Displacement Analysis and Rigid Body Guidance in Spherical Linkages Using SU(2) and Homotopy Continuation

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This work seeks to efficiently and systematically model and solve the equations associated with the class of design problems arising in the study of spherical kinematics.

Introduction

SU(2) is used to analyze and synthesize the kinematics of a variety of systems including the three-roll wrist, the spherical four-bar mechanism, and the spherical Watt I linkage.

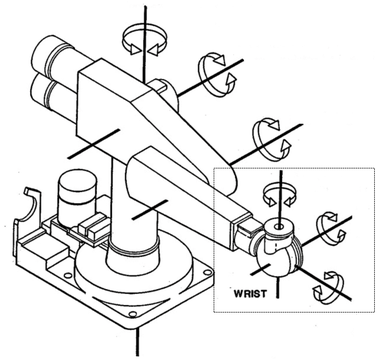
Two methods of formulating the synthesis problem are considered. Specifically, the five orientation synthesis of a spherical four-bar mechanism and the eight orientation task of the Watt I linkage are solved using both the loop closure equations and an approach derived from the dot product that recognizes physical constraints within the linkage.

Inverse Kinematics

3-Roll Wrist

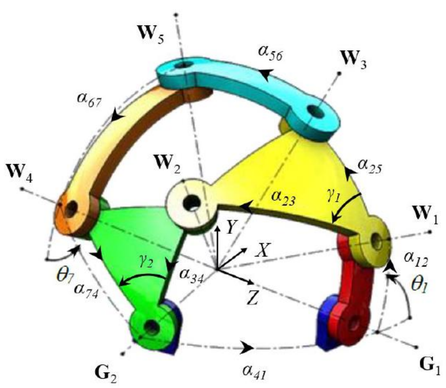
$$Q_{3RW} = \begin{bmatrix} M_1 & \overline{M}_2 \\ -M_2 & \overline{M}_1 \end{bmatrix}.$$

$$Q_{3RW} = Q_z(\theta_1)Q_x(\theta_2)Q_z(\theta_3).$$

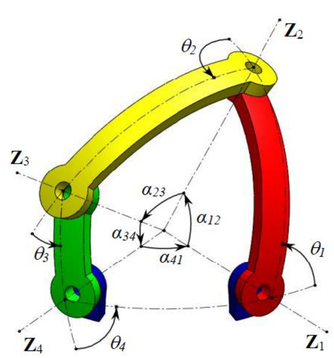


3-Roll Wrist connected to a robotic arm.

Displacement Analysis and Singularities



Spherical Watt I linkage schematic diagram



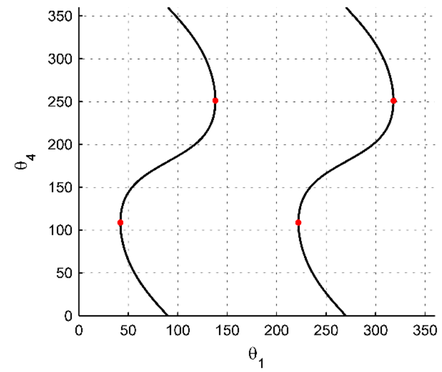
Spherical four-bar linkage schematic diagram

Forward Kinematics Four-Bar

$$Q_z(\theta_1)Q_y(\alpha_{12})Q_z(\theta_2)Q_y(\alpha_{23})$$
$$Q_z(\theta_3)Q_y(\alpha_{34})Q_z(\theta_4)Q_y(\alpha_{41}) = I$$

Singularity Points Four-Bar

$$\begin{bmatrix} -\mathbf{U}_2 & -\mathbf{U}_3 & \mathbf{U}_4 \end{bmatrix} \begin{bmatrix} \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \end{bmatrix} + \mathbf{U}_1\dot{\theta}_1 = 0.$$



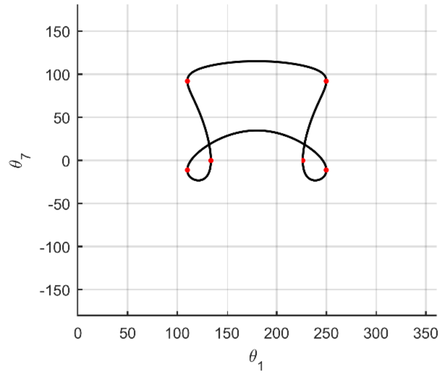
Motion curve for spherical four-bar projected onto θ_1-θ_4

Forward Kinematics Watt I

$$Q_z(\theta_1)Q_y(\alpha_{12})Q_z(\theta_2-\gamma_1)Q_y(\alpha_{25})Q_z(\theta_5)Q_y(\alpha_{56})Q_z(\theta_6)$$
$$Q_y(\alpha_{67})Q_z(\theta_7)Q_y(\alpha_{74})Q_z(\theta_4-\gamma_2)Q_y(\alpha_{41}) = I.$$

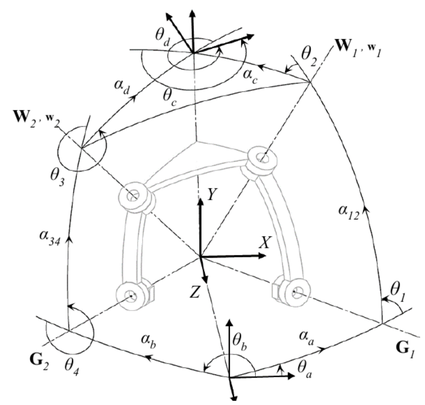
Singularity Points Watt I

$$\begin{bmatrix} -\mathbf{U}_2 & -\mathbf{U}_3 & \mathbf{U}_4 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{U}_2^{\dagger} & \mathbf{0} & \mathbf{U}_4^{\dagger} & -\mathbf{U}_5 & -\mathbf{U}_6 & -\mathbf{U}_7 \end{bmatrix} \begin{bmatrix} \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \\ \dot{\theta}_6 \\ \dot{\theta}_7 \end{bmatrix} + \begin{bmatrix} \mathbf{U}_1 \\ \mathbf{U}_1^{\dagger} \end{bmatrix} \dot{\theta}_1 = 0$$

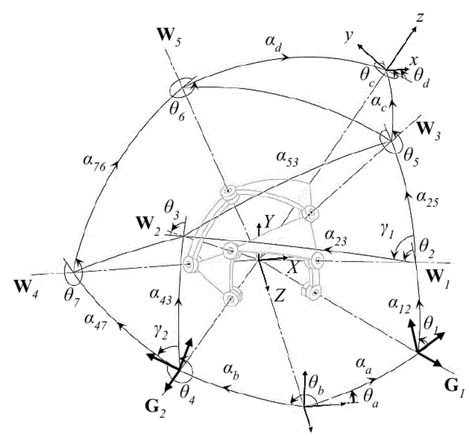


Motion curve for spherical Watt I projected onto θ_1-θ_7

Synthesizing Spherical Linkage



Parameters for solving synthesis problem of spherical four-bar



Parameters for solving synthesis problem of spherical Watt I.