Determination of Interest Rate's Behavioral Movement Utilizing the Brownian Motion

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Abstract: Here is a look into the Brownian Motion, and how it is able to portray the erratic movement over time of stock prices and interest rates. Further, is a look into how different financial models such as the Ho-Lee Model and Vasicek Model are able to utilize the Brownian Motion in order to describe the movement of short term interest rates and thus can be used to carry out various financial valuations, such as bond option pricing and evaluating interest rate futures.

Standard, Non-Standard, Geometric Brownian Motion

A stochastic process W = {W(t): t ≥ 0} possessing continuous sample paths is called a Standard Brownian Motion if

W(0) = 0

W has both stationary and independent increments

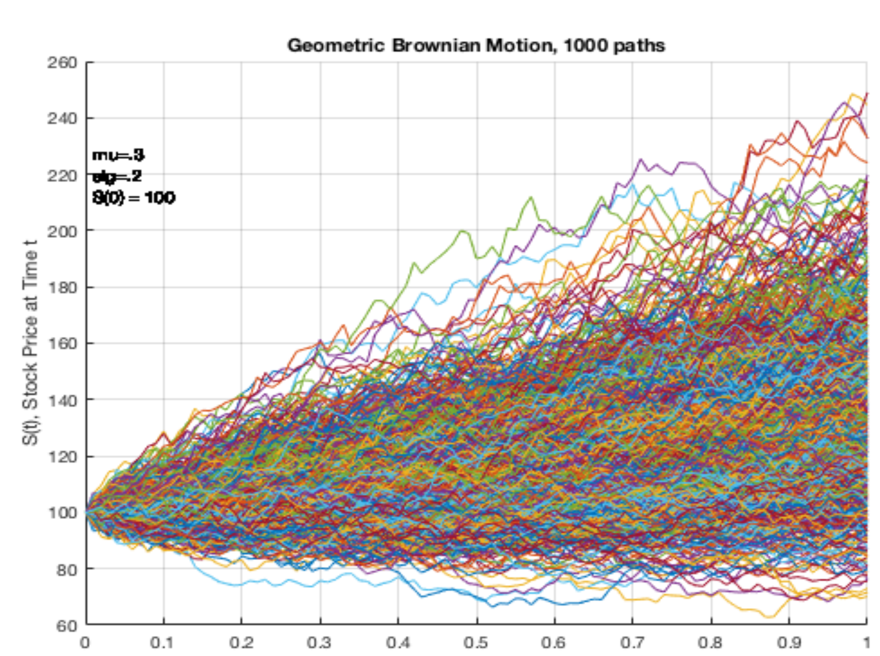
W(t) – W(s) has a normal distribution with mean 0 and variance t – s, 0 ≤ s < t

For a non standard Brownian Motion with variance $\sigma 2$ and and drift μ has the same definition except 3. Must be modified such that W(t) – W(s) has a normal distribution with mean μ(t-s) and variance σ2(t-s).

Similar to the Standard and Non-Standard BM, the geometric BM is a continuous time stochastic process S(0)*exp{μ - σ2/2)t + σWt} where W(t) is the BM. The Geometric BM model for market prices is preferred over the standard and non-standard models because it is everywhere positive (with probability 1), unlike the latter two, which can take on negative values.

<u>Example:</u> Stock Price

S(t) = S(0)*exp{μ - σ2/2)t + σWt}



Algorithm:

S(0) = S0

lnS(t1) = lnS(0) + (μ - σ2/2)t1 + σW(t1)

» lnS(t2) = lnS(0) + (μ - σ2/2)t2 + σW(t2)

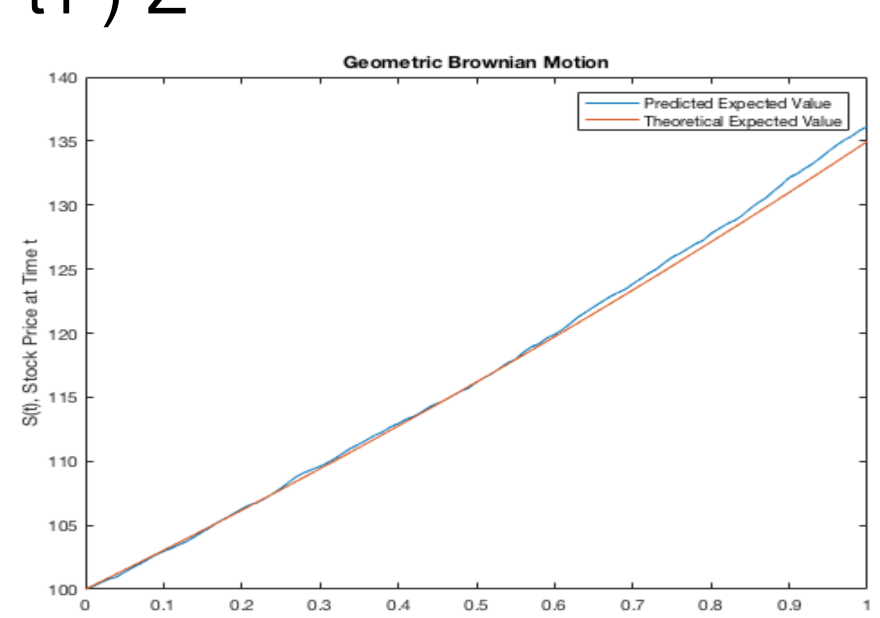
» lnS(t2) - lnS(t1) = (μ - σ2/2)(t2 – t1) + σ (W(t2)– W(t1)),

