafter, the scattered output is fed back to the acoustic driver, and the resulting nonlinear dynamics are examined to establish novel monostable, bistable, multistable and chaotic regimes

## Summary:

A series of recent studies involving hybrid acousto-optic (AO) scattering in the Bragg domain under first-order feedback have shown the ability of the AO feedback system to encrypt, transmit and decrypt RF information applied via the sound driver. The basic premise of this operation is founded on the chaotic nature of the hybrid Bragg cell under feedback [1-3].

It turns out that the conventional operation of the hybrid Bragg cell makes the following assumptions:

- (1) The device is operating in the pure Bragg regime, i.e., the incident beam angle matches the Bragg angle ( $\theta_B$ ) of the device, and that the Klein-Cook parameter  $Q \gg 8\pi$  [4];
- (2) Both the input optical beam and the acoustic beam in the sound cell are *uniform plane waves*, thereby being characterized by single wave vectors; and
- (3) Even under feedback, with the system parameters under allowable range of variations, the device continues to operate in the pure Bragg regime.

As a result of the above assumptions, the two diffracted orders from the (open loop) Bragg cell may be expressed as:

$$E_0 = E_{inc} \cos\left(\frac{\hat{\alpha}_0}{2}\right), \text{ and } E_1 = (-j) E_{inc} \sin\left(\frac{\hat{\alpha}_0}{2}\right), \text{ where } E_{inc} \text{ is the incident phasor}$$

electric field amplitude, and  $\hat{\alpha}_0$  is the effective optical phase shift in the Bragg cell during propagation through the acoustic grating.

With the above first-order light under intensity feedback, typically the hybrid cell dynamics are assumed to follow the following nonlinear dynamical equation for intensity:

$$I_1(t) = I_{inc} \sin^2 \left[ \frac{1}{2} \left( \hat{\alpha}(t) + \tilde{\beta} I_1(t - TD) \right) \right], \text{ where } I_1 \text{ is the diffracted first-order intensity at}$$

time  $t$ ,  $I_{inc}$  is the input light intensity,  $\hat{\alpha}$  is the total optical phase shift, influenced generally by the net bias voltage applied to the RF acoustic generator (this may also be time varying);  $\tilde{\beta}$  is the effective gain in the feedback loop and  $TD$  is the delay time in the feedback loop.

While one of the current researchers has been extensively involved with the analytic and numerical examination of the above problem, including signal encryption and recovery from the above device under chaos, the work presented here approaches the same problem from the perspective of a *profiled* instead of a *uniform* input optical beam. Clearly, for such a beam, the standard zeroth- and first-order solutions above are no longer valid. In general, this problem is more complex, and finding the output profile involves decomposition of the input profile into its angular spectra, and then applying a transfer function formalism developed some years ago [5] to properly (numerically in this paper) evaluate the corresponding first-order output profile. This approach has been recently studied for the open-loop problem with results that are unexpected in some cases [6].

Having established numerically the diffraction behavior in the first-order for the open-loop Bragg cell under (as discussed here) a Gaussian input profile, we next use the scattering data (instead of the analytic *uniform* plane wave solution) via a closed-loop model in order to examine the resulting closed-loop behavior of the hybrid device for *profiled* input beams.

Results from the preliminary simulations using the above approach indicate the following characteristics:

- (1) For relatively low Qs (around 50 or less), the plot of  $I_1$  vs.  $\hat{\alpha}_0$  has the appearance of a  $\sin^2$ -type behavior, even though this is non-intuitive;
- (2) At higher Qs, the behavior of  $I_1$  relative to  $\hat{\alpha}_0$  begins to deviate from the  $\sin^2$ -type characteristic;
- (3) There is an axial shift of the profile at large (asymptotic) values of  $\hat{\alpha}_0$  (this confirms analytic predictions);
- (4) The nonlinear dynamical thresholds of the hybrid cell seem to be significantly different for the profiled propagation problem than the uniform case. Thus, the mono- and bistable- regimes, even for low Qs where the scattered output appears to be  $\sin^2$ -type, so not coincide with the well-known uniform plane wave results. Likewise, and perhaps more importantly, the chaotic thresholds, which are of special interest in this research, are altered noticeably. Preliminary findings indicate onset of chaos for much lower feedback gain; additionally, chaotic bands appear to migrate into the originally bistable/hysteretic regions.

The current work will explore further the implications of the above preliminary results, and in particular examine the nonlinear dynamics of the profiled beam propagation under feedback using the techniques of Lyapunov exponents and bifurcation maps that have been used before [3].

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