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A handbook on the construction and use of manipulative math materials

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A HANDBOOK ON THE CONSTRUCTION AND USE OF MANIPULATIVE MATH MATERIALS

MASTER'S PROJECT

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Master of Science in Education

by

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Approved By:

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Signature of Advisor

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My sincerest gratitude is extended to my husband John for his patience, understanding and technical advice during all stages of this project.

I would also like to thank Dr. Paul Lutz for his support and cooperation.

TABLE OF CONTENTS

CHAPTER I

INTRODUCTION TO THE PROBLEM

The inclusion of mathematics in the elementary curriculum has long been considered to be a fundamental and essential condition for a well developed educational program. Indeed, the three r's--reading, 'riting, and 'rithmetic--have become identified as the cornerstones upon which sound basic education rests.

In the instruction of any subject matter, the cognitive level (or ability) of the students should be of primary consideration when developing a course of study. Determining where best to begin and how best to proceed with instruction is critical to the development of an optimum educational strategy.

Piaget (1958) described three major periods in the cognitive development of a child: sensorimotor, concrete operations and formal operations. While the study of mathematics involves the understanding and mastery of concepts and abstractions, the cognitive stage of the average primary school student has advanced only to the level of concrete operations. During this stage, the youngster thinks or perceives through the manipulation or organization of the

world around him/her. For this reason, many educators have resorted to utilizing concrete objects (i.e., visual aids) and/or manipulatives in the instruction of mathematics. Commercial enterprises, upon sensing a demand for such instructional aids, have developed materials for classroom use, but often at prices which extend beyond the average classroom budget.

Justification of the Problem

The use of physical and mental interaction in the teaching of mathematics to young children is a practice which is encouraged and supported by the National Council of Teachers of Mathematics (NCTM) (1980). The Arithmetic Teacher, a NCTM monthly publication which addresses aspects of elementary-level mathematics instruction, strongly recommends using manipulative devices for the instruction of abstract mathematical concepts, where and when possible. Each monthly issue includes at least one article dedicated to the either the design or use of a concrete teaching tool.

In 1986, while teaching the course entitled Mathematics in the Elementary School at the University of Dayton, Dr. Daniel Niswonger, Darke County mathematics curriculum advisor, introduced his graduate students to many commercially-manufactured manipulatives and offered suggestions for their

use in the classroom. The graduate students--most of whom were practicing teachers--recognized the instructional potential of such materials, but were discouraged by the prices of many of the manufactured items. As a whole, the class demonstrated as enthusiasm for using manipulatives in the arithmetic classroom, but an anxiety over the prices of many of the commercially-made tools. Many students indicated that they would use these instructional aids if the manipulatives were available at lower costs. Upon examination of the manufactured manipulatives, the author concluded that the average individual could inexpensively construct many of these durable devices by using simple hand tools and locally available materials.

Statement of the Problem

The purpose of this project is to compose a handbook specifically designed to assist elementary teachers in the construction and use of self-made, concrete teaching devices. This book will also provide local access to and identification of the proper materials for construction of such manipulatives, and easy-to-follow, step-by-step instructions for the construction of these devices.

Procedures

Subjects

This handbook is designed for use by teachers instructing students in grades kindergarten through third in mathematics.

Setting

The manipulatives constructed through the use of this handbook can be adapted for use in any kindergarten through third grade classroom.

Data Collection

The researcher compiled, selected and arranged the subject matter in the following manner:

1. Located and cited research findings to support the premise that the use of manipulative materials in the mathematics classroom is an effective method by which to teach abstract mathematical concepts.
2. Gathered and evaluated information about various manipulative objects to determine the feasibility of including some of these materials (or adaptations of these materials) in a "how-to" construction handbook.

3. Developed original ideas for construction of projects and selected those projects which were to be included in the "how-to" handbook.

4. Documented which tools and construction techniques were necessary for the complete assembly of all projects included in the handbook.

5. Identified construction material sources located in the Dayton area and collected pricing data.

6. Wrote step-by-step procedures (including the generation of necessary diagrams) for the assembly of each project.

7. Constructed prototypes of each project to test feasibility, utility, and adequacy of instructions.

8. Furnished suggestions for integrating the manipulative materials into the mathematics curricula.

Format

The handbook is divided into individual project construction sections. Each section includes a list of building materials, fabrication directions and suggested curriculum activities for each finished product. Two Appendices are included: a bill of materials and a list of suggested readings. A Table of Contents is provided to assist the reader in locating instructions and uses for each project.

Definition of Term

Manipulative

A manipulative will be defined as a three-dimensional object (or a collection of three-dimensional objects) by means of which learning is enhanced through the senses of touch and/or sight. For the purposes of this project, the cost of construction for any manipulative unit (i.e., complete teaching aid) shall not exceed twenty dollars; and, when properly constructed, each unit should be durable enough to withstand at least two full school years of reasonable classroom use.

Results

The result of this study is a handbook which can be used by teachers to construct low cost, durable mathematics manipulatives to be used in grades kindergarten through third.

