

# Model of Interplanetary Gravity Assist Fly-By

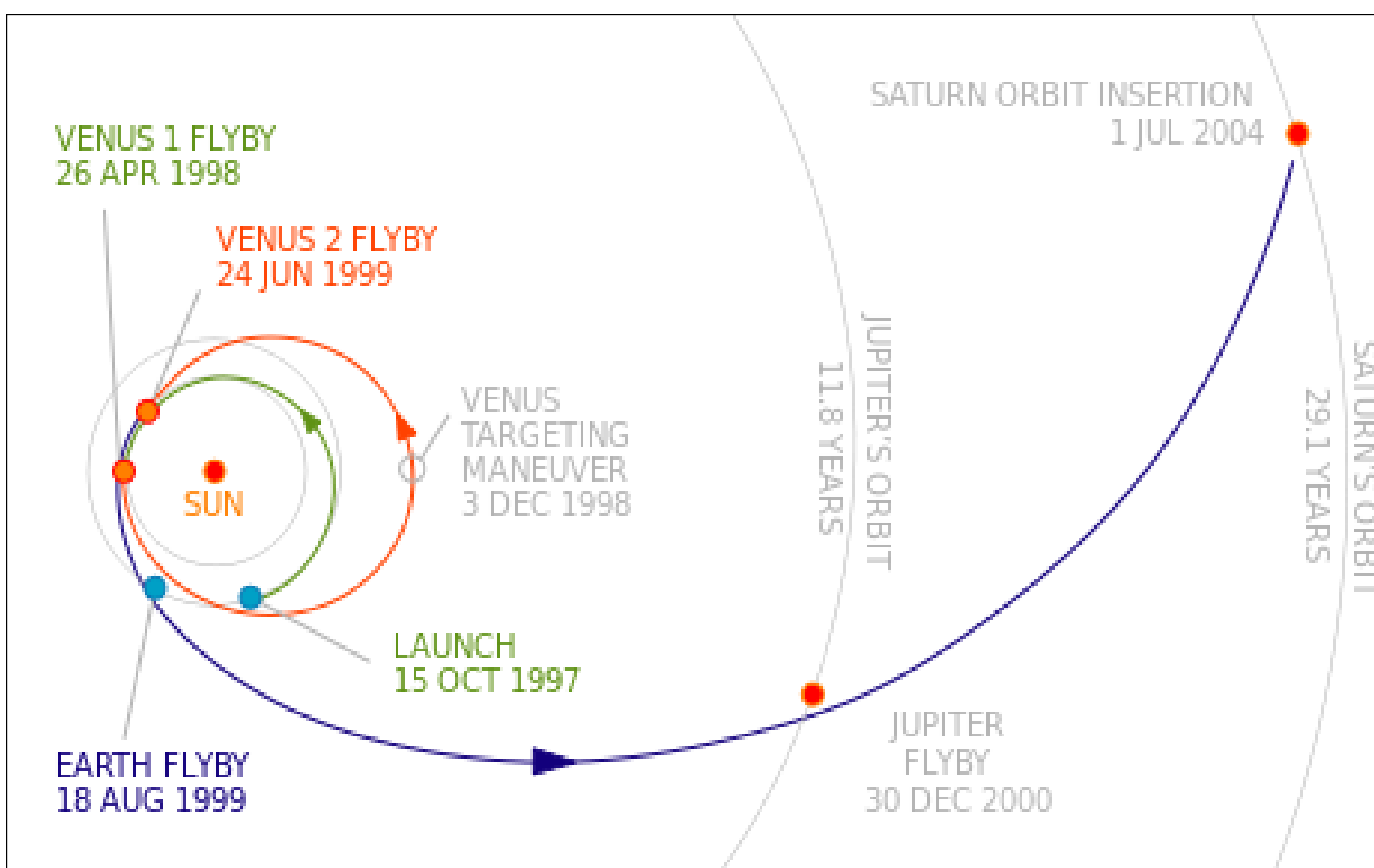
Joshua Lynch

Advisor: Ivan Sudakov, Ph.D.

**Research Objective:** To computational model of two body motion of a gravity assist; analysis of bodies energies as measure of effectiveness.

