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Research exercise: A Mathematical Model to Calculate an Animals Equilibrium Temperature based on the Environmental Temperature

A Mathematical Model to Calculate an Animals Equilibrium Temperature based on the Environmental Temperature

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Abstract:

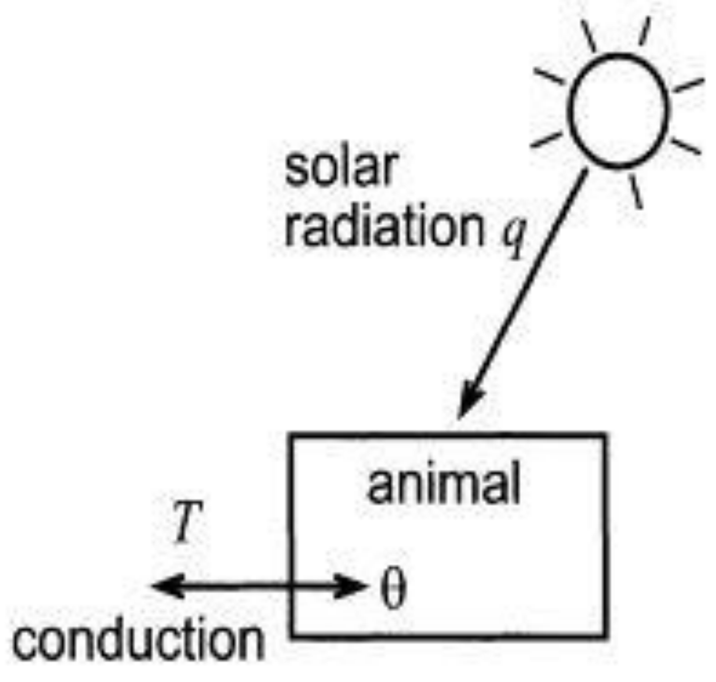
Every animal's temperature is directly correlated to the temperature of the environment that the animal lives in. The animal's equilibrium temperature can be modeled using mathematical tools based on the temperature of the environment, the amount of solar radiation, and the heating characteristics of the specific animal. Newton's Law of Cooling can be used to model this sort of phenomena of temperature changes. This mathematical model provides a relation between the unknown temperature and the derivative of this unknown temperature. In this work we will solve this model numerically using different techniques such as the Euler method, the three-term Taylor method, and the Runge-Kutta method. Using these three different mathematical methods, the animal's body temperature due to the environment can be determined. We use Matlab for such numerical computations.

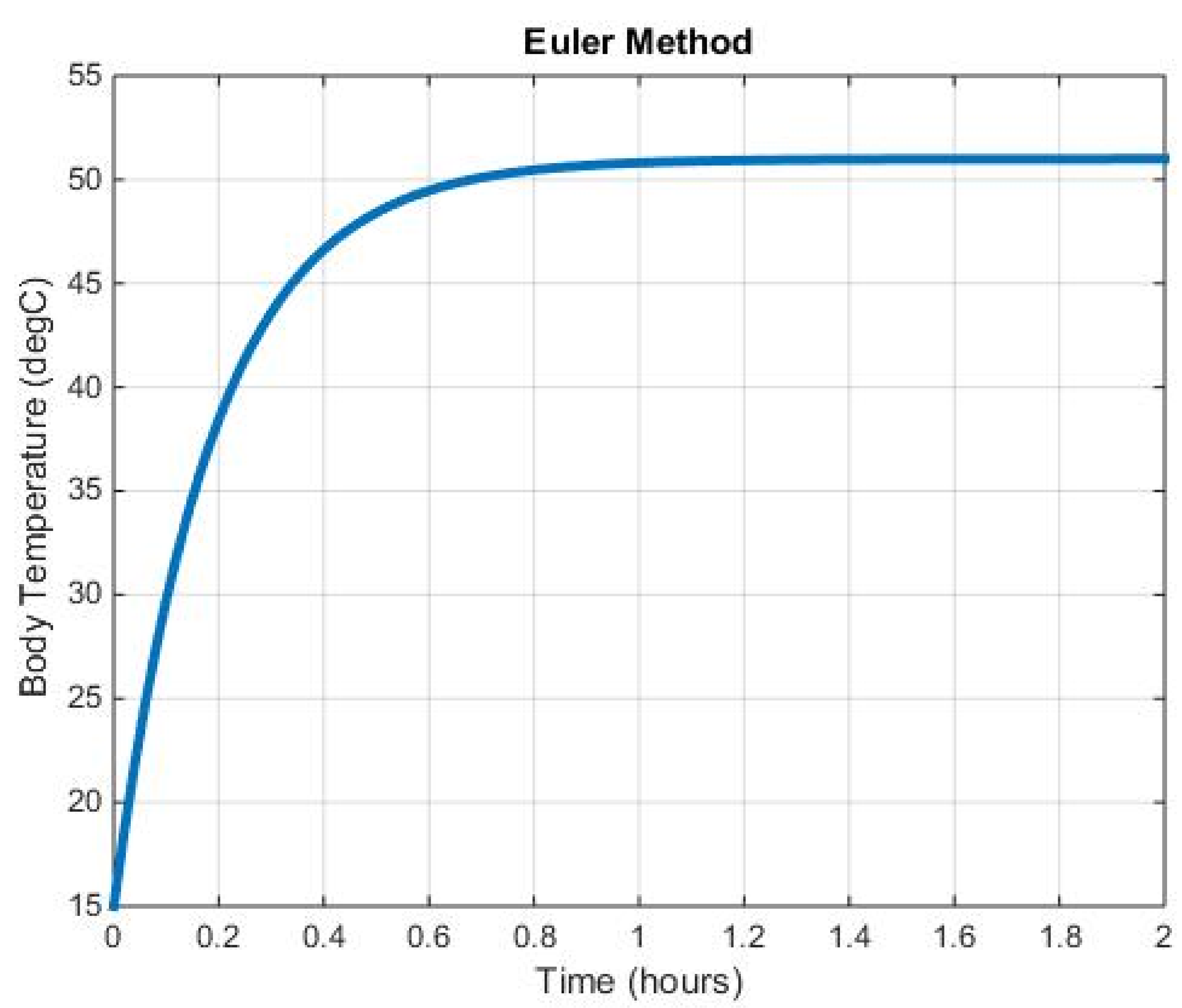
Mathematical Model:

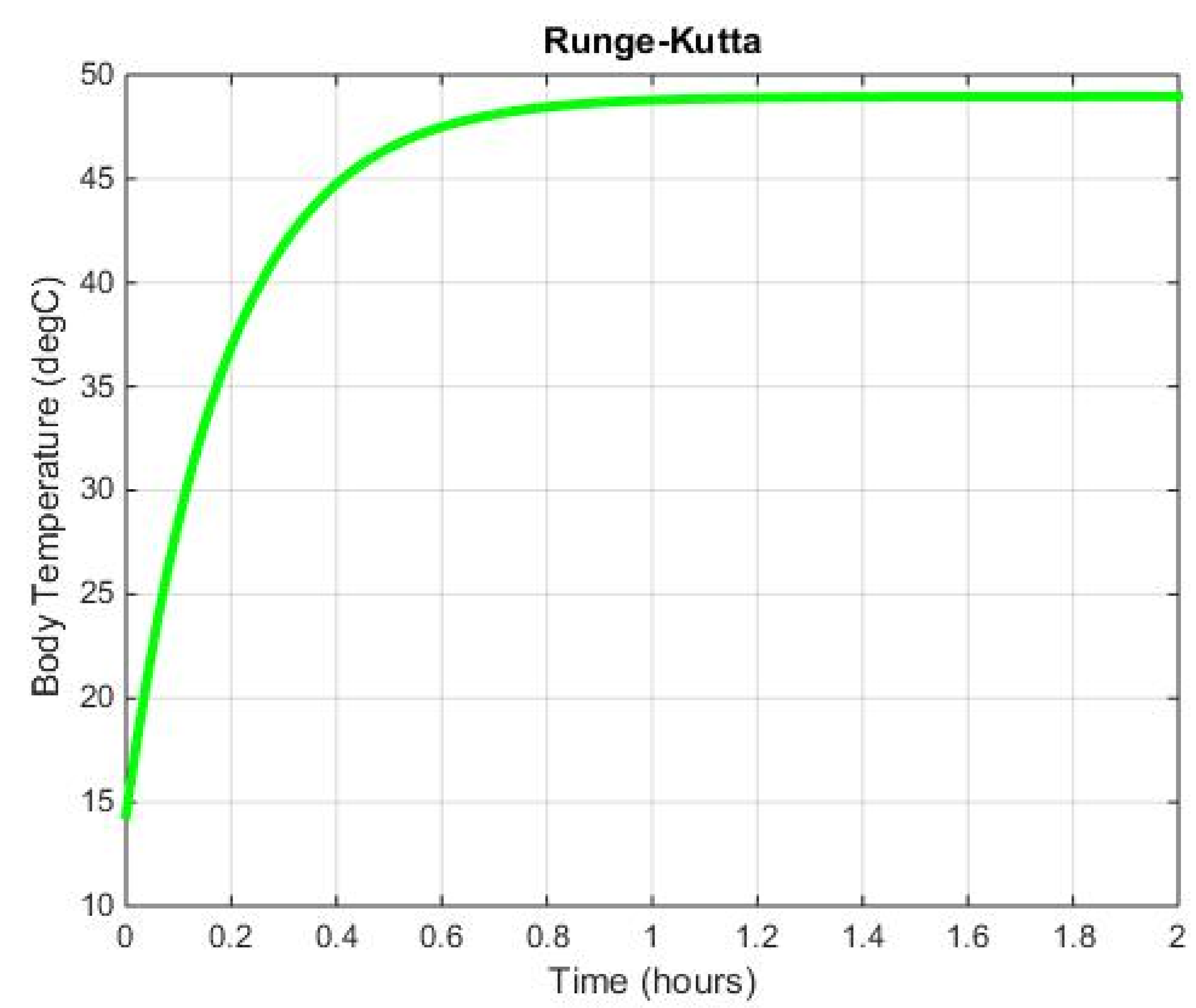
It can be seen from the image that the animal at temperature θ is receiving solar energy q, and giving off heat energy to surroundings. The rate is modelled by using Newton's Law of cooling.