CHAPTER II

REVIEW OF THE RELATED LITERATURE

Mathematics itself has existed from that point in time when prehistoric man first distinguished between one and more than one. While early civilizations limited mathematical use to a practical or applied level, the Greeks advanced the science to a pure form through the formulation of the two mental processes of abstraction and proof (Bergamini, 1963). It was the Greeks who introduced the formal study of mathematics into the educational system. Plato himself founded the first university known to history, The Academy, over the portals of which was allegedly inscribed, "Let no one destitute of geometry enter my doors." (Newman, 1956, p. 95).

From the Hellenic university to the twentieth century classroom, the learning of mathematics has been recognized as necessary for the development of a well-rounded individual. However, teaching the abstract concepts to young children presents a most interesting challenge. Abstraction is defined as, "the art of perceiving a common quality or qualities in different things and forming a general idea therefrom." (Bergamini, 1963, p. 39). However, studies

suggest that most elementary school-aged students think and learn at the concrete operational, representational levels (Piaget, 1963; Bruner,1966; Dienes, 1960). In light of such significant findings, the question of how best to teach abstract concepts to concretely-thinking youngsters should first involve a review of some of the important educational and developmental theories relating to the child-learning process.

Early educators approached instruction from an adult perspective. In instructional settings, children were regarded as small adults. The young mind was conceived as being a less mature version of the adult mind. In the seventeenth century, the educational applecart was upset when it was proposed that children should not memorize what they were incapable of understanding (Comenius, 1910). Later, in the eighteenth century, it was advocated that the young should learn through self-initiated activities (Rousseau, 1965). This theory of active participation in the learning process was advanced on a practical level by Pestalozzi (1898), Hebart (1896), and Froebel (1912). The latter resorted to the use of concrete geometrical forms in the classroom for the purpose of teaching the abstract concept of unity.

In America, the functional school of psychology regarded education as the continuing re-creation of experience (Dewey, 1910). In his work, Dewey stressed that going from the concrete to the abstract required an intellectual emphasis on the word "go." Thus, he did not regard the transition from concrete to abstract cognition as automatic--it required a mental leap. McLellan and Dewey (1909) defined a sound educational process as one which assisted in the successive maturation of psychical functions. Both concluded that only through a knowledge of psychology could a sound educational program be developed.

Research in the areas of cognitive and learning development has intensified in the twentieth century. Cognitive psychologist Piaget (1958) characterized a stage theory of development in children's thought which implied that elementary school-aged children think on a concrete operational level which is limited to physical reality. He concluded that manipulation of objects or body parts **must** precede any mental development. Zoltan Dienes (1960) emphasized the need for the elementary-aged child to explore various physical manifestations of a concept and to synthesize these experiences for the purpose of forming a concept. Bruner (1964) presented a theory of cognitive development which was characterized by three modes of representation in a develop-

mental sequence. In the first mode, called "enactive," learning is achieved through action. In the second, or "iconic," mode, cognitive development is based on representation through perceptual means. Finally, in the "symbolic" mode, experience is translated into words. The concrete operational level of Piaget paralleled Dienes' representation level and Bruner's iconic (or ikonic) thinking level for young children (Dienes, 1960, 1971; Bruner, 1964). The similarity of these cognitive models suggests that, from age seven to age eleven, most children are only capable of reasoning logically about ideas that can be represented in the real world. Each of these models includes an action-image-symbol sequencing of mental activity which further suggests a natural order in the development of human understanding. Englehardt (1977) wrote that intellectual development and concept attainment is the result of an ordered process of sensory-motor activity, visual imagery and, finally, symbolization. Skemp (1971) asserted that children in this age range are incapable of understanding concepts unless they are expressed by means of sensory and motor examples. Hawkins (1965) concluded that a "messing about" stage during which children first use materials and equipment without formal instruction created an educational environment which provoked student-initiated probing,

questioning and discovery. Hubbard (1972) noted the success of the learning sequenced Developing Mathematical Processes mathematics program and wrote that the best way to provide mathematics instruction was through the natural sequencing of learning from manipulation-to-pictures or diagrams-to-symbols.

A review of psychological and educational theories relating to the mental development of children indicates that, during the elementary school years, students think on a concrete, representational level (Piaget, 1958; Bruner, 1964; Dienes, 1960). In consideration of these theories, some mathematics educators have proposed that elementary level arithmetic instruction should be approached through the use of hands-on, concrete manipulative devices (Reys, 1973; Copeland, 1974; Holt and Dienes, 1973). Research findings generally support the hypothesis that proceeding from the concrete to the abstract enhances the mathematics comprehension of elementary school-aged children. Howden (1989) wrote that,

> Experiences with concrete materials... help students to visualize characteristics of all sorts of relationships from their everyday experiences and help them to recognize that in generating, organizing, and analyzing data to identify and generalize patterns, mathematics applies to any situation that can be quantified. (p. 23)

Fennema (1972) concluded that the appropriate use of concrete manipulatives makes meaningful learning of concepts more likely. Suydam and Higgins (1977), upon completing a thorough review of the literature on activity-based instruction in mathematics, recommended that manipulatives be an integral part of elementary education because "lessons using manipulative materials have a higher probability of producing greater mathematical achievement than do non-manipulative lessons" (p. 83).

