The Walking Dead: Don't Run, Use Math!

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Abstract

To study the effect of a zombie outbreak, our team used several differential equations and techniques learned in class to predict the population of humans and zombies during a zombie outbreak. It is important to be able to study the population of both humans and zombie to understand the odds of getting infected and to predict how long the outbreak will last for. This information could then be given to the Center of Disease Control for proper defensive measures to ensure the survival of humans. If there is an outbreak, it is best to be prepared.

Background

When people hear zombie, they hear interpretations of the fictional creature hobbling around with a death stare look on his/her face. The face is probably pale and wrinkly, head is probably to the side because their body cannot support all that is left of them. Zombies are a cannibalistic creature spawning from the dead corpse of a human [1]. A zombie is a resurrection of the human after death and tends to act in an aggressive and instinctive behavior. Zombies feed off the living and won't stop at nothing for human flesh. However this portrayals come from none other than the imagination of the human mind. How the whole zombie outbreak starts off is usually the random spawning of the dead coming back to life as it is said in the famous movie Dawn of the Dead "When there is no more room in hell the dead will walk the Earth."[9] This quote shows that the zombie outbreak starts with the dead and the dead feed off of the living. Nowadays, zombie outbreaks in movies usually start off with a virus or a dangerous takeover of fungi which is seen in games such as "The Last of Us." The kind of outbreak that we assumed to happen is the typical virus outbreak. Movies, shows, video games, and even comic books are chalked filled with zombie apocalypses such as Dawn of the Dead, 28 days later, World War Z, even comedies like Shaun of the Dead. All of these movies portray different types of zombies: ones slow but aggressive and ones that are fast and cunning [13.41…]). For this project, the zombie we have chosen is the slow but aggressive zombie somewhat like the ones in games such as Dead Rising and the popular show the Walking Dead. All of these fads have a few things in common: the setting is post-apocalyptic, zombies are the dominant species and everyone is trying to survive some way or another. Why we chose the slow zombie is because most zombie entertainment often show the slow but vigorous zombie. Also, as it is not obvious enough but a zombie apocalypse is a fantasy setting. Therefore, assumptions must be made and the knowledge of these fictional creatures will have to be based on movies and books. In addition, the slower zombie is a more balanced variable slow moving but deadly at the same time, easy enough to keep track of. Even though zombies are fictional creatures, it would never be considered too careful to be ready if there was an outbreak. And no I am not just talking a stockpile of food, supplies and weapons, I am talking about the rate the zombie outbreak would be at from the start of the spread to the end of it all. In order to find out how quick this zombie outbreak can infect everyone, mathematics is going have to be involved heavily.

The SZR Model

To start off the solution, a mathematical model must be constructed so that every variable and number may be organized according to the model. This model can be called the Basic Model. Within the Basic Model, we consider three basic classes: Susceptible (S), Zombie (Z), and Removed (R). The point of this model is to show the statistical number of those who are living and those who are infected. If a person is completely dead, the person would be thrown in the removed class. Susceptible can only be turned into Zombies through transmission. Zombies can be killed which means they can fall into the removed class. So given this can be the parameter 'δ'. The Susceptible that are transmitted through zombie infection can be known as the parameter 'β'. It is assumed that the birth rate of humans does not come into play during the apocalypse. Therefore the equation of how many Susceptible are alive can come to this equation: $\frac{ds}{dt} = \beta SZ - \delta S$, Where $\frac{ds}{dt}$ is the rate of change of how many Susceptibles are alive. The amount of people surviving through the apocalypse which are subtracted by the amount of Susceptible affected by transmission and subtracted by those who die by natural causes. Now let's look at the amount of zombies roaming the Earth. First off the Zombie population will most likely be growing but will be decreasing quite some. While looking at the zombie count a few more parameters will have to be taken into account. Since Zombies are resurrected Humans, then humans that are in the removed class can resurrect and become a Zombie. This resurrected Zombie can be known as the 'ζ' parameter. Zombies can also be taken down by other human beings but not by other Zombies, so this parameter will be known as the 'α' parameter. The amount of zombies existing can be predicted by this equation: $\frac{dz}{dt}=\beta SZ+\zeta R-\alpha SZ$. Here $\frac{dz}{dt}$ is the rate of change in zombies and that is equal to the addition of transmitted human being plus the resurrected and subtracted by the disabled zombies. The last group to be taken into account is the removed. Like what is said before, the susceptible are able to become a part of the removed by natural case and zombies however those moved into removed class can become a zombie and that is what would get our removed equation: $\frac{dR}{dt}=\delta S+\alpha SZ-\zeta R$. The rate of change in the removed shows the addition of Susceptible lost through natural causes as well as the Susceptible list through Zombie attacks and this is subtracted by Resurrected.

