# Production Chain Analysis using Markov Chains

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## Objective of the Research

Analyze production systems using structured Markov Chains to determine production decisions.

Goal: Evaluate performance of the system in steady state.

Why: Those probabilities can be used to determine peak times for the company as well as provide an idea for the amount of inventory the company should have with each restock

How: By analyzing Artelejo's model, we can find probabilities of unsatisfied demands when the system is experiencing different levels of usage.

## (*S*, *s*) Inventory Model

Demands arrive ~ PP(λ) to a facility that dispenses that product, which is stored in that facility. Max inventory is *S*, min is *s*.

*Restocking Time:* $T_r$ ~ Exp(ν) when *s* < *S*.

*Provision Time of Product:* $T_d$ ~ Exp(μ).

*Model*: Level-dependent quasi-birth-and-death (**LDQBD**) process modeling a **retrial queue**.

Φ = { (R(*t*), I(*t*)): *t* ϵ Set of nonnegative integers }
where:
R(t) = # of Unfilled Demands in Orbit at time t ϵ $Z^+$
I(t) = Inventory at time t ϵ {0, …, S}
States: {(*i*,*j*): *i* ϵ $Z^+$, j={0,…,S} }

*i*=number of people in orbit
*j*=number of items in inventory

## Rates of Orbit Size

**Infinitesimal Generator** (Transition Rates)

$$\begin{bmatrix} A_1^0 & A_0^0 & 0 & 0 & 0 \\ A_2^1 & A_1^1 & A_0^1 & 0 & 0 \\ 0 & A_2^2 & A_1^2 & A_0^2 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

where

$$A_0^i = \begin{bmatrix} \lambda & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$A_2^i = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ i\mu & 0 & 0 & \cdots & 0 & 0 \\ 0 & i\mu & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & i\mu & 0 \end{bmatrix}$$

$$A_1^i = \begin{bmatrix} D_1^i & 0 & \Delta_v \\ L & C_2^i & 0 \\ 0 & L' & D_2^i \end{bmatrix}$$

**Steady State Probability of Inventory**:

$$P_i = \lim_{t \to \infty} P(R(t) = i)$$

**Traffic Intensity Formula**:

$$\lambda < (S - s)\nu$$

Computed from:

$$\pi^* A^* = \pi^*$$

$$\pi^* e = 1$$

$$D^* = \lim_{i \to \infty} \pi^i (A_0^i - A_2^i) e = \pi^* (A_0^* - A_2^*) e$$

$$D^* = \lambda - (S - s)\nu < 0$$

## Conclusions

Contributions:

- Model implementation without random environment

Suggestions for Future Research:

- Adding multiple products and batch arrivals
- Adding a random environment to modulate the exponential states
- Introducing service of the demands that is allowed to fail

## Reference

- J. Artalejo, A. Krishnamoorthy, and M. Lopez-Herrero. Numerical analysis of (s, S) inventory systems with repeated attempts. Annals of Operations Research, 141(1):67–83, 2006.
- J. Cordeiro, J. Kharoufeh, M Oxley: On the ergodicity of a class of level-dependent quasi-birth-and-death processes. Advances in Applied Probability, 51, 1109-1128(2019)

## Acknowledgement

University of Dayton Department of Mathematics