Numerical Examination of the Nonlinear Dynamics of a Hybrid Acousto-Optic Bragg Cell with Positive Feedback under Profiled Beam Propagation

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A Numerical Examination of the Nonlinear Dynamics of a Hybrid Acousto-Optic Bragg Cell with Positive Feedback Under Profiled Beam Propagation

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Abstract:

In standard weak interaction theory, acousto-optic Bragg analysis typically assumes that the incident light and sound beams are uniform plane waves. Acousto-optic Bragg diffraction with non-uniform profiled input beams is numerically examined under open loop via a transfer function formalism. Unexpected deviations in the first-order diffracted beam from the standard theory are observed for high Q values. These deviations are significant because the corresponding closed-loop system is sensitive to input amplitudes and initial conditions, and the overall impact on the dynamical behavior has not been studied previously in standard analyses. To explore the effect of such non-uniform output profiles on the feedback system, the numerically generated scattered output is fed back to the acoustic driver, and the resulting nonlinear dynamics are manipulated to create novel monostable, bistable, multistable and chaotic regimes. The effects of the non-uniform input on these regimes are examined using the techniques of Lyapunov exponents and bifurcation maps. The orbital behavior is characterized

with quadratic maps, which are an intuitive method of predicting the parametric behavior of the system. The latter trajectory-based approach offers yet a third arm in the process of developing fuller understanding of the profiled output beam under feedback. The results of this work indicate that the nonlinear dynamical thresholds of the hybrid cell are significantly different for the profiled propagation problem than the uniform case. The mono- and bistable- regimes do not coincide with the well-known uniform plane wave results and the chaotic thresholds, which are critical to understanding encryption applications, are altered noticeably.

Keywords: Acousto-optics**,** nonlinear dynamics, bistability, chaos, Bragg regime**,** profiled beams, Gaussian, Klein-Cook parameter, optical phase shift, angular spectra, transfer function, scattered profiles.

1. Introduction

Acousto-optic (A-O) diffraction, used in many areas of signal processing, refers to the interaction of light and sound waves in order to controllably diffract light beams. A-O cells are characterized by the Klein-Cook parameter (Q), whose value determines the regimes of operation [1]. Most applications utilize the Bragg regime where there is only one diffracted order, and Chatterjee and Chen showed that Q should be larger than 8π in this case [2]. The mode of operation when Q is much smaller than one is called the Raman-Nath regime, characterized by multiple diffracted orders [3]. Both regimes are studied using weak interaction theory, which assumes *uniform* plane waves of sound and light. Although these assumptions are not physically realistic, they allow for tractable analysis and lead to well-known expressions for scattering under Bragg and Raman-Nath diffraction. In order to examine non-uniform optical beams, Chatterjee *et*. al developed a transfer function formalism for characterizing diffracted output profiles for arbitrary input profiles [4]. This approach utilizes a plane wave angular

spectrum of the field distribution, which allows the scattered fields to be represented by Fourier integrals in the angular domain.

Using this transfer function technique, the first-order diffracted light has been recently studied for Gaussian, third-order Hermite-Gaussian, and zeroth-order Bessel input beams. With each of these profiles, the behavior of the first-order diffracted beam of scattered light is thoroughly explored as a function of various parameters, including the Klein-Cook parameter Q, the effective profile width, and the optical phase-shift parameter ($\hat{\alpha}_0$) in the sound cell. The standard Bragg theory indicates a $\sin^2$ shape in the output intensity profile along the optical phase shift dimension. With a non-uniform plane wave input, the scattered first-order profiled output versus the optical phase-shift also appears to maintain a |sin| shape, but only at relatively small values of Q (typically about 20-50). At higher Qs, on the other hand, it is found that the first-order intensity deviates substantially from the expected pattern. Additionally, there are distortions along the second (spatial) dimension of the output profile at higher Qs, and an axial shift for a Gaussian input optical profile is observed as predicted by the asymptotic wave theory [4].