Constants and Supporting Functions

N = 501
β = .095
α= .005
ζ= 0.0001
Δ=0.0001

S(0) = 500
Z(0) = 1
R(0) = 0
ρ= .005

$$\frac{dS}{dt} = -\beta ZS - \delta S$$
$$\frac{dZ}{dt} = \beta SZ + \zeta R - \alpha SZ$$
$$\frac{dR}{dt} = \delta S + \alpha SZ - \zeta R$$

$$(\delta * N)(Z/N)S = \delta SZ$$

$$S(t)+Z(t)+R(t)=N$$

$$S+Z+R \rightarrow \infty \text{ as } t \rightarrow \infty \text{ if } D \neq 0$$

Numerical Methods Used for Simulations

The SZR Model was solved numerically using Three different techniques the Euler Method, ODE23 Method, and the ODE45 built-in functions of MATLAB.

Euler Method:
Euler's method is the simplest method to use and gives an answer quickly. However, this procedure is the least accurate method among the three.

ODE23:
The method of using ODE23 is a more accurate approach to solving ODE's. This function that is run internally in MATLAB is more precise (as seen in the plots).

ODE45:
The method of using the internal MATLAB function of ODE45 is the most accurate of the three methods. This process, however, will take longer to run, but will give the most precise answers.

Results

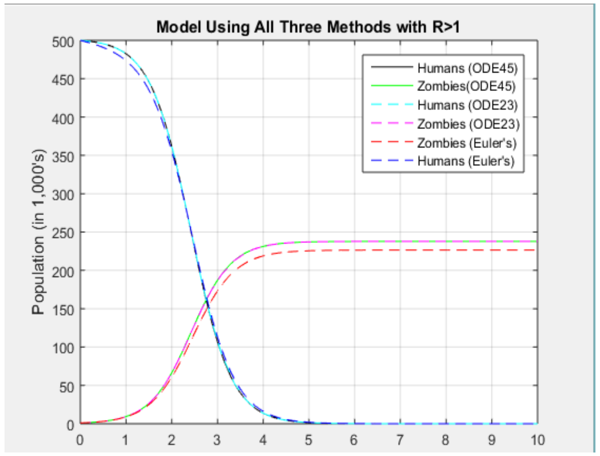

Figure 1

Figure 1 shows the plot of all three ODE problem solving methods; Euler's Method, ODE23 and ODE45. Figure 2 shows the zoomed in region of Figure 1 at approximately (3,150). Figure 3 is showing that when there is a starting human population of 501 and there is a starting population of 0 zombies, the zombie population stays at 0, and the human population stays at 501.

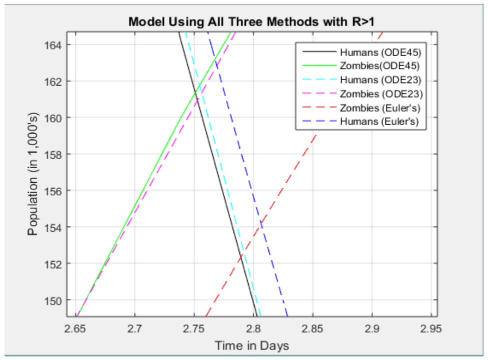

Figure 2

Plot of Simulation with Observed Data

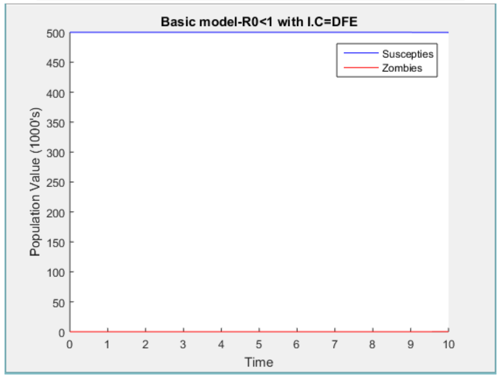

Figure 3

Conclusion

Being prepared for an epidemic takes more than stocking up on food and weapons and making an underground bunker. With the use of calculus, the zombie outbreak with the right precautions can be prepped at the perfect time. This report has explained the outcomes of the zombie outbreak and separated the different types of classes into other variables. From here this made a system of equations. As the report progressed graphs were made to show in a short amount of time (which was about 10 days) the possible outcome of the outbreak. From Figure 1 we can see that the Human population rapidly decreases throughout the first few days and is nearly 0 by day 5. Likewise, the Zombie population increases rapidly the first few days and steadies off around day 5 as well. This makes sense because as there are less humans to infect, there will be less increase of Zombie population. Unfortunately from this model, all of the Susceptible Humans die, and the Zombies population claims half of the Human population.

References and Acknowledgments

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