Distance Between Graphs

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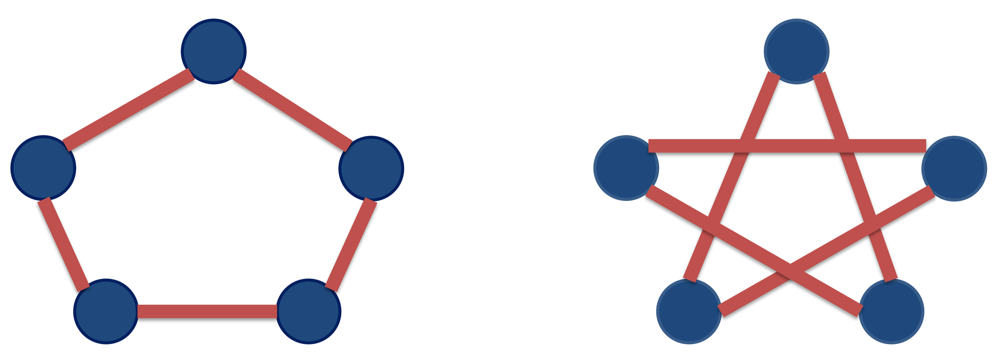
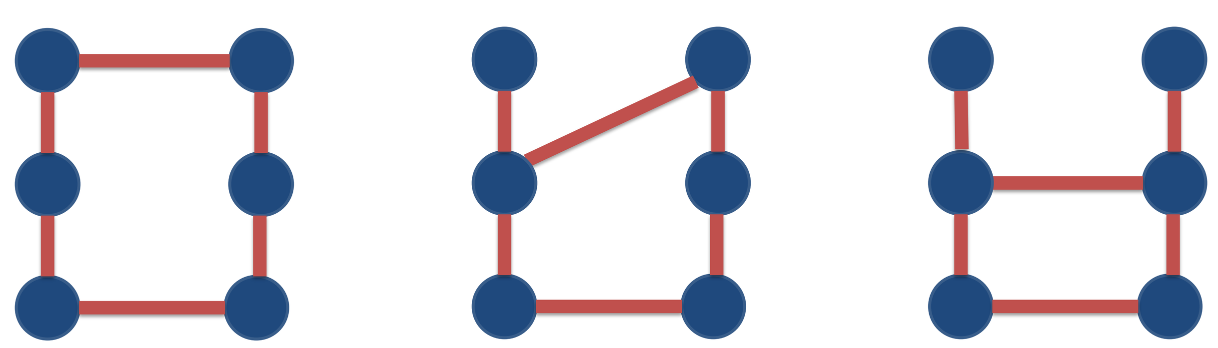
Introduction

Two graphs G and H are said to be **isomorphic** if there exists a bijection ϕ from the vertex set of G to the vertex set of H such that uv is an edge in G if and only if $\phi(u)\phi(v)$ is an edge in H. Isomorphic graphs can be relabeled and redrawn to look identical.



Two isomorphic graphs



A B C

Non-isomorphic graphs of same order and size

How "far" are graphs A, B, and C from being isomorphic to each other?

A graph G can be **rotated into** a graph H if, for vertices u, v, w in G, $G - uv + uw$ is isomorphic to H. The **rotation distance** $d(G,H)$ between G and H is the smallest number of graphs needed to transform G into H by rotations. For example, $d(A,B) = 1$ and $d(A,C) = 2$.