Using uniform input beams, researchers have studied the hybrid closed-loop system created by detecting the first-order scattered output with a photodetector and feeding the resulting signal into the (external) bias input of the RF generator. The resulting nonlinear dynamics exhibit mono-, bi-, multistable and also chaotic behavior [5,6]. These properties have been exploited for a variety of signal processing applications including signal encryption and decryption [7,8]. The understanding of the nonlinear dynamics thus far has been limited by the assumption of uniform input beams. Practical optical beams are more likely to be non-uniform in profile, and it has been shown that significant, unexpected deviations in the first-order output

occur for open loop A-O systems [9]. Since chaos is extremely sensitive to amplitudes, it becomes necessary to examine the consequences of specific profiled light beams upon the feedback system.

An overview of A-O Bragg diffraction, including the transfer function formalism, of profiled beam under open-loop propagation is presented in section 2 [9]. Section 3 details the nonlinear dynamics of profiled beams propagation under positive feedback. Simulations, numerical results, and interpretation of the nonlinear dynamics are discussed in section 4. Specifically, we include plots of monostable, bistable, multistable and chaotic behavior under profiled beam propagation in subsection 4.1, a study of the nonlinear dynamics using the techniques of Lyapunov exponent and bifurcation maps in subsection 4.2, and an analysis using quadratic maps in subsection 4.3. Section 5 provides concluding remarks along with plans for future work.

2. Overview of A-O Bragg diffraction of profiled beams under open-loop propagation

Fig.1 illustrates an A-O modulator with first-order feedback showing two scattered orders created by an arbitrary input profile, assuming upshifted operation at the (exact) Bragg incidence. Here we focus on only the open-loop operation of this device such that the feedback loop may be considered disconnected. We take the profiled beam to be incident nominally at the Bragg angle. The coordinates r and r' represent the transverse radial coordinates with respect to the direction of the incident (or zeroth-order) field and the diffracted field, respectively. $E_0(r)$ and $E_1(r')$ are the zeroth- and first-order scattered outputs, $\delta\phi_B$ is the angular deviation from the nominal Bragg angle $\phi_B(\approx K/2k)$, and $\overline{K}$ is the acoustic wave vector [4].

Both diffracted orders can be described by a set of coupled differential equations given by [7], which leads to the two scattered orders $\tilde{E}_0$ and $\tilde{E}_1$ for near-Bragg diffraction. The transfer function formalism developed by Chatterjee *et*. al [4] follows from the solutions of these equations. This leads to the following two transfer functions for the zeroth- and first-order fields in the δ domain under arbitrary angular deviations from the Bragg angle [4] :

$$\tilde{H}_0(\delta) = \frac{\tilde{E}_0(\zeta)_{\xi=1}}{\tilde{E}_{inc}} = -\frac{e^{-j\frac{\delta Q}{4}}}{\sqrt{\left(\frac{\delta Q}{4}\right)^2 + \left(\frac{\hat{\alpha}}{2}\right)^2}} \left(\sqrt{\left(\frac{\delta Q}{4}\right)^2 + \left(\frac{\hat{\alpha}}{2}\right)^2} \cos\left(\sqrt{\left(\frac{\delta Q}{4}\right)^2 + \left(\frac{\hat{\alpha}}{2}\right)^2} \right) + j\frac{\delta Q}{4} \sin\left(\sqrt{\left(\frac{\delta Q}{4}\right)^2 + \left(\frac{\hat{\alpha}}{2}\right)^2} \right) \right) , \quad (1)$$

$$\tilde{H}_1(\delta) = \frac{\tilde{E}_1(\zeta)_{\xi=1}}{\tilde{E}_{inc}} = -j\left(\frac{\hat{\alpha}}{2}\right) \frac{e^{j\frac{\delta Q}{4}}}{\sqrt{\left(\frac{\delta Q}{4}\right)^2 + \left(\frac{\hat{\alpha}}{2}\right)^2}} \left(\sin\left(\sqrt{\left(\frac{\delta Q}{4}\right)^2 + \left(\frac{\hat{\alpha}}{2}\right)^2} \right) \right) . \quad (2)$$

In these equations, $\hat{\alpha}\ (= kC|A|L/2)$ is the peak phase delay, $\xi(= z/L)$ is the normalized propagation distance in the sound cell, and Q is the Klein-Cook parameter [4].