## Motivation/Introduction

- A satellite can perform a trailing fly-by on a hyperbolic trajectory to “steal” energy from a sun orbiting planet
- Gravity assist interplanetary maneuvers allow for drastic reduction in propulsion requirements for deep space missions
- Computational approach can be applied, as computation becomes more efficient, to create more robust models

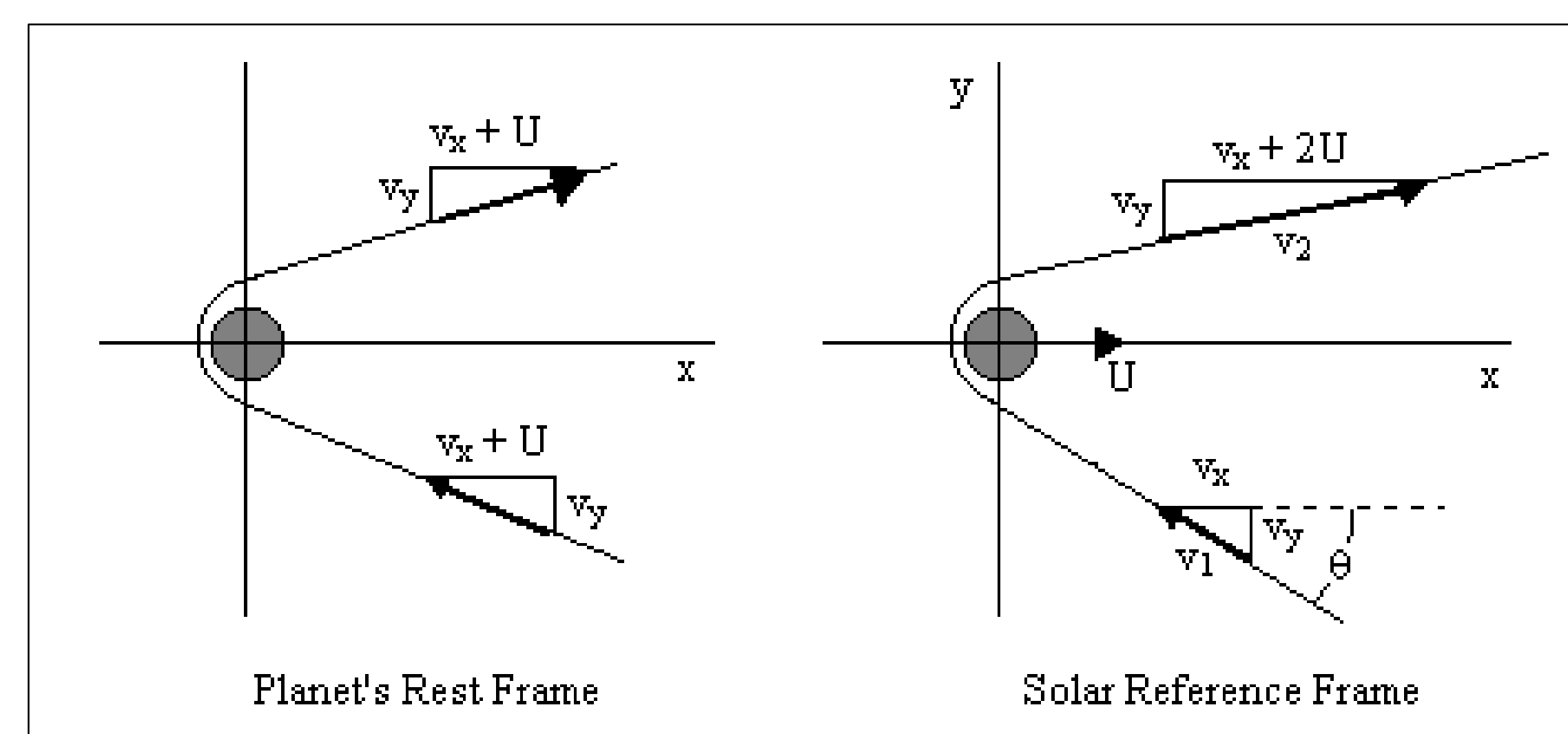


Source: <https://saturn.jpl.nasa.gov/resources/1776/>

## Methodology

- Jupiter Fly By
  - Assume/set: Straight trajectory, satellite in Jupiter SOI (Sphere of Influence), 2 Body Problem, Jupiter velocity constant
  - Three passes modeled: Far, medium, and close approach
- Driving Equation -  $\ddot{\mathbf{r}} = \frac{GM\mathbf{\hat{r}}}{r^3}$
- Runge Kutta 4<sup>th</sup> order numerical method used
  - Allows for improved accuracy of the higher order system; step size decreased (by ¼ size) near planet as this is where most errors occur.
- Initial Conditions
  - Jupiter – 778 x10<sup>9</sup> km semi major axis, 13.1 km/s orbital speed, 1.898x10<sup>27</sup> kg mass
  - Satellite – Start at SOI, 2 km/s perpendicular to Jupiter flight path
- Analytical Solution
  - Equation for velocity after assist