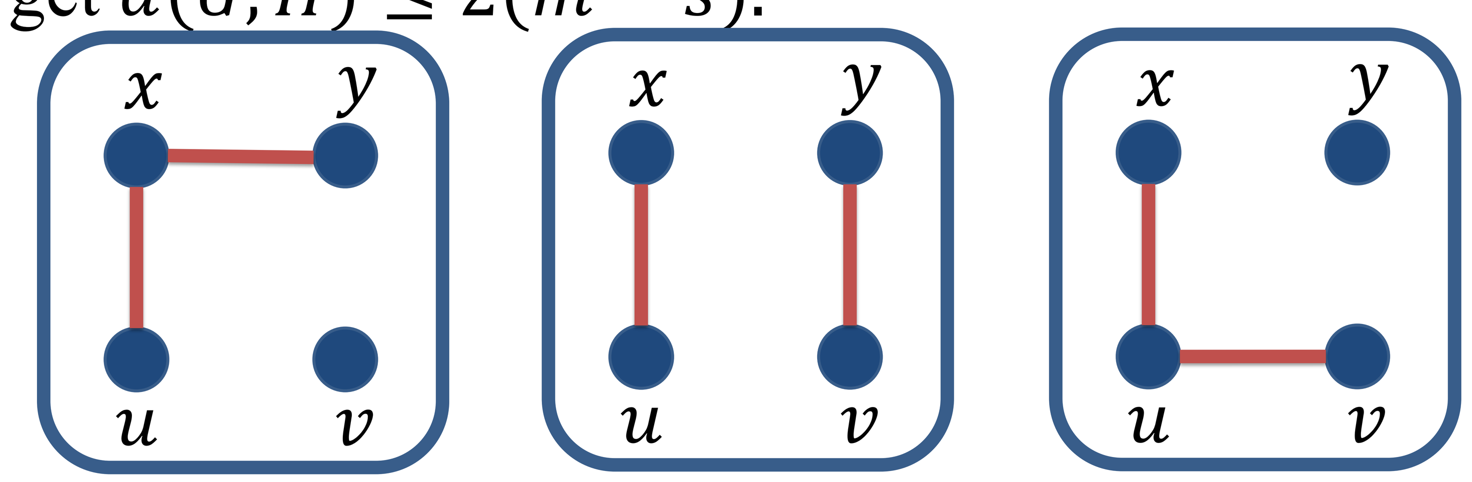
A **greatest common subgraph** of graphs G and H is a graph of maximum size that is isomorphic to edge-induced subgraphs of both G and H.

A **rotation distance graph** has a vertex set made up of graphs where two vertices are adjacent if their rotation distance is 1.

Theorem

Let G and H be graphs of order n and size m, and let F be a greatest common subgraph of G and H with size s. Then $d(G,H) \leq 2(m-s)$.

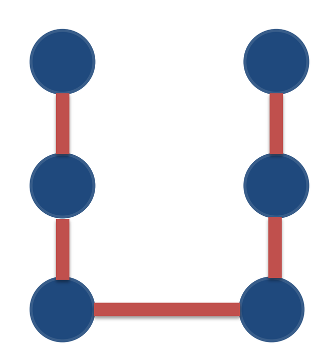
Why? If G and H are not isomorphic, then there exists an edge uv in H but not in G and an edge xy in G but not in H. We can construct a graph $H' = G - xy + uv$ so that $d(G,H') \leq 2$, and H and H' have $s+1$ edges in common. Repeating this process $m-s$ times, we get $d(G,H) \leq 2(m-s)$.



G H_0 H'

Example case in proof of theorem

The bound $2(m-s)$ is sharp. Consider graphs A and C to the left. The graph D (see below) is a greatest common subgraph of A and C with size 5. Our theorem states that $d(A,C) \leq 2(6-5) = 2$, and $d(A,C) = 2$.