The scattered beams are numerically computed using an inverse Fourier transform applied to the product of the incident spectrum $\tilde{E}_{\mathrm{inc}}(\delta)$ and the corresponding transfer function $\tilde{H}(\delta)$ [4]. This process is shown in eq.3, where $E_{out}(r';r)$ is either the first- or zeroth-order output, depending on the transfer function used, and ϕ_B is the Bragg angle of the sound cell:

$$E_{out}(r) = \int_{-\infty}^{\infty} \tilde{E}_{inc}(\delta)\, \tilde{H}(\delta)\, e^{-j\frac{2\pi}{\lambda}\delta\phi_B r} \left(\frac{\phi_B}{\lambda}\right) d\delta \ . \quad (3)$$

To generate the output profiles, the angular spectrum of the incident light is inserted into this equation, and the output field spectra are numerically computed. These fields are functions of the peak phase delay $\hat{\alpha}_0$ and the Klein-Cook parameter Q. Three different incident profiles

$E_{inc}(r)$ have been tested for determining the output profile characteristics: the Gaussian, third-order Hermite-Gaussian, and zeroth-order Bessel profiles. Fig.2 displays the results for the Gaussian case (which is of relevance to the current work), showing the output profile along the peak phase delay $\hat{\alpha}_0$ for three different values of Q.

As is clearly visible in Fig.2, unexpected deviations from the standard theory occur at higher Q values. The curve for Q=20 appears consistent with uniform plane wave inputs, although the impact is significant in the closed-loop system (primarily due to the non-uniform output amplitudes) as is discussed in the results. As seen, the $\hat{\alpha}_0$ intensity profile begins to deviate from the standard $\sin^2$ behavior as Q increases; thus, the curve for Q=533 is clearly no longer similar to uniform plane wave input results. Another observation, fully described in a recent paper [9], is a spatial shift in the output profile for Gaussian incident beams for high sound pressure (i.e., large $\hat{\alpha}_0$) as predicted by the asymptotic spectral theory in [4]. The fact that these deviations are due to specific profiled incident beams is confirmed by the results for zeroth-order Bessel and Airy beams, which are not significantly shifted or diffracted off-axis during diffraction at high $\hat{\alpha}_0$ values [9]. Overall, the diffracted output for non-uniform incident beams exhibits significant deviations from uniform plane wave behavior. Since the nonlinear dynamics of closed-loop feedback systems is sensitive to such changes and depends on the diffracted amplitudes, it is necessary to study the impact on the performance of the A-O hybrid feedback system.

3. Nonlinear dynamics of profiled beam propagation under positive feedback

If the first-order diffracted light is picked up by a photodetector, and its output is amplified and fed back into the acoustic driver, then a familiar closed-loop system is created as

shown in Fig.1. The resulting nonlinear system displays complex bistability behavior which was first observed in 1978 [7]. Under the assumption of a uniform plane wave input, the output from the photodetector follows the nonlinear equation:

$$I_1(t) = I_{inc} \sin^2 \left\{ \frac{1}{2} \left[\hat{\alpha}_0(t) + \tilde{\beta} I_1(t - TD) \right] \right\} \quad . \tag{4}$$

In this equation, I_{inc} is the amplitude of the incident light, $\hat{\alpha}_0$ is the peak phase delay, $\tilde{\beta}$ is the feedback gain, and TD is the feedback time delay. Depending on the values of these parameters, this equation leads to *monostable*, *bistable* and *multistable* behavior, as well as chaos. Simulations of such behavior are shown in Figs.3 and 4, and consist of plots of intensity versus the optical phase shift at a fixed value of the feedback gain. Fig.3 illustrates bistability and the beginning of chaos with a feedback gain of 2.42, which is just above the nominal threshold value between bistability and chaos under the assumption of a uniform plane wave input. Fig.4 illustrates stronger chaotic behavior of the closed loop system for feedback gain $\tilde{\beta}$=3.

