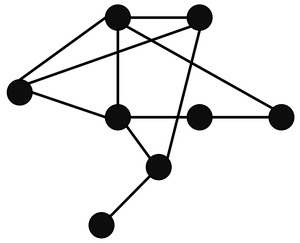
Covering Your Roots: Root Cover Pebbling on Graphs

Name: Claire Sonneborn

Advisor: Dr. Aparna Higgins

Vertex
Edge



Components of a Graph

- Vertices- dots of the graph, represent objects
- Edges- lines of the graph, represent relationship between two vertices

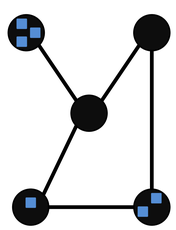
Adjacent Vertices

- Two vertices are **adjacent** if they are connected by an edge.
- Two vertices are not adjacent if there is no edge between them

Pebbling

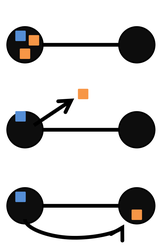
Place pebbles on the vertices of a graph.

Pebble



Pebbling Move

1. Pick up 2 pebbles from one vertex
2. Remove 1 pebble from graph
3. Move 1 pebble to adjacent vertex



Cover Pebbling Number

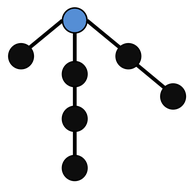
The minimum number of pebbles needed so that, no matter where they are originally placed, we can use pebbling moves to place one pebble on each vertex.

Root Cover Pebbling Number

The minimum number of pebbles needed so that, **if all pebbles are initially placed on the root vertex**, we can use pebbling moves to place one pebble on each vertex.

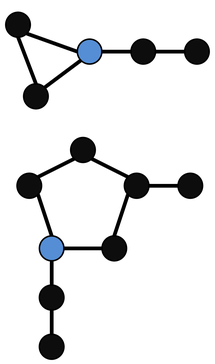
Types of Graphs we Consider

Root vertex



One root vertex with pendants (paths) hanging off.

One cycle with one pendant or multiple pendants



Root Cover Pebbling

Fix an initial configuration

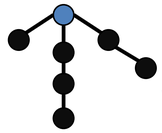
Place all pebbles on one vertex (root)

Use pebbling moves to move one pebble to each vertex

Finding Root Cover Pebbling Numbers

Let q = number of pendants.

And $n_1, n_2, \ldots, n_q$ be the lengths of pendants.



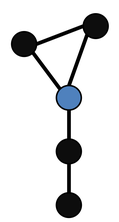
q = 3

$n_1 = 1$
$n_2 = 3$
$n_3 = 2$

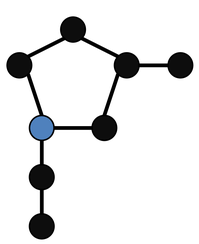
$$R = 1 + \Sigma_{i=1, 2, \ldots q} (2^{ni+1}-2)$$

Let c be the length of the cycle and R(c) be the root cover pebbling number of the cycle

$R(c)=(2^{\lfloor c+1/2 \rfloor}-2)+(2^{\lceil c+1/2 \rceil}-2)$



$$R = R(c) + (2^{n+1}-2) + 1$$

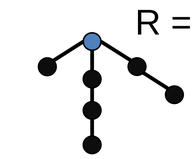


Let d1, d2, … , dq be the distance on the cycle from the root to each pendant.

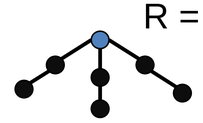
$$R = 1 + R(c) + \Sigma_{i=1, 2, \ldots q} (2^{ni+di+1}-2^{di+1})$$

Minimizing Root Cover Pebbling Number

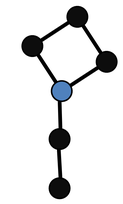
When the number of vertices remains constant, R is minimized when the pendants are equal length.



R = 23



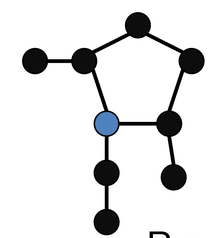
R = 19



R = 15

In the case of one pendant, if the number of vertices remains fixed, R is minimized when the cycle is twice as long as the pendant.

When the number of pendants remains constant, R is minimized when the pendants are close together with the root in the middle.



R = 27

Additional Work

Relationship between number of pendants and root cover pebbling number

How to choose the placement of the root vertex on a cycle with multiple pendants