The effect of motivation upon student achievement has been the subject of many educational research studies. Research by Johnson (1967) and Hughes (1970) suggested that there is a positive relationship between academic learning and learning enjoyment. Findings within the Vancouver Public Schools (1963) indicated that the use of activity-based instruction aided in the development of positive attitudes toward subject content. In his work, Herbert (1985) noted that there were significant motivational advantages to using manipulatives in mathematical instruction. A positive correlation among motivation, manipulative use, and student achievement has been demonstrated. The National Council of Teachers of Mathematics (1953) found that student achievement in mathematics increased as positive student attitude increased.

Based upon the responses to a teacher-questionnaire, the findings of Harvin (1965) supported her hypothesis that frequency in the use of manipulative materials in the classroom is a contributing factor to elementary mathematics achievement. In researching the relationship between the hands-on use of concrete teaching devices and the mathematics scores of primary school-aged children, Steffe and Johnson (1970) found that first graders using manipulatives scored significantly higher than their manipulative-deprived peers when administered a problem solving test. Parham (1983) analyzed 64 studies which compared the effects of use or non-use of manipulative materials on student achievement. She found that those students who used the manipulative devices scored at approximately the 85th percentile on achievement tests, while those who were deprived of these devices scored at about the 50th percentile. Parham concluded that use of such materials has a positive effect on student achievement in mathematics. Labinowicz (1985) discovered that manipulatives often served as communication tools by which teachers could find out what students thought. These materials also helped teachers to sort out student misconceptions. When Nicols (1971) compared two teaching approaches to the instruction of multiplication and division to third grade students, she found that the

manipulative approach with student discovery resulted in a significant achievement gain over a semi-concrete, abstract approach.

In comparing a manipulative, conceptual approach to math instruction with a procedural approach, Lampert (1985) observed that students of the procedural approach learned the mechanics but not the theory of mathematics. Pupils who learned through physical experience demonstrated a greater understanding of the "whys" of mathematics. Gunderson and Gunderson (1957) observed that second grade students are capable of formulating the concept of fractions if they are instructed both orally and through the use of manipulatives materials.

Research studies (Wheatley, 1978) suggest that different parts of the human brain have specialized orientations: the left side is linguistically and algorithmically oriented, while the right side is spatially and problem-solving oriented. A study by Van Devender and Rice (1984) suggested that the right-brain, manipulative instructional approach to teaching geometry and measurement to second graders results in the greatest gain in both achievement and attitude when compared to the left-brain textbook and left/right-brain textbook-and-manipulative approaches. The findings of Post (1980) support other findings that two-dimensional textbooks

may be unable to furnish the concrete experiences which students require to gain understanding of concepts.

Based upon their research findings, Kamaii and Williams (1986) stated that early childhood educators realize that youngsters learn better with hands-on experiences than with worksheets. Teachers' responses to a survey conducted by Scott (1983) to gain information on the current use of manipulatives in a large urban school district indicated that the teachers with more recent training tended to use more manipulative materials. However, a majority of all the teachers who were surveyed wanted additional concrete teaching materials in their classrooms. As a result of their survey of teachers, Perry and Grossnickle (1987) discovered that, despite research findings which indicate that use of manipulatives enhances understanding of math concepts, not all classrooms are stocked with an adequate supply of manipulatives.

The availability of concrete teaching materials may be directly related to the expense involved in acquiring such materials. Fey (1979) reported that many teachers cited insufficient funding as a reason for inadequate supplies of manipulative teaching aids in their classrooms. Kloosterman and Harty (1987) sent questionnaires to 301 Indiana school administrators concerning the use of manipulative materials

in science and mathematics teaching. The respondents mentioned that commercially made, hands-on materials were beyond the average elementary school budget.

A review of related literature from both psychological and clinical perspectives appears to indicate that three conclusions may be drawn relative to the desirability of using concrete manipulative teaching tools in the elementary mathematics classroom. First, most elementary school-aged children cogitate at a level which can be described as concretely oriented. Second, the use of concrete objects in the instruction of elementary-level mathematics can result in significant achievement gains in the mathematics scores of students. Finally, teachers recognize the instructional potential of manipulative aids in the teaching of arithmetic and desire the availability of these tools.

CHAPTER III

A HANDBOOK ON THE CONSTRUCTION AND USE OF MANIPULATIVE MATH MATERIALS

M^3

(Making Math Meaningful)

A HANDBOOK ON THE CONSTRUCTION AND USE OF MANIPULATIVE MATH MATERIALS

TABLE OF CONTENTS

TABLE OF CONTENTS

INTRODUCTION

In the English language, the verb "to see" is synonymous with the verb "to understand." Indeed, one of the most pleasant experiences which a teacher encounters in the classroom is to hear an enthusiastic student exclaim, "Yeah, I can see it now!" (translation: "Yes, I understand!"). Unfortunately, in the past, many educators have equated the seeing process with the understanding process, to the exclusion of the other senses.