Most of the literature on nonlinear dynamics (based on eq.3) assumes a uniform plane wave input in order to make the analysis tractable. Since practical optical beams are more likely to be non-uniform in profile, and nonlinear dynamics are extremely sensitive to amplitudes, it becomes necessary to examine the consequences of specific profiled light beams upon the feedback system under examination. Therefore, the transfer function technique described earlier is used to model the scattered fields created with a Gaussian input profile [4]. Differences in the scattered fields, as previously discussed, are observed between the Gaussian assumption and uniform assumption. This causes the above equation for photodetector current (derived from the first-order optical intensity) to become invalid for Gaussian input profiles. The effect of non-uniform inputs on nonlinear dynamics to the current authors' knowledge has not been previously

studied. This work builds on the results of section 2 to simulate nonlinear dynamics for profiled input beams.

As one would expect, changes in the scattered beam caused by a profiled input create corresponding changes in the nonlinear dynamics for the close loop system. The threshold value of the feedback gain $\tilde{\beta}$, at which the system become monostable, bistable, and chaotic, changes for profiled inputs. There are also changes in the Lyapunov exponents and bifurcation maps which will have implications for encryption and decryption of RF signals applied via the bias loop. These differences are described and shown in the results. To generate these characteristics, the simulated first-order diffracted beam collected by the photodetector (which creates a current $I_{ph}(t)$) is applied to the feedback system as a set of numerical data derived via the open-loop analysis instead of the standard analytic expression known for the uniform profile case. This photon current is amplified and time-delayed, and then fed back into the acoustic driver, as is done in the standard case. The following equation represents the photodetector current, where the function f represents the observed output for a Gaussian input profile (which is not the same as the sine-profile that occurs under the standard analysis), where the argument of f represents one-half of the equivalent sound pressure applied to the Bragg cell through the feedback. There is no closed form expression for f in this case, since it is related to Q and effective beam width in a complicated way, as was numerically examined for the open-loop. Thus,

$$I_{ph}(t) = \left| f\left(\frac{1}{2}\left[\hat{\alpha}_0(t) + \tilde{\beta}\left(I_{ph}(t - TD)\right)\right]\right)\right|^2 \quad . \tag{5}$$

The theory of Lyapunov exponents (LE), developed by Ghosh and Verma to predict the nonlinear dynamics of the A-O feedback system, is briefly outlined here, beginning with the

discretization of the photodetector current. Ghosh and Verma showed that the incremental change in the feedback intensity after *n* iterations can be represented by an exponential function [10]:

$$\frac{\Delta I_1}{\varepsilon} \rightarrow e^{n\lambda} \qquad , \tag{6}$$

where ε represents a small random perturbation to the initial condition and λ is the LE. This equation is true in the limit as $n \rightarrow \infty$ and $\varepsilon \rightarrow 0$. This leads to the fact that if $\lambda > 0$, the iterations diverge leading to chaotic behavior and if $\lambda < 0$, chaos does not occur. Ghosh and Verma also derived a necessary but not sufficient condition for chaos, given by $\left|\tilde{\beta} I_{inc}\right| > 1$. The behavior of the LE as a function of the feedback system parameters has been explored numerically with the assumption of uniform input beams [8,10]. In this work, we explore the effect of non-uniform Gaussian input beams on the LE and chaos, a necessary and novel development because realistic input beams are not truly uniform.

