Spherical Linkages Analysis and Synthesis by Special Unitary Matrices for Solution via Numerical Algebraic Geometry

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Introduction

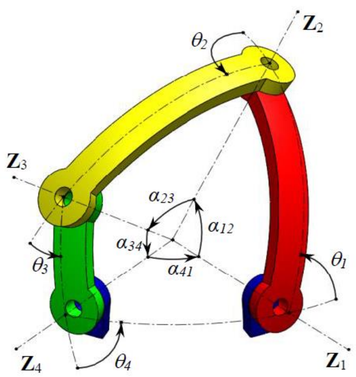
This research seeks to develop methods to solve analysis and synthesis problems for spherical linkages. Spherical linkages meet a variety of common needs in mechanical engineering practice. However, due to the intense mathematics associated with their design, these linkages have received little attention in literature when compared to planar linkages.

Method

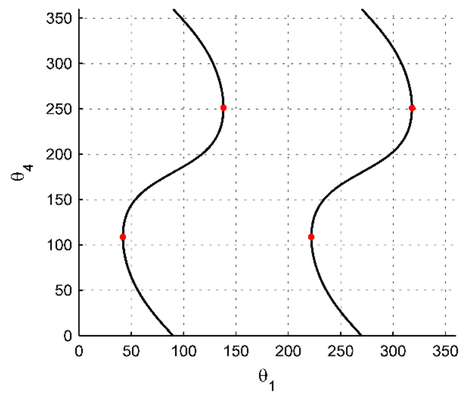
The method used in this research exploits the structure of the special unitary matrices *SU(2)* in forming a system of equations that govern the behavior of spherical mechanisms. Special unitary matrices can be used to represent spatial displacement in an efficient and compact form that is suited to employ the tools of numerical algebraic geometry. The general form for an element of *SU(2)* to describe a rotation about an angle ϕ and **s**-axis of rotation is given as

$$Q(\mathbf{s},\phi) = \begin{bmatrix} \cos\frac{\phi}{2} + is_z \sin\frac{\phi}{2} & s_y \sin\frac{\phi}{2} - is_x \sin\frac{\phi}{2} \\ -s_y \sin\frac{\phi}{2} - is_x \sin\frac{\phi}{2} & \cos\frac{\phi}{2} - is_z \sin\frac{\phi}{2} \end{bmatrix}$$

Spherical four-bar

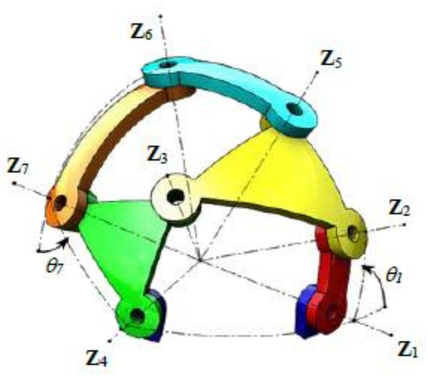


Spherical four-bar linkage schematic diagram

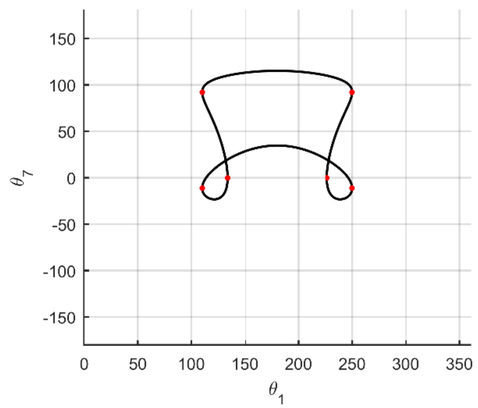


Motion curve for spherical four-bar projected onto θ_1-θ_4

Spherical Watt I



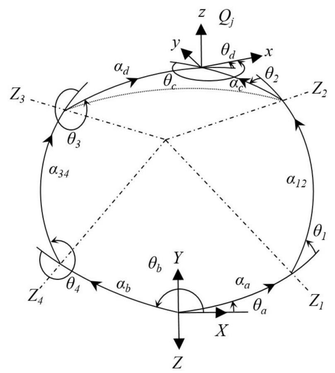
Spherical Watt I linkage schematic diagram