The little red schoolhouse utilized a two-dimensional teaching approach where information was usually presented on a X' x Y' slateboard. Learning was perceived as the assimilation of information from the blackboard (or text) directly to the brain.

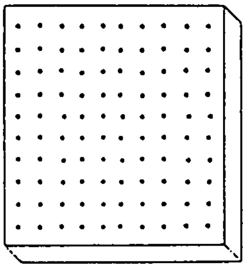
However, in modern times, educational researchers have come to understand that the cognitive process of the child is one which develops in stages (sensorimotor, concrete operations, and formal operations) and that the cognitive stage of the average primary school student has advanced only to the level of concrete operations. During this stage, the youngster **thinks** or **perceives** through the manipulation or organization of the world around him/her.

The study of mathematics involves the understanding and mastery of concepts and abstractions. In light of the latest findings in cognitive research, educators are beginning to realize how difficult it is for primary-aged students to take the giant leap from the chalkboard to the brain in mastering arithmetic. For this reason, many teachers have resorted to utilizing concrete objects (i.e., visual aids and/or manipulatives) in the instruction of mathematics. Commercial enterprises, upon sensing a demand for such instructional aids, have developed materials for classroom use, but often at prices which extend beyond the average classroom budget.

The purpose of this guide book is to assist elementary school teachers in the construction and use of simple, self-made, concrete teaching devices which are both inexpensive and durable. The step-by-step assembly instructions are designed to help those teachers who are "all-thumbs." Construction is limited to the use of simple hand tools and locally available materials. An Appendix provides both pricing and source information for each project in this book. Additionally, each project includes an ACTIVITY section which suggests various uses for the assembled manipulative.

It is hoped that, through the use of these concrete learning materials, teachers may hear "Yeah! I can see it now!" because their students have had the opportunity not only to see, but to touch (manipulate) as well.

THE GORGEOUS GEOBOARD



Geoboards are usually wooden, square-shaped boards with pegs or nails arranged in square arrays. Colored rubber bands are sometimes stretched across the pegs to form shapes. The versatility of these boards and the ease of their construction make them excellent candidates for homemade assembly. Geoboards can be introduced to students in grades K-1 and can be adapted for use in each of the succeeding elementary grades. Shape recognition and composition, number line placement, bar graphing, multiplication, and Cartesian plotting activities are just a few of the many uses to which the geoboard can be put to use.

Examples of three variations of geoboard construction are provided in this guide. Geoboard #1 is considered to be the "classic" example of the geoboard. However, Geoboard #2 offers students more manipulative opportunities, more hands-on experience. While Geoboards #1 and #2 are specifically designed for individual use, Geoboard #3 is intended for teacher-whole class activities.

Pegboard is an excellent medium for construction of this manipulative. The pre-drilled holes are usually exactly one inch apart, creating 1"-square arrays. By leaving a 3/8" border around all four edges of the geoboard, it is possible to group boards together and still maintain the 1"-square array.

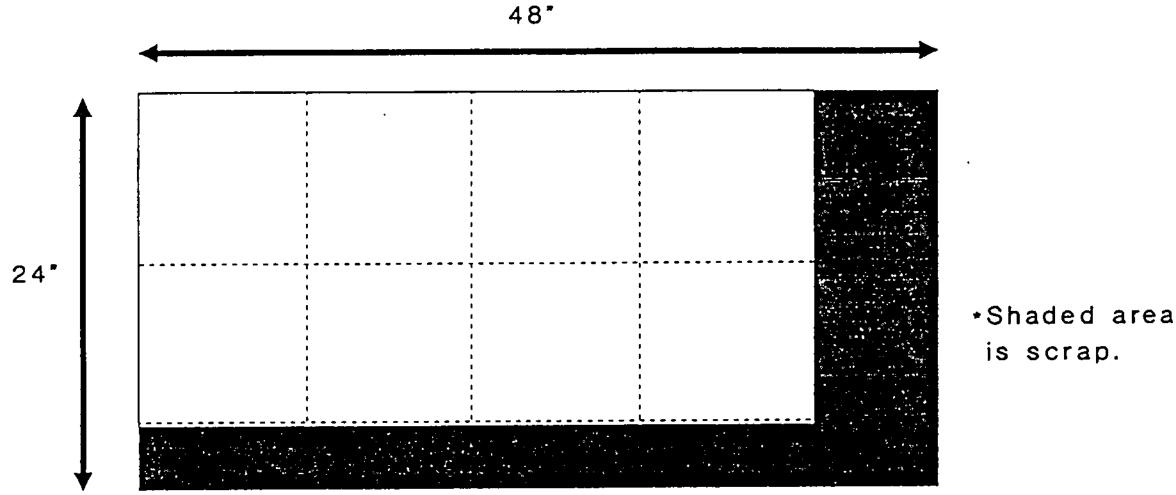
GEOBOARD #1

MATERIALS

3	¼" x 48" wooden dowels
1	2' x 4' x ¼" pegboard sheet
	wood glue
1 pkg	rubber bands (assorted colors)

DIRECTIONS

1. Using a fine tooth saw, cut two 10"x10" squares of pegboard, each with a 3/8" solid border on all four edges (see diagram). **NOTE:** It is possible to construct four geoboards from one 2' x 4' x ¼" sheet of pegboard.



