Star Decompositions of the Complete Split Graph

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Introduction
A graph is a discrete mathematical structure that consists of a set of vertices and a set of edges between pairs of vertices. A graph decomposition is a partitioning of the edges of a graph into disjoint sets in such a way that the induced subgraphs produced are isomorphic to each other. The graphs we focus on here are stars and complete split graphs (see below).

Special Cases
- \( m = n - 1 \): decomposable if and only if \( t \mid m \)
- \( t = 1 \): trivial
- \( t = 2 \): decomposable if and only if total number of edges is even \([3]\)

Necessary Conditions
- If \( G \) can be decomposed into \( t \)-stars, then \( t \left( \frac{n-m}{2} \right) + m(n-m) \)

Casework and Results
- \( n - m < t \): decomposable if and only if
  - \( t \left( \frac{n+m-1}{2} \right) \)
- \( n - m = t \): decomposable if and only if
  - \( t \) is odd and \( m \geq \frac{t+1}{2} \)
- \( t < n - m < 2t \): decomposable if
  - \( t \left( \frac{n+m-1}{2} \right) \) or
  - \( t \) is odd, \( t \mid m \), and \( n - m = t + 1 \)
- \( n - m \geq 2t \): decomposable if
  - \( t \left( \frac{(n-m)(n+m-1)}{2} \right) \) and \( t \mid m(n-m) \),
  - \( t \left( \frac{n+m-1}{2} \right) \), or
  - \( n - m \) is odd and \( m \equiv -1 \mod t \)

Future work
Since we were unable to completely solve the problem for two of our cases, this is one place to begin.

We could also consider a more general problem by removing a subgraph \( H \) belonging to a different class of graphs.

Rather that limiting the size of stars to be a fixed value, we could consider decomposing a graph into stars of size \( t \) where \( t \) comes from some finite set of positive integers.

References