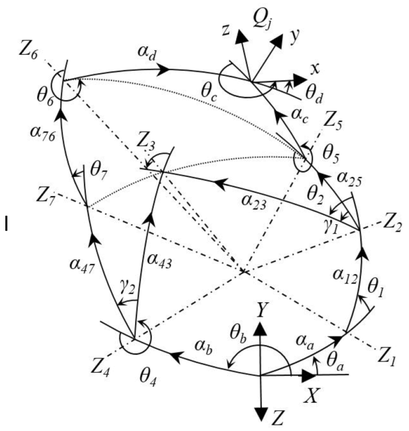
Motion curve for spherical Watt I projected onto θ_1-θ_7

Synthesis loop closure approach

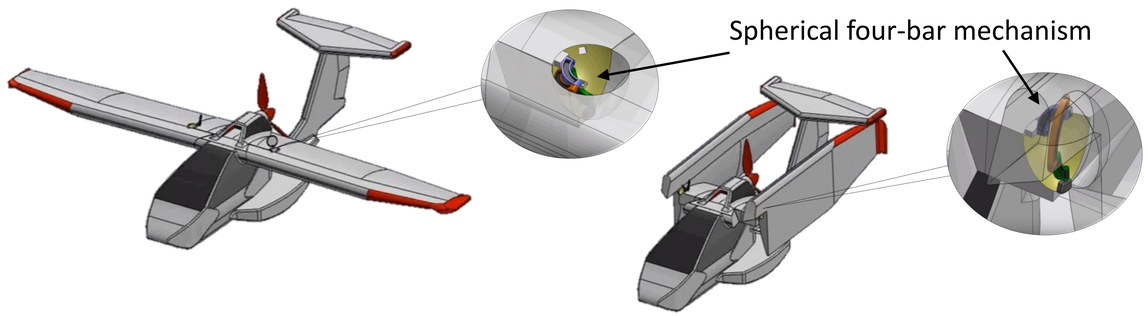
Spherical four-bar linkage



Spherical Watt I linkage

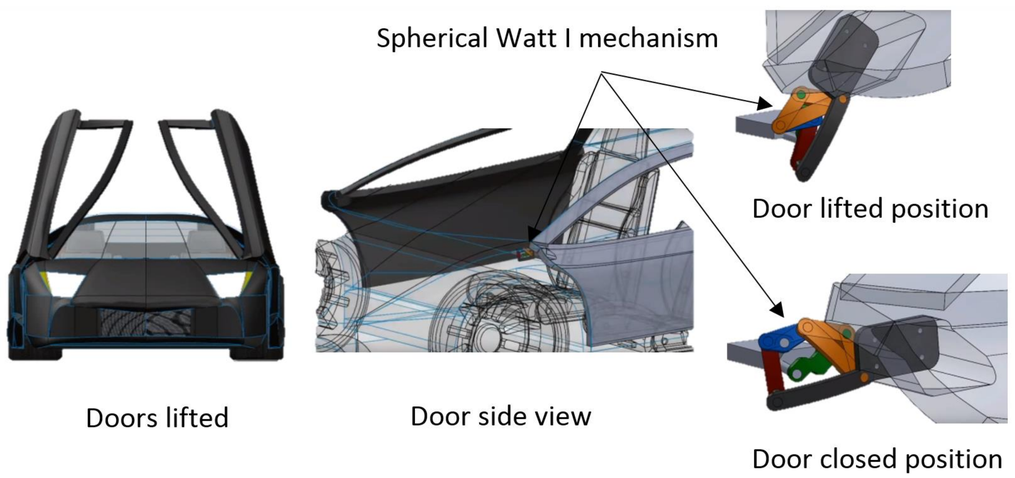


Spherical four-bar example



Spherical four-bar linkage is used to fold the wing of an airplane [1].

Spherical Watt I example



Spherical Watt I linkage is used to lift the doors of an automobile [2].

References

[1] Mechanical Design 101. (2017, March 02). Icon A5 spherical linkage. Retrieved from https://www.youtube.com/watch?v=wj-rHROUa3o

[2] Mechanical Design 101. (2017, March 01). Spherical Watt I Six-bar linkage. Retrieved from https://www.youtube.com/watch?v=N7nDlAAOd5U