2'x4' PEGBOARD CUTOUTS (10" SQUARES)

2. With the fine tooth saw, cut each of the four dowels into pieces measuring 1 ¼" each. (Yield = more than 100 pieces)

3. Next, liberally cover the top smooth surface of one of the 10" x 10" pegboard squares with wood glue.

4. Position the remaining 10" x 10" pegboard square, smoother surface up, directly on top of the glued surface, ensuring that the holes on both surfaces are in alignment.

5. Now, drive a 1 ¼" dowel piece into each of the four corner holes. Make sure that the dowel is driven through <u>both</u> layers of pegboard. (This anchors the two pieces of pegboard in place and assists in the adhesion process.)

6. Set this project on a piece of aluminum foil or wax paper. Place a heavy object (e.g. book or rock) in the center of the board. Let stand until glue is dry (per glue manufacturer's instructions).

7. When the boards are dry, glue 1 ¼" dowel pieces into the remaining holes. (**Suggestion:** work from center outward.) Dip each dowel piece into glue approximately ¼". Dowels may need to be tapped through <u>both</u> layers of pegboard using a small hammer. Consequently, it is recommended that assembly be performed on a surface which is expendable or mar-proof.

8. With a damp rag, wipe excess glue from both surfaces.

9. Turn peg board upside down, allowing board to rest on dowel pieces until glue is dry. If dowels are properly seated, both dowels and pegboard surface will be flush. If not, adjust position of dowel pieces accordingly. Allow to dry per glue manufacturer's instructions.

<u>ACTIVITIES</u>

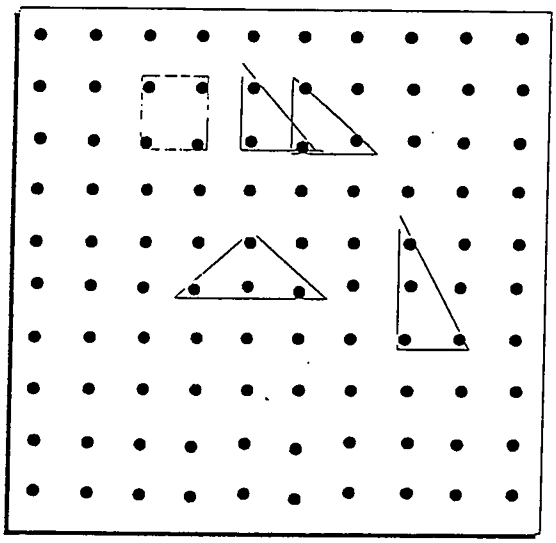
1. For younger children, the geoboard may be used for shape identification instruction. Ask the students to "draw" a house or other familiar object, using the colored rubber bands. Next, point out the different shapes which were used by the students. (If blocks are available, ask students if they can match up the shape of the blocks to the shapes which they used on the geoboard.) As the students become more familiar with shape identification, reverse the procedure. Name a shape and then ask the pupils to "draw" them on the geoboards with the colored rubber bands. Combine shape and color identification by directing the children to "make a square, using a red rubber band; draw a triangle, using a yellow rubber band, etc."

2. Direct the students to "draw" a familiar object on their geoboards, using only one large rubber band. Next, ask the children to use the smaller colored rubber bands to form shapes <u>within</u> the borders of the larger rubber band. (Just like in coloring, they cannot go outside the lines.) How many different shapes can they fit into the larger one? Do they see that a bigger shape can be made up of many smaller shapes?

3. COMPLETE THE SQUARE: The object of this game is to complete more squares than one's opponent. The first player stretches a rubber band across two adjacent pegs, either horizontally or vertically. The second player connects two other pegs anywhere on the

square. That player receives a "chit" or token for completing the square, then continues to play until he/she is unable to complete a square. Then the other player takes a turn. When no more sides can be drawn, the player with the most chits wins. (**NOTE:** This game encourages strategic planning.)

4. With older students, the teacher places a rubber band over some pegs as represented by the dotted figure in the diagram. The students are instructed that this is a square unit. As part of a group activity, the students are challenged to form other shapes which measure a square unit, using the rubber bands on the geoboard. Encourage the pupils to count the number of unit regions needed to cover an area. Try this with other shapes as well.



GEOBOARD #2

MATERIALS

3	¼" x 48" wooden dowels
1	2' x 4' x ¼" pegboard sheet
	wood glue
1 pkg	rubber bands (assorted colors)

DIRECTIONS

The directions for GEOBOARD #2 are identical to those for GEOBOARD #1, with one exception: the dowel pegs are not glued into the board. This permits the students to enjoy more manipulation of board pieces and to exercise creativity.

