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Extremum Seeking Control Observer Design for Multiple-Input Multiple-Output Linear Time-Invariant Systems

Abdulhakim A. Daluom
University of Dayton, stander@udayton.edu

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Abstract

In this proposal a control strategy, we try to address the problem of output (performance) function by applying the Extremum Seeking Control (ESC) approach to the observer of the Single-Input Single-Output (SISO) and Single-Input Multiple-Output (SIMO) Linear Systems. By this control approach, we drive the performance function to its maximum or minimum value. The construction of a seeking algorithm is used to drive the system states to the desired set-points that maximize or minimize the value of an objective (performance) function. This controller is designed in case of the states are known by applying the observer of the system. Also, Lyapunov's stability theorem and the perturbation theory including the averaging method is used in the design of the extremum seeking controller structure to check the stability of the system.

Keywords: Linear System, Observer, and Extremum Seeking Control.

Controller Design

The controller design will be
\[ u' = -x_{si} + v, \quad \text{where} \quad v = -(\phi + \alpha \cos(\alpha t)) \]

Error dynamics:
\[ \dot{\hat{x}} = \hat{z} - \hat{x}, \quad \dot{\hat{x}} = \hat{z} - x \]
and \( \phi = \phi' \) where \( x_{si} = -\phi' \) is constant.

Then, the system dynamics is
\[ \dot{\hat{x}} = \hat{\theta} + Fv \]
\[ \dot{z} = \hat{\theta} + Ft - Lz \]
\[ \dot{\phi} = -c(\phi - \phi') - \alpha + (z_{si} - \hat{z}) \]
\[ \dot{\theta} = -c\dot{\theta} + \alpha + \beta \]
\[ \dot{\delta} = -c\dot{\delta} - \beta - \Delta \alpha \sin(\alpha t) \]
\[ \zeta = \alpha \Delta \]
where \( c, z_{si}, l, l_1, l_2, \) and \( \alpha \) are positive numbers and should be tuned well. \( \alpha \) and \( \alpha \) are frequency and amplitude perturbation respectively, and \( \Delta = J^* - J(z) \cdot \zeta \) where \( J^* \) is the maximum of the cost function. With the time scale \( \tau = \alpha t \)
we apply the averaging model
\[ \dot{\hat{x}}_{avg} = \hat{z} - \hat{x}, \quad \dot{\hat{x}}_{avg} = \hat{z} - x \]
\[ \dot{\phi}_{avg} = \dot{\phi}_{avg} \]
\[ \dot{\theta}_{avg} = \dot{\theta}_{avg} \]
\[ \dot{\delta}_{avg} = \dot{\delta}_{avg} \]
The block diagram for the ESC is shown below.

Problem Statement

Consider the Linear time-invariant system
\[ x = Ax + Bu \]
\[ y = Cx + Du \]
where \( x \in \mathbb{R}^n \) is the system states, \( u \in \mathbb{R} \) is the input, and \( y \in \mathbb{R}^m \) is the output. \( D \) is assumed to be zero. Then, we add another state to the system by letting
\[ u = x_{si} \quad \text{and} \quad \dot{x}_{si} = u - x_{si} + v \]

Then, the system becomes
\[ \begin{bmatrix} \dot{x} \\ \dot{x}_{si} \end{bmatrix} = A \begin{bmatrix} x \\ 0 \end{bmatrix} + B egin{bmatrix} x \\ u \end{bmatrix} + Fv, \quad y = C \begin{bmatrix} x \\ 0 \end{bmatrix} \]

where \( F = 0 \) and \( B_{avg} \).

The observer (estimator) of the system is
\[ \begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{x}}_{si} \end{bmatrix} = A \begin{bmatrix} \hat{x} \\ 0 \end{bmatrix} + B \begin{bmatrix} \hat{x} \\ \hat{x}_{si} \end{bmatrix} + Fv + L(x - \hat{x}), \quad \gamma = C \begin{bmatrix} \hat{x} \\ 0 \end{bmatrix} \]

where \( L = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}, \quad \hat{x} = \begin{bmatrix} \hat{x} \\ \hat{x}_{si} \end{bmatrix} \), \( \hat{\theta} \) is Hurwitz, \( (A, F) \) is controllable, and \( (A, C) \) is observable.

Stability Analysis: To prove the stability of the system, we use Lyapunov candidate function which is
\[ V(\hat{x}, \hat{\theta}, \hat{\delta}) = \frac{1}{2} \left[ (\hat{x}^T \hat{x} + \hat{\theta}^T \hat{\theta} + \hat{\delta}^T \hat{\delta}) \right] \geq 0 \]
So, \( V(\hat{x}, \hat{\theta}, \hat{\delta}) \) should satisfy the function
\[ \dot{V}(\hat{x}, \hat{\theta}, \hat{\delta}) \leq 0 \]

Plant and Results

Consider the linear time-invariant system
\[ \dot{x} = \begin{bmatrix} -1 & 4 & 1 \\ 0 & -3 & 2 \\ 1 & 0 & 1 \end{bmatrix} u, \quad y = \begin{bmatrix} 1 \\ 0 \end{bmatrix} x \]

where \( x_1 = -2, x_2 = \frac{2}{3} x_2, \) and \( x_3 = \frac{11}{3} x_3 \) are the maximizer of the cost function. The cost function is
\[ J(z) = J^* - (\dot{z} - x^*)^T Q (\dot{z} - x^*) \]
where \( J^* = 1000 \) is given for testing the maximum of the cost function, and \( Q \geq 0 \).

Next figures show the results of the work.

Conclusion

We conclude that this control design is working perfectly by letting the system states track the desired points or the maximiser of the cost function. As we see in the Fig. 1 that the observer is estimating the real states very well. In Fig. 2, the estimated states are driven to the desired set-points with very small error. Also, the cost function is driven to the extremum (maximum) as shown in Fig. 3. Figs. 4 and 5 show the controller and its states results respectively.

From all these results, we can say that this controller is working very well under the given conditions.

Bibliography