In order to use an AO feedback system for encryption and decryption of data, it is necessary to characterize the chaotic behavior of such systems using the Lyapunov exponent or bifurcation maps. Both approaches depend on the nonlinear system parameters, including bias, gain, input intensity and $I_{\mathrm{ph}}(0)$. Chaos can also be explored using plots of the photodetector output versus gain or peak phase delay while holding other parameters constant. These plots, known as bifurcation maps, are generated from the simulation and can be compared to the behavior predicted by the LE theory. Bifurcation maps illustrate sudden changes in the dynamic behavior as a system parameter crosses a threshold.

A third method for analyzing dynamic behavior is quadratic maps [11]. This is a discrete, iterative graphical method produced by initially starting at zero, moving vertically to

intersect the nonlinear intensity curve (when available), and then moving horizontally to intersect the next slope. This process is repeated to create typically a spiraling, nested discrete curve that indicates whether chaos or convergence occurs. This is yet another technique for studying the nonlinear dynamics of closed-loop systems, and some results pertinent to this approach will also be provided in section 4.

4. Simulation, numerical results and interpretations

The motivation for this work is to explore Gaussian input beams propagating through a closed-loop system, in order to better simulate actual laser beam profiles. As was mentioned, the nonlinear dynamics of closed-loop systems have been studied by others but only for a uniform input beam assumption. Building upon recent open-loop simulations for non-uniform profile beams [9], the corresponding closed loop characteristics are derived. The results presented in section 4.1 include plots of monostable, bistable, multistable and chaotic behaviors under profiled beam propagation. Section 4.2 presents an examination of the nonlinear dynamics using the techniques of Lyapunov exponent and bifurcation maps. The final subsection 4.3 provides analysis of the system using quadratic maps.

4.1 Examination of the nonlinear dynamics under profiled beam propagation

One of the main effects of a non-uniform input profile is that the threshold value of $\tilde{\beta}$ for transition between bistability and chaos becomes a function of the Klein-Cook parameter Q. As Q increases, the threshold value of $\tilde{\beta}$ also increases. This is demonstrated by Figs.5-7 which show plots of the intensity versus the peak phase shift (immediately prior to reaching chaos) for three values of Q, where the value of $\tilde{\beta}$ in each case is chosen to be just below the threshold of transition between bistability and chaos. In Fig.5, this sub-threshold value of $\tilde{\beta}$ is 1.28

corresponding to Q=20. At this low value of Q, even though the scattered output appears to be $\sin^2$, this value of $\tilde{\beta}$ for the hybrid cell is significantly different from that for the uniform case. Fig.6 illustrates the hysteresis loop for Q=177, with a corresponding threshold $\tilde{\beta}$ = 1.7. Fig.7 shows a similar result for Q=533, where the corresponding threshold $\tilde{\beta}$ increases to 1.92. Not only does the threshold $\tilde{\beta}$ change with Q, Figs.5-7 also show that the peak intensity drops as Q increases. This is most likely due to the deviations in the scattered output profile, as was demonstrated in Fig.2.

From this point forward, the Klein-Cook parameter Q is fixed at 20, where the scattered output $\hat{\alpha}_0$-profile appears similar to that for a uniform beam. Fig.8 illustrates the intensity versus the peak phase shift with three different values of $\tilde{\beta}$ {0.9, 1, 1.2}, all below the threshold value of 1.28. These demonstrate the feedback gain tuning sensitivity of this hybrid closed-loop system. As $\tilde{\beta}$ is increased above the threshold, chaotic behavior occurs as shown in Figs.9-10. Fig.9 uses a $\tilde{\beta}$ value of 1.6 where the transition to chaos is evident. When $\tilde{\beta}$ is increased to 2.2 as in Fig.10, chaos is strongly present. The chaotic thresholds, which are of special interest for encryption applications, are significantly lower due to the non-uniform profiled beam. Additionally, as can be observed in Figs.9-10, the chaotic bands appear to migrate along the peak phase delay into the originally bistable/hysteretic regions. For encryption applications, it is necessary to predict where the chaotic bands occur; this is explored in the next subsection.