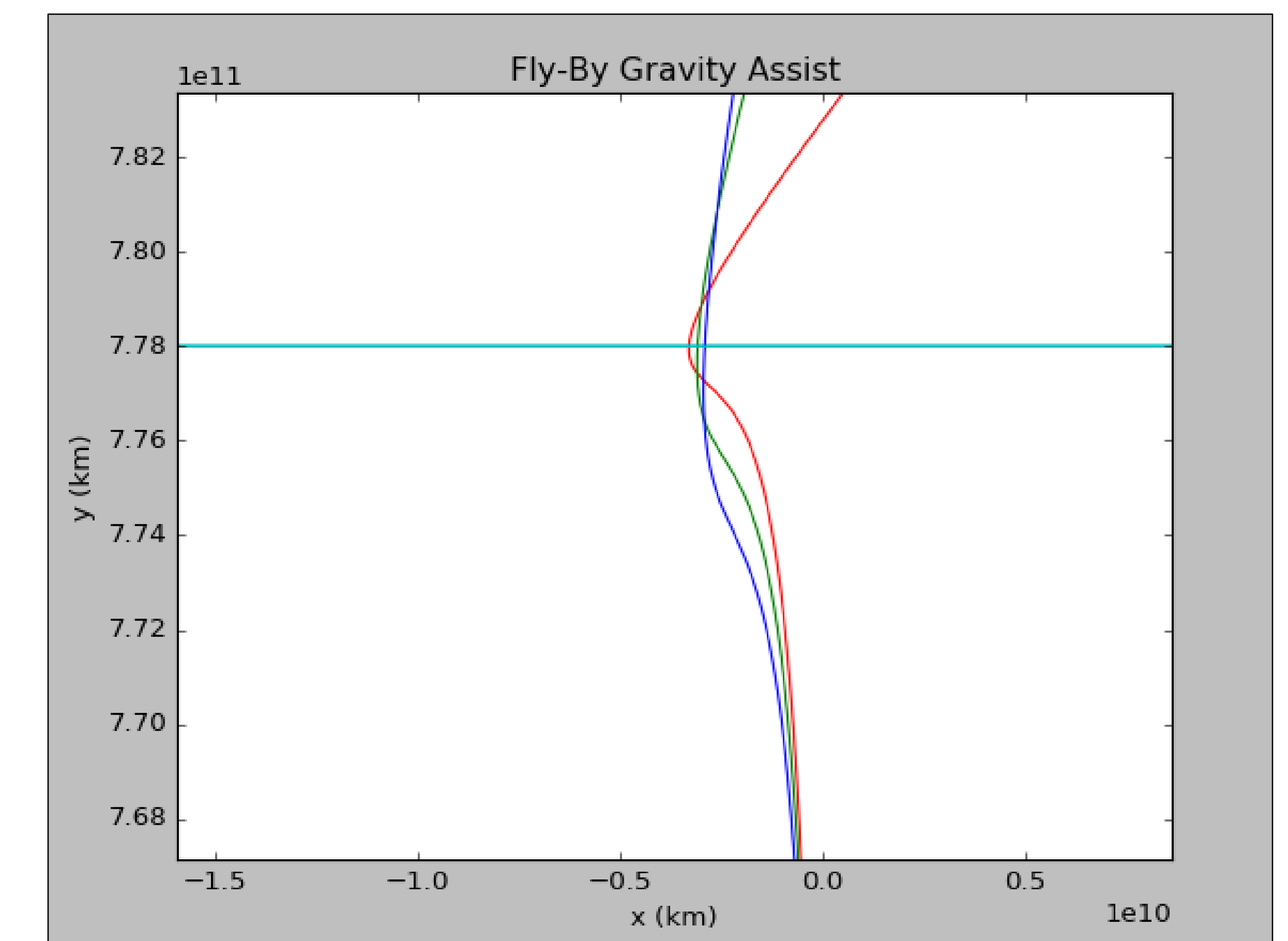
$$v_2 = (v_1 + 2u) \sqrt{1 - \frac{4uv_1(1 - \cos(\theta))}{(v_1 + 2u)^2}}$$



Source: <http://www.mathpages.com/home/kmath114/kmath114.htm>

## Results

Approach	Closest Distance	Initial Energy	Final Energy	Energy Change
Close	0.388 x 10 <sup>6</sup> km	-1.798 x 10 <sup>8</sup> J	1.221 x 10 <sup>7</sup> J	1.921 x 10 <sup>8</sup> J
Medium	1.741 x 10 <sup>6</sup> km	-1.798 x 10 <sup>8</sup> J	-1.189 x 10 <sup>8</sup> J	0.608 x 10 <sup>8</sup> J
Far	3.211 x 10 <sup>6</sup> km	-1.798 x 10 <sup>8</sup> J	-1.434x 10 <sup>8</sup> J	0.363 x 10 <sup>8</sup> J
Analytical, 90° turn	N/A	-1.798 x 10 <sup>8</sup> J	-1.569 x 10 <sup>8</sup> J	0.229 x 10 <sup>8</sup> J



## Conclusion

- Model behaves as anticipated
- Error likely near intersect, even with scaling integrator
- Results similar to analytical solution; potential error sources:
  - Lack of Sun in system
  - Model error

## Recommendations/Future Work

- Further investigation into analytical model vs. numerical method and reason for difference in answers.