**Title:** Star Decompositions of the Complete Split Graph  
**Name:** Adam Volk  
**Advisor:** Atif Abueida, Ph.D.

### Introduction
A graph is a discrete mathematical structure that consists of a set of vertices and a set of edges between pairs of vertices. A graph decomposition is a partitioning of the edges of a graph into disjoint sets in such a way that the induced subgraphs produced are isomorphic to each other. The graphs we focus on here are stars and complete split graphs (see below).

![A complete split graph as the join of a complete graph and independent set](image)

### Special Cases
- **$m = n - 1$:** decomposable if and only if $t | m$
- **$t = 1$:** trivial
- **$t = 2$:** decomposable if and only if total number of edges is even [3]

### Necessary Conditions
- If $G$ can be decomposed into $t$-stars, then
  $$t \left( \binom{n-m}{2} + m(n-m) \right)$$

### Casework and Results
- **$n - m < t$:** decomposable if and only if
  - $t \left( \binom{n-m}{2} \right)$
  - $n - m = t$: decomposable if and only if
    - $t$ is odd and $m \geq \frac{t+1}{2}$
  - **$t < n - m < 2t$:** decomposable if
    - $t \left( \binom{n-m}{2} \right)$
    - $t$ is odd, $t|m$, and $n - m = t + 1$
  - **$n - m \geq 2t$:** decomposable if
    - $t \left( \frac{(n-m)(n-m-1)}{2} \right)$ and $t|m(n-m)$
    - $t \left( \frac{n+m-1}{2} \right)$, or
    - $n - m$ is odd and $m \equiv -1 \pmod{t}$

### Future work
Since we were unable to completely solve the problem for two of our cases, this is one place to begin.

We could also consider a more general problem by removing a subgraph $H$ belonging to a different class of graphs.

Rather than limiting the size of stars to be a fixed value, we could consider decomposing a graph into stars of size $t$ where $t$ comes from some finite set of positive integers.

### References