ACTIVITIES

1. Use GEOBOARD #2 to emphasize place values. The far right row is named the UNITS column, the row to its left is the TENS column, etc. Permit the students to place the pegs in the appropriate holes, reminding them that when the first column is filled, a peg must jump to the next column.

2. Reinforce the concept of ordinal numbering by asking the students to place pegs in specified holes [e.g., "Using only the bottom row of holes (for younger children, the teacher may have to show the students where "1" begins), place a peg in the Fourth Hole, Seventh Hole, etc."]

3. Help students to discover the commutative property of addition and multiplication. Ask one student to place 4 pegs anywhere on the geoboard. Then ask the same student to place another 5 pegs on the board. Ask the second student to position 5 pegs anywhere on the board and then to add another 4 pegs. Direct each student to add up the pegs on his/her board. Compare the results of each student. What do they find?

Older students can study the commutative property of multiplication by first filling in 4 rows and 5 columns with pegs and counting the total number of pegs. Then they can fill in 5 rows and 4 columns with pegs, count the total number of pegs, and compare their results with the previous exercise. What conclusion can be made about 4 x 5 and 5 x 4?

4. The concepts of perimeter and area can be emphasized by using the geoboard. Students may be instructed to "surround" areas (perimeters) or to fill them in with their pegs (areas).

5. Exercises in Cartesian plotting can be introduced to students by using the geoboard. Instead of x,y coordinates, the teacher can direct the students to move over,up. Using the bottom left peg as the 0,0 point, the teacher may ask the children to "move over" 5 and "up" 4, etc. and insert a peg in the corresponding hole.

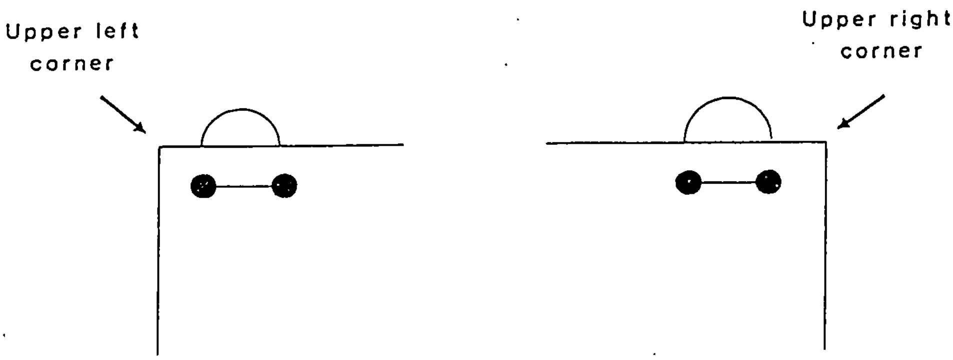
GEOBOARD #3

MATERIALS

3	1/4" x 48" wooden dowels
1	2' x 4' x 1/4" pegboard sheet
4 yds	3/4" velcro hook and loop tape
1 ft	string
2	cloth measuring tapes
1	Elmer's glue

DIRECTIONS

1. Cut string into two 6" pieces. Designate one 48" edge of pegboard as the top of the geoboard. Along this edge, lace a 6" piece of string through the fist two holes in the upper left hand corner; tie ends of string together to form a loop. Lace the remaining 6" string in the last two holes in the upper right hand corner; tie ends together to form a loop. (See diagram) These loops may be used to suspend the geoboard.



2. Velcro tape consists of two pieces "stuck" together. The sticky side is called the hook side and the smooth side is called the loop side. Separate the four yards of velcro tape into 4 yards of hook side and 4 yards of loop side. Cut the velcro tape hook side into two pieces 48" long and two pieces 22½" long. Glue the hook side of the longer pieces to the top and bottom edges of the geoboard. Glue the hook side of the shorter pieces to the side edges. The velcro should now "frame" the geoboard.

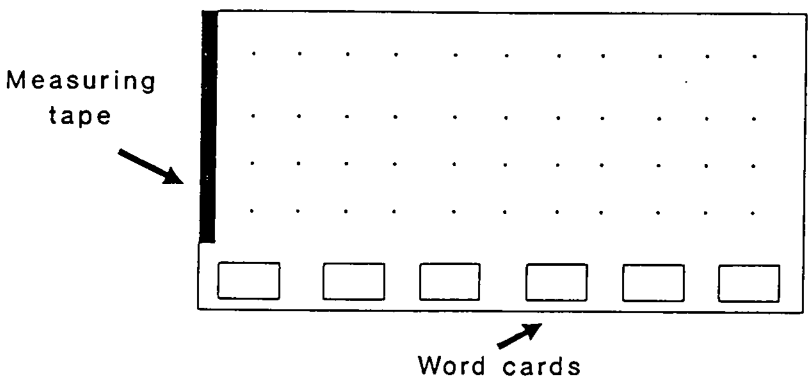
3. With a fine tooth saw, cut each of the four dowels into pieces measuring 1¼" each. (Yield = more than 100 pieces)

4. Sew or glue the remaining loop side of the velcro tape to the back side of each of the two 60" cloth measuring tapes.

ACTIVITIES

1. Since the holes in the 2' x 4' x ¼" pegboard are exactly one inch apart, the measuring tape may be fastened to the geoboard with the inch markings corresponding exactly with the holes on the pegboard. One measuring tape may be mounted along the left edge of the board (↑) and another measuring tape may be mounted along the bottom edge of the board (→) to form x,y coordinates. Use the geoboard as a point plotter for the purpose of introducing graphing to students. Substitute the words "over" and "up" for x,y.