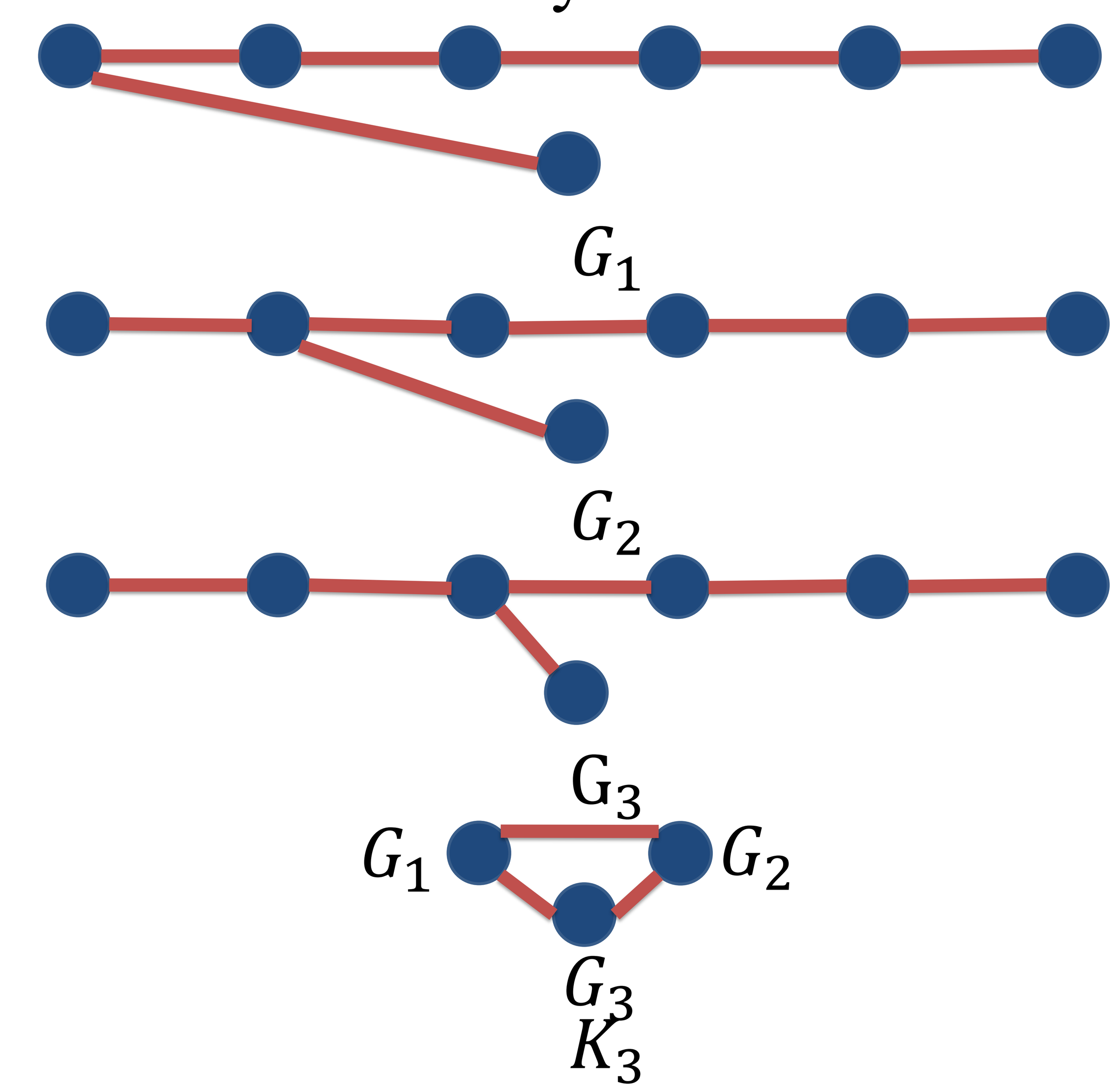
D

References

- Chartrand, Gary, and Ping Zhang. "Distance Between Graphs." *A First Course In Graph Theory*. Dover Publications, 2012, pp. 356-359.
- Faudree, RJ, et al. "On the Rotation Distance of Graphs." *Discrete Mathematics*, vol. 126, pp. 121–135.

