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Distance Between Graphs
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Introduction
Two graphs $G$ and $H$ are said to be \textbf{isomorphic} if there exists a bijection $\phi$ from the vertex set of $G$ to the vertex set of $H$ such that $uv$ is an edge in $G$ if and only if $\phi(u)\phi(v)$ is an edge in $H$. Isomorphic graphs can be relabeled and redrawn to look identical.

A graph $G$ can be \textbf{rotated into} a graph $H$ if, for vertices $u,v,w$ in $G$, $G - uv + uw$ is isomorphic to $H$. The \textbf{rotation distance} $d(G,H)$ between $G$ and $H$ is the smallest number of graphs needed to transform $G$ into $H$ by rotations. For example, $d(A,B) = 1$ and $d(A,C) = 2$.

A \textbf{greatest common subgraph} of graphs $G$ and $H$ is a graph of maximum size that is isomorphic to edge-induced subgraphs of both $G$ and $H$.

A \textbf{rotation distance graph} has a vertex set made up of graphs where two vertices are adjacent if their rotation distance is 1.

Theorem
Let $G$ and $H$ be graphs of order $n$ and size $m$, and let $F$ be a greatest common subgraph of $G$ and $H$ with size $s$. Then $d(G,H) \leq 2(m - s)$.

Why? If $G$ and $H$ are not isomorphic, then there exists an edge $uv$ in $H$ but not in $G$ and an edge $xy$ in $G$ but not in $H$. We can construct a graph $H' = G - xy + uv$ so that $d(G,H') \leq 2$, and $H$ and $H'$ have $s + 1$ edges in common. Repeating this process $m - s$ times, we get $d(G,H) \leq 2(m - s)$.

The \textbf{bound} $2(m - s)$ is sharp. Consider graphs $A$ and $C$ to the left. The graph $D$ (see below) is a greatest common subgraph of $A$ and $C$ with size 5. Our theorem states that $d(A,C) \leq 2(6 - 5) = 2$, and $d(A,C) = 2$.

Existence of Distances
For each positive integer $k$, there exist two graphs $G$ and $H$ such that $d(G,H) = k$. For any given $k$, construct $H$ as shown below, and let $G = P_{k+3}$.

Rotation Distance Graphs
Every complete graph $K_n$ is a rotation distance graph. For example, the graph $K_3$ is a rotation distance graph of graphs $G_1, G_2, G_3$ below for $n = 3$. Similar graphs can be constructed for any $n$.

References
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