2. To use this geoboard for displaying bar graphs, vertically mount the cloth measuring tape along the left hand edge of the board. After each spelling quiz, have an assigned "statistician" write all the words the class misspelled on separate 3" x 5" unlined index cards. On the back side of each of these cards, glue a small piece of the loop side of velcro. (These cards can now be affixed to the hook side of the velcro which is mounted to the geoboard.) Place each spelling word horizontally along the bottom edge of the board. To form the graph, have the statistician add the number of times each word was misspelled and plot the results with a peg dowel. Using this method, the students should be able to determine which words were most often misspelled and which words were generally mastered.



MAGIC VELCRO CARPET BOARD

For many years, the felt board has served as the classroom's primary display surface. However, the biggest limitation of this type of board is basically that only felt "sticks" to felt. Now, using velcro and carpet, it is possible to construct a classroom display board which is capable of supporting various fabrics of various weights. Even three-dimensional objects can be positioned on this surface!

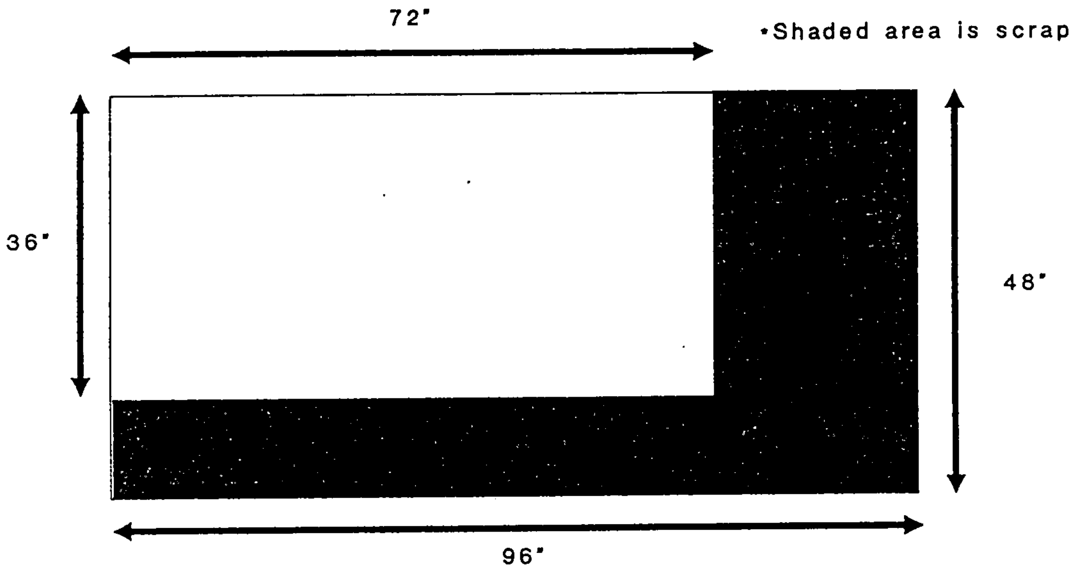
For years, frame shop owners have advertised their wares by affixing a strip of velcro hook tape to the back of their frames and placing them on a carpeted display. Likewise, fine china departments of major retailing establishments have exhibited their flatware and silverware selections by placing a small piece of hook tape to the back of their place settings and pressing them against a carpeted background. It's a puzzlement that educators have not "hooked" on to the use of velcro and carpet in the classroom.

MATERIALS

1	4' x 8' sheet of inexpensive panel board
1	3' x 6' automobile carpet
1	12" heavy string
1 quart	carpet cement