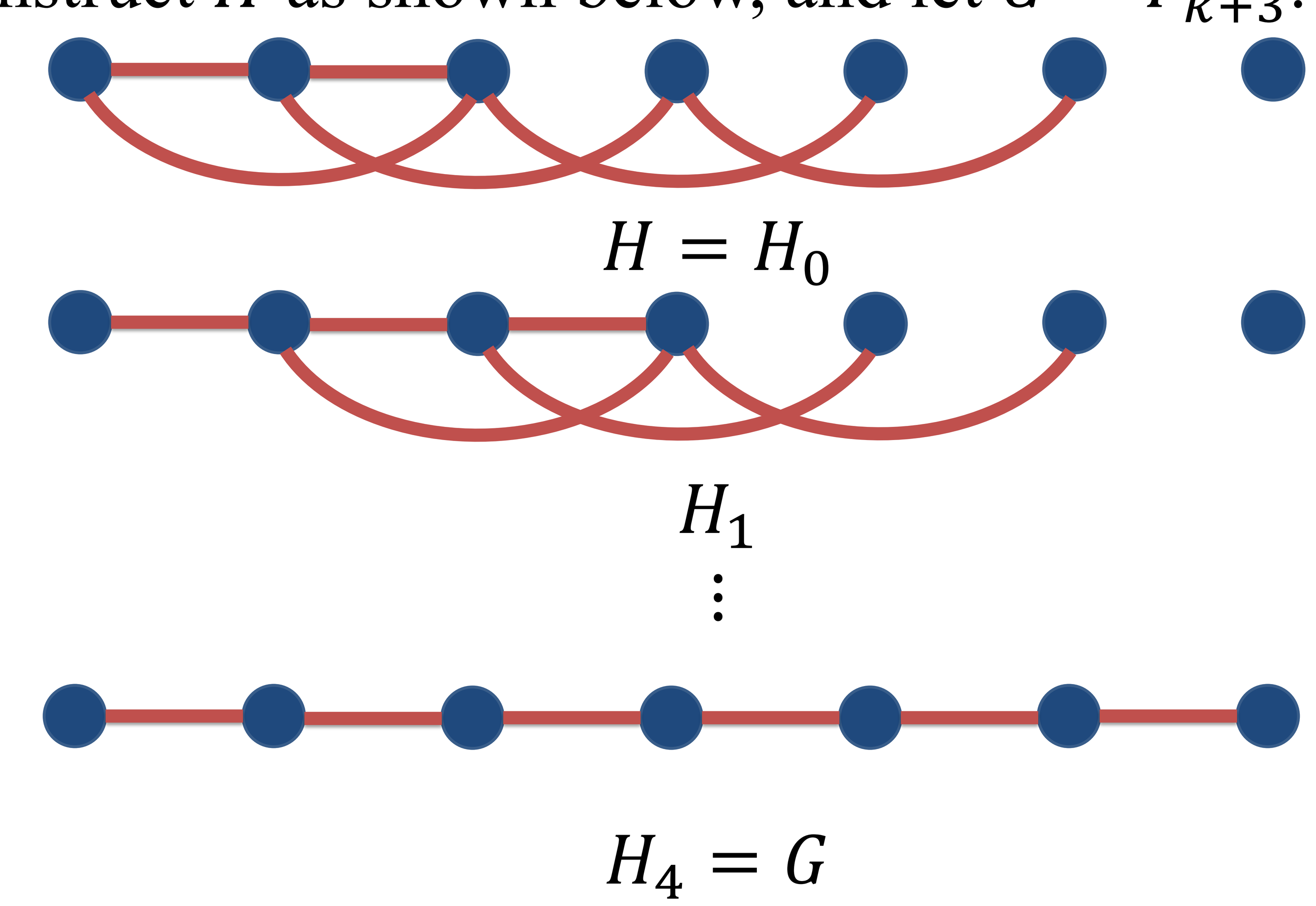
Rotation Distance Graphs

Every complete graph K_n is a rotation distance graph. For example, the graph K_3 is a rotation distance graph of graphs G_1, G_2, G_3 below for $n = 3$. Similar graphs can be constructed for any n.



Existence of Distances

For each positive integer k, there exist two graphs G and H such that $d(G,H) = k$. For any given k, construct H as shown below, and let $G = P_{k+3}$.



Example where $d(G,H) = k = 4$

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