4.2 Examination of dynamical behavior based on both LE and bifurcation maps

Figs.11-14 consist of bifurcation maps, which show photodetector intensity versus bias $\hat{\alpha}_0$, alongside the Lyapunov exponent characteristics. These two types of results independently describe the state of the system as a function of system parameters, and they are

seen to be in close agreement. In Fig.11, for $\tilde{\beta}$ = 1, the LE is negative for all $\hat{\alpha}_0$ except at 1.7, where the LE is zero. The corresponding bifurcation map shows a period-doubling at this peak phase shift value. In Fig.12, where $\tilde{\beta}$ is 1.2, there are multiple locations where the LE is zero and the corresponding bifurcation map shows consistent period-doubling at each location. As $\tilde{\beta}$ is increased above the threshold for chaos, the LE becomes positive for some bands of peak phase delay. This is clearly shown in Fig.13, with $\tilde{\beta}$=1.5, where the bifurcation map shows bands of chaos whenever the LE is positive. The locations and strengths of these bands change with $\tilde{\beta}$, as evident from Fig.14 for $\tilde{\beta}$=2.

Figs.15-18 use LE and bifurcation maps to explore the dynamic behavior as a function of the feedback gain for fixed peak phase delays. Fig.15 uses $\hat{\alpha}_0$ =1 and illustrates bands of chaos along the gain dimension. Figs.16-17 use $\hat{\alpha}_0$ =1.2 and $\hat{\alpha}_0$ =2, respectively, and these show the migration of the bands. For $\hat{\alpha}_0$ =3, and for a doubled input intensity, the location of the chaotic bands are as shown in Fig.18, indicating a sensitivity to input intensity as well as feedback gain, peak phase delay and initial condition. In addition, to better understand the effect of non-uniform profiled incident beams, Fig.19 shows the LE and bifurcation maps using all the same parameters as Fig.14, but this time for a *uniform* plane wave input. Comparing the two figures, it is clear that a non-uniform input produces significantly more passbands of chaos potentially useful for encrypted communication.

4.3 Dynamical analysis using quadratic maps

Along with LE and bifurcation maps, quadratic maps are another way to examine the nonlinear dynamics. Fig.20 shows the dynamical analysis of points of intersections showing the orbit along the intensity curve with non-uniform Gaussian input with Q=20 for $\tilde{\beta}$=1. At these

values the orbit converges to a single point and there is no chaos. For $\tilde{\beta}$=2.3, as shown in Fig.21, the orbit in the quadratic map does not converge indicating chaos. These results are consistent with the LE and bifurcation map analyses.

5. Concluding remarks

This work builds upon previous work which established the open loop characteristics of the scattered first-order output for profiled inputs. This output is incorporated into a closed-loop system in order to examine the nonlinear dynamics of profiled beam propagation under feedback and thereby realize and derive new insights into encrypted signal transmission and recovery. This work is necessary due to the significant deviations from the standard analysis which assumes uniform input beams of light and sound. These deviations are shown to cause the threshold feedback gain parameter to become a function of Q. For a value of Q fixed at 20, chaos occurs at lower feedback gain values for non-uniform profiled input beams. In order to predict and examine the chaotic bands, LE and bifurcation maps are used, showing consistent results. Further analysis using quadratic maps is also used to examine these nonlinear dynamics, confirming the parallel observations. Since the application motivating this work is the transmission of encrypted signals, future work will include AC signal modulation at the bias input of the RF driver. As has been shown in recent work [7], the first-order intensity becomes an amplitude-modulated chaotic wave. This process for secure communication has been shown to be feasible for uniform plane wave inputs; the impact of non-uniform plane wave inputs, however, needs to be studied further. Future work will present simulations of encryption and decryption of high frequency signals, including stationary images and also video signals.

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