Note that W(t2)– W(t1) – N(0, t2 – t1) = √(t2 – t1)*Z

» lnS(t2)/S(t1) = (μ - σ2/2)(t2 – t1) + σ √(t2 – t1)*Z

» lnS(t2) = S(t1)[exp{(μ - σ2/2)(t2 – t1) + σ √(t2 – t1)*Z}]

E(S(t)) = S(0)*exp{μt}



Ho-Lee and Vasicek Model

Ho-Lee Model is a simple arbitrage-free interest rate model, which allows for the perfect matching of the initial term structure. This means that the theoretical zero bond prices are the same as the market prices at the initial date. The idea is to model the uncertain behavior of interest rates or bond prices at different terms or maturities.

The Vasicek interest rate model is a method of modeling interest rate movement that describes the movement of an interest rate as a factor of market risk, time and equilibrium value that the rate tends to revert towards. Essentially, it predicts where interest rates will end up at the end of a given period of time given current market volatility, the long-run mean interest rate value, and a given market risk factor. It is important to note that the equation can only test one market risk factor at a time.

Algorithm:

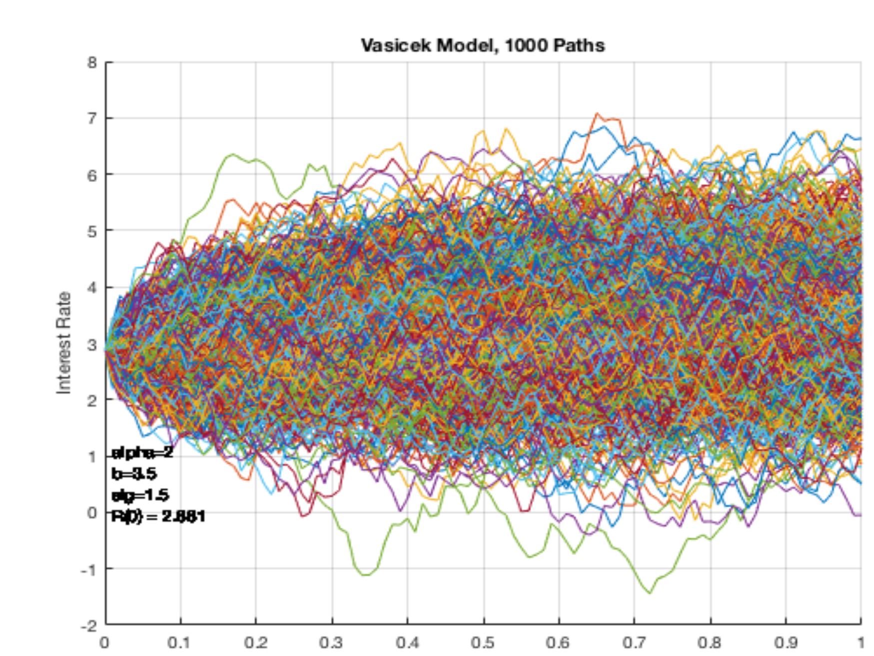
$dr_t = \alpha\,(b_t - r_t)dt + \sigma dW_t$, $(\alpha, b_t, \sigma > 0)$

» $r_t = e^{-\alpha t} r_0 + \alpha \int_0^t e^{-\alpha(t-s)}\,ds + \sigma \int_0^t e^{-\alpha(t-s)} dW_s$

» $r_{t+1} = e^{-\alpha(ti+1-ti)} r_0 + \alpha \int_{ti}^{ti+1} e^{-\alpha(t(i+1)-s)} b_t ds + \sigma \int_{ti}^{ti+1} e^{-\alpha(t(i+1)-s)} dW_s$

Note: $\alpha \int_{ti}^{ti+1} e^{-\alpha(t(i+1)-s)} ds = b_t*(1 - e^{-\alpha t1})$

Note: $\sigma \int_{ti}^{ti+1} e^{-\alpha(t(i+1)-s)} dW_s = \sigma\sqrt{}(1 - e^{-\alpha t1}) * Z$



Bond Price with Vasicek Model

Algorithm:

$B(0,T) = \exp\{-\int_{t0}^{tN} r_u\,du\}$

$B(0,T) = \exp\{-(r_{ti}(t_{i+1}-t_i) + r_{ti+1}(t_{i+2}-t_{i+1}) + \ldots)\}$

$B(0,T) = \exp\{-\Delta t * \sum r_{tk}\}$