The Centrality of Trees

Morgan Ball

Senior Mathematics Capstone

Abstract

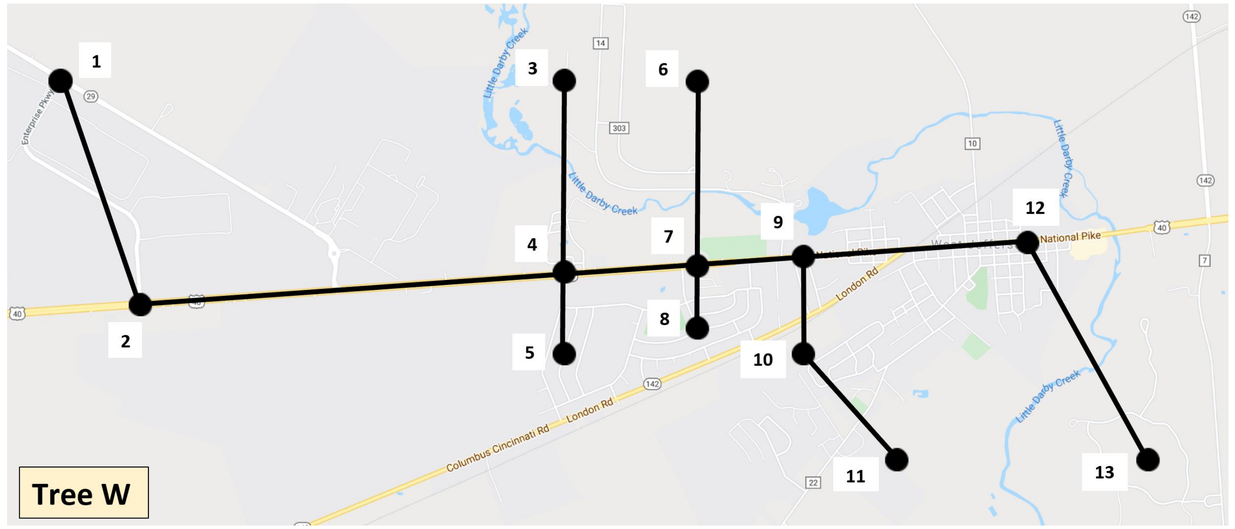
The study of the middle of graphs has numerous real-life applications. In this presentation I will focus on identifying the most logical place to put a Hospital in West Jefferson, OH, using the different measures of the middle of trees. As part of this discussion, I will introduce the importance of the processes of eccentricity, radius, diameter, and status in regard to the graph, in addition to the reasoning behind why each vertex of the graph was chosen. Lastly, I will discuss the significant different between the center of a graph and the median of a graph and distinguish the correct time to use each.

Academic Language

Order – the number of vertices in graph W
Path – a finite, alternating sequence of vertices and edges of W, beginning at any one vertex and ending at any other vertex, in which no vertex is repeated.
Cycle – a path whose beginning and ending vertices are the same
Tree – a graph which contains no cycles
Degree – the number of vertices to which one vertex is adjacent to in graph W, denoted deg(W)
Endvertex – when deg(x)=1
Eccentricity – the eccentricity of vertex x, denoted by e(x), is the distance to a vertex farthest from x
Radius – the minimum eccentricity of the vertices of W, denoted rad(W)
Diameter – the maximum eccentricity of the vertices of W, denoted diam(W)
Center – the set of vertices of graph W with minimum eccentricity
Status – the sum of all distances from each vertex of W to a vertex (i.e., s(x) = $\sum_{y \in V(W)} d(x,y)$
Median – the set of vertices of the graph W with minimum status

Materials

The figure below displays a map of West Jefferson, OH. On top of the map lies Tree W, which contains 13 vertices that will be used to apply the numerous methods to find the middle of the graph. Each vertex represents an area that is heavily populated and consistently hosts large gatherings of people for sporting events or work.



Results

Eccentricity:

e(1) = 6
e(2) = 5
e(3) = 5
e(4) = 4
e(5) = 5
e(6) = 4
e(7) = 3
e(8) = 4
e(9) = 4
e(10) = 5
e(11) = 6
e(12) = 5
e(13) = 6

Radius:

rad(W) = 3

Diameter:

diam(W) = 6

Status:

s(1) = 46
s(2) = 35
s(3) = 37
s(4) = 26
s(5) = 37
s(6) = 34
s(7) = 23
s(8) = 34
s(9) = 26
s(10) = 35
s(11) = 46
s(12) = 35
s(13) = 46

Center:

The center of Tree W is at vertex 7

Median:

The median of Tree W is at vertex 7

Therefore, the hospital should be placed at vertex 7.

Conclusions

Based on the results, if a hospital were to be built in West Jefferson, OH, it should be located at vertex 7. In this situation, many trips will be made from the hospital to the different vertices on the tree, and vice versa. Therefore, we want to minimize the sum of the distances from the hospital to the vertices. Hence, we want the hospital to be located at the vertex with the minimum status, or the median, which I found to be vertex 7. However, vertex 7 is also the center of Tree W. Thus, this also ensures that any emergency medical services will have the smallest response time to any other vertex, because the center is the vertex with the minimum eccentricity.

Sustainability

There is a hospital directly east of the village and another directly west of the village. Thus, from any vertex on Tree W, the closest hospital is no more than 20 minutes away. Therefore, there is no pressing need for a hospital to be built in West Jefferson, OH. Additionally, the population of West Jefferson is 4,411 as of 2018.. There are 3 fast food chain restaurants, few locally owned businesses, and an elementary, middle, and high school. In the past 20 years there have been two different grocery stores that have closed due to lack of business, along with numerous other smaller businesses. In such a scarcely populated area, the monetary capability to support the building, and sustainment, of a hospital is questionable.

References

Abueida, Atif, et al. “Module 08-3: Centrality and Anticentrality in Trees.” *DIMACS Center for Discrete Mathematics & Theoretical Computer Science*, May 2008.