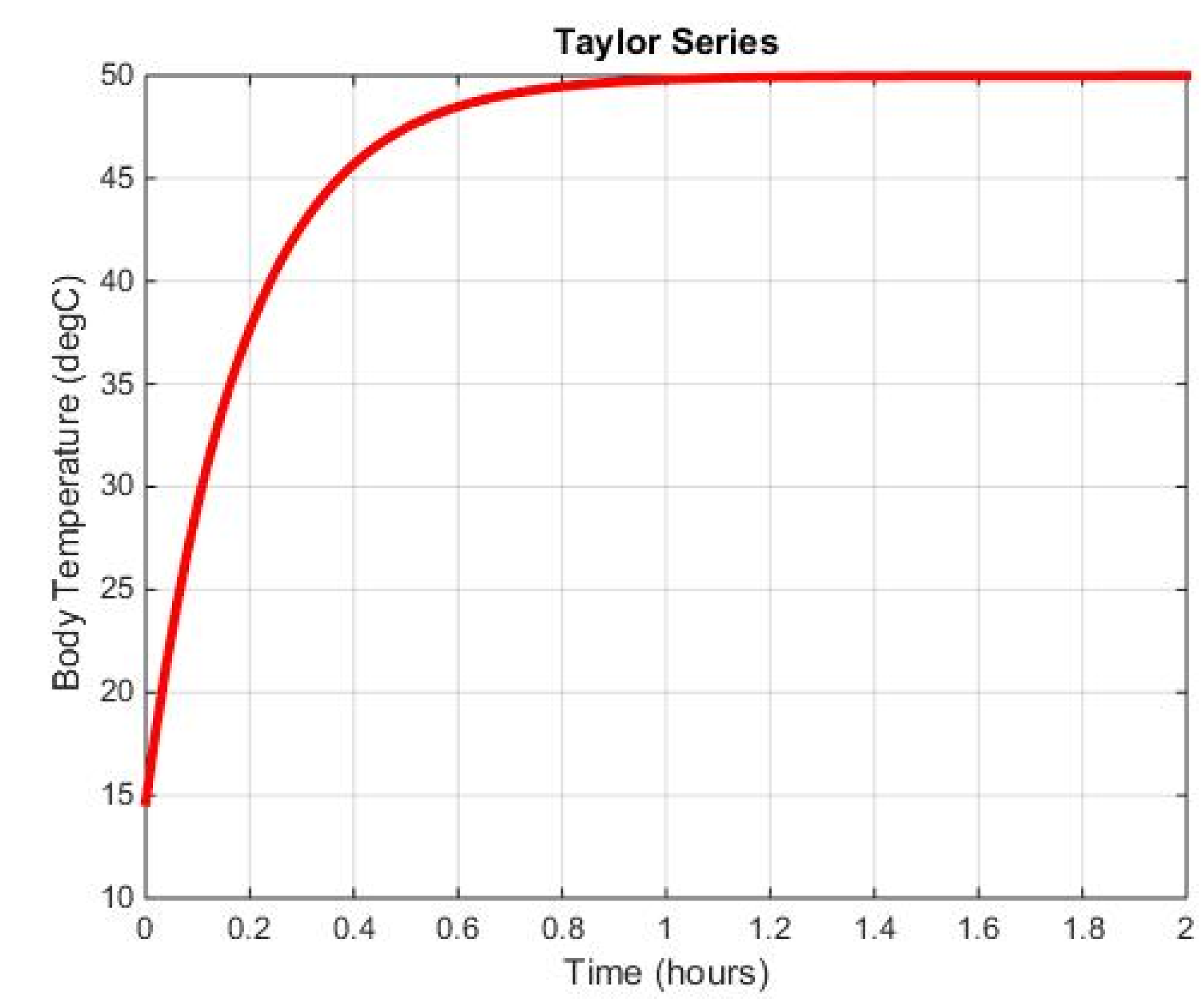
$$\frac{d\theta}{dt} = \frac{q}{mc} - \frac{k}{mc}(\theta - T)$$

Where θ is the animals temperature, q is the solar energy, m is the mass of the animal, c is the specific heat, and k is a constant.









Euler	Runge-Kutta	Taylor Series
• $y_{n+1} = y_n + hf(x_n, y_n)$	• $y_{n+1} = y_n + \frac{1}{6} + k_1 + 2k_2 + 2k_3 + k_4$	• $y_{n+1} = y_n + y_n' h + y_n'' \frac{h^2}{2}$

Conclusion:

After seeing Newton's Law of cooling on an animal visually, we can see that as time passes throughout the day, the temperature of the animal rises until it reaches it's steady temperature. The increase in temperature of the animal is caused by solar radiation allowing more heat to enter the animal than what is leaving the animal. This causes the equilibrium to become unbalanced and the temperature to rise.

Resources: Mathematical Methods in Biology- J. David Logan, William Wolesensky, A First Course in Differential Equations –Dennis G. Zill