Runge-Kutta Methods to Explore Numerical Solutions of Reactor Point Kinetic Equations

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Abstract

This work is the study of Reactor Point Kinetic equations. This is a system of seven coupled ordinary differential equations, one for neutron density and six for delayed neutron precursors.
The application of Runge-Kutta methods is used to study the system numerically. Solutions were then compared for different values of reactivity using MATLAB built-in functions ode23 and ode45. There are graphs and tables presented to compare these methods; theoretically ode45 is of higher-order than ode23.

Introduction

Reactor point kinetic equations arise in mathematical models of nuclear reaction in the core of the reactor to study:

- When a nuclear power plant is under the condition to start up
- When a nuclear power plant is in the condition to shutdown
- The solutions to the equation is a factor of how much power is produced Safety analysis
- Optimize the nuclear fuel rod placement in the core

http://theconversation.com/small-nuclear-power-reactors-future-or-folly-81252

Mathematical Model

Reactor point kinetic equations have been solved in many different ways in the past [1, 2]. In this work, the system of seven ordinary differential equations (henceforth ODEs) is solved numerically using numerical methods called the Runge-Kutta methods.

Reactor Point Kinetic Equations:

$$\frac{dN(t)}{dt} = \frac{\rho(t) - \beta}{\Lambda} N(t) + \sum_{i=1}^{6} \lambda_i C_i(t)$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\Lambda} N(t) - \lambda_i C_i(t)$$

Runge-Kutta Methods to solve system of ODEs

Runge-Kutta methods are derived using Taylor's series. The algorithms are:

Second Order Runge-Kutta Method [3]

$$y_{n+1} = y_n + \frac{1}{2}(k_1 + k_2)$$

$$with\ k_1 = hf(x_n, y_n)$$

$$k_2 = hf(x_n + h, y_n + k_1)$$

for n = 0, 1, 2, . . . , N

Fourth Order Runge-Kutta Method [3]

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$with\ k_1 = hf(x_n, y_n)$$

$$k_2 = hf\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1\right)$$

$$k_3 = hf\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2\right)$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

for n = 0, 1, 2, . . . ,N

Conclusion

For Reactor Point Kinetic Equations numerical results obtained by ode45 and ode23 are in accordance with analytical solutions ode45 is more accurate than ode23. The CPU time of these ODEs in MATLAB was less than 0.5 seconds allowing for high efficiency to obtain answers to the reactor point kinetic equations. One might ask 'Why should we use numerical solutions when exact solutions are available?' The answer: This is the simplest nuclear point kinetic model, in practice the model may consist of hundreds of equations and the only way to solve is the numerical approximation. Furthermore, other models such as infectious diseases models can be solved using the same idea.

Numerical Results and Analysis

Parameters for Case I and Case II:

λi = 0.0127, 0.0317, 0.155, 0.311, 1.4, 3.87

βi = 0.000266, 0.001491, 0.001316, 0.002849, 0.000896, 0.000182

Δ = 0.00002

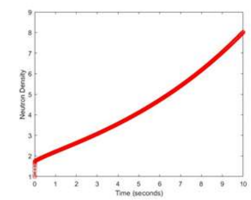
<u>Case I</u>

ρ = 0.003

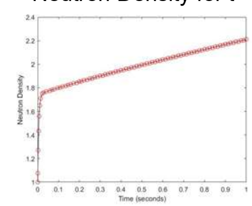
Time (seconds)	Exact	ode45	ode23
t = 1	2.2098	2.20948	2.20948
t = 10	8.0192	8.01879	8.01879
t = 20	28.297	28.29476	28.294762

on the graphs, ode23 solutions are represented by 'o'

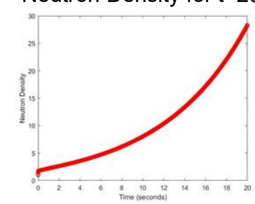
Neutron Density for t=10



Neutron Density for t=1



Neutron Density for t=20



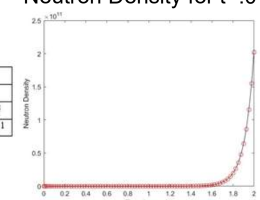
<u>Case II</u>

ρ=0.007

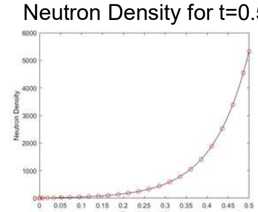
Time (seconds)	Exact	ode45	ode23
t = 0.01	4.5088	4.50885	4.50885
t = 0.5	5.3459 * 10^3	5.34605 * 10^3	5.34605 * 10^3
t = 2	2.0591 * 10^{11}	2.05950 * 10^{11}	2.05950 * 10^{11}

on the graphs, ode23 solutions are represented by 'o'

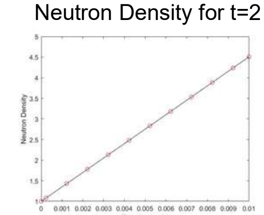
Neutron Density for t=.01



Neutron Density for t=0.5



Neutron Density for t=2



References

1.McMahon, D., Pierson, A., 2010. A taylor series solution of the reactor point kinetics equations.
2.Kinard, M., Allen, E.J., 2004. Efficient numerical solution of the point kinetics equations in nuclear reactor dynamics. Annals of Nuclear Energy 31, 1039-1051.
3. Zill, D., 2001. A first course in differential equations the classic fifth edition.