DIRECTIONS

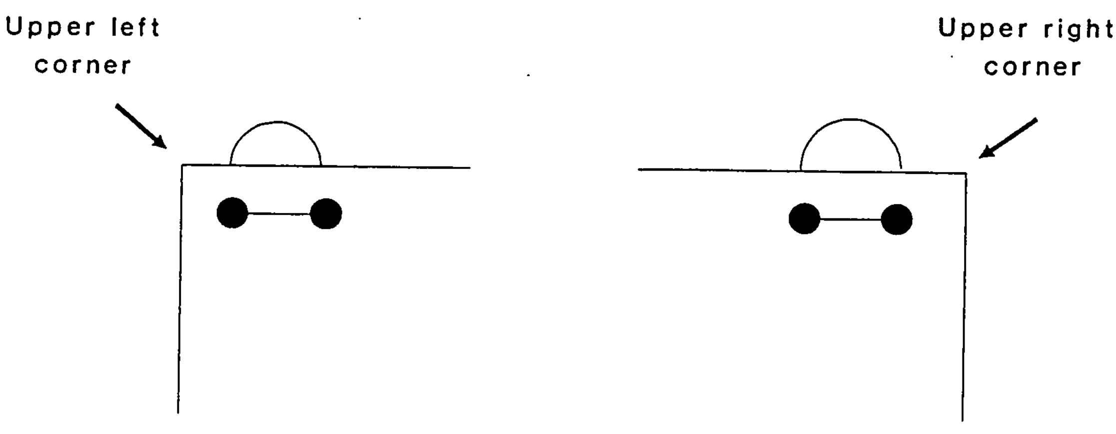
1. Using a fine tooth saw, trim the panel board to a piece measuring 3' x 6'.



Panel Board Cutout

2. Designate one of the longer edges to be the top of the board. On each of the top corners of the panel board, drill a hole which is 1" down from the top and 2" in from the side. Drill a second hole which is 1" down from the top and 3" in from the side. (There should now be two holes in each top corner, 1" apart from each other.)

3. Cut the 12" heavy string in half. Use a 6" piece to lace in each of the top corner holes as shown in the diagram. Tie the ends of each 6" string together to form a loop. These loops may be used to suspend the board from nails or hooks.



4. Following the manufacturer's directions, apply the carpet cement to the 3' x 6' panel board.

5. Glue the 3' x 6' piece of automobile carpet to the panel board according to the instructions on the cement can. (Make sure that the loop stings protrude from the back side of the board.)

6. Place some heavy books on the carpet surface to ensure contact between the carpet and the board. Allow the carpet cement to dry per manufacturer's directions.

ACTIVITIES

1. **Shape Identification:** Attach the velcro hook tape to different familiar objects of the same shape (e.g., circle, square, rectangle). Use both two-dimensional and three-dimensional objects (multi-embodiment) on the carpet board. Ask the students to identify what each of these objects has in common.

   Examples:

Concept:	Circle
Objects:	paper plate, ping pong ball, washer, toilet paper core, coin, etc.
Concept:	Square
Objects:	block, die (dice), square wrapped present, pizza box top, etc.
Concept:	Rectangle
Objects:	license plate, 2" x 4" wood scrap, playing card, 5" x 7" picture frame, etc.

   Later, when students are more comfortable with shape identification, combine shapes on the carpet board and ask students to explain how they <u>differ</u> from each other.

2. **Word Problems:** Enact word problems using the carpet board. Select problems which involve manipulation of available materials. For example:

a. Subtraction: Mary has 10 forks and 7 spoons. How many more forks than spoons does Mary have?

Directions: Place velcro hook tape on the backs of plastic spoons and forks. Ask a student to attach the forks to the carpet board. Ask him/her to place a spoon next to each fork on the board (1:1 correspondence). By taking away all the sets of forks and spoons, how many extra forks remain?

b. Addition: Two ducks swam in the farmer's pond. When spring arrived, 7 more ducks came to enjoy the water. How many ducks swam in the pond in the spring?

Directions: Mount duck figures to cardboard. Glue velcro hook tape to the back of the cardboard. Place the 2 ducks on the carpet board. Then ask a student to attach 7 more figures to the board. How many ducks are in the pond?

SUGGESTION: Wallpaper is an excellent source of visuals. Ducks, houses, cartoon characters, unicorns, etc. are colorfully displayed on wall coverings. Just cut them out, paste them to cardboard, trim, and affix a piece of velcro hook tape to the back. Voila, a handy manipulative!

c. Multiplication: Snow White cooked breakfast, lunch and dinner for her friends, the Seven Dwarfs. She placed 7 plates on the table for breakfast, 7 for lunch, and 7 for dinner. How many plates did she set each day?

Directions: Ask a student to place enough velcro-backed paper breakfast plates for each of the dwarfs on the carpet board. (Have her/him arrange these in a row.) Next, have the child set the lunch plates beneath the breakfast plates. Do the same with the dinner dishes. The student has created an "array" of 7 x 3. Ask the class to count how many dishes were set in one day.

d. Division: Grandma gave a crayon to each of her 4 grandchildren. She had 12 pieces of paper and she wanted to give each grandchild the same amount of paper. How many sheets of paper did each grandchild get?

Directions: Place 4 velcro-backed crayons on the carpet board. Ask a student to place one velcro-backed sheet of paper next to each crayon, continuing round robin until all the paper is gone. How many pieces of paper did each grandchild receive? Suggestion: Try this with number combinations which are <u>not</u> wholly divisible (e.g., 3 crayons, 10 sheets of paper). How many pieces of paper are left over?

3. Place Value:

MATERIALS
tongue depressors/craft sticks or popsicle sticks small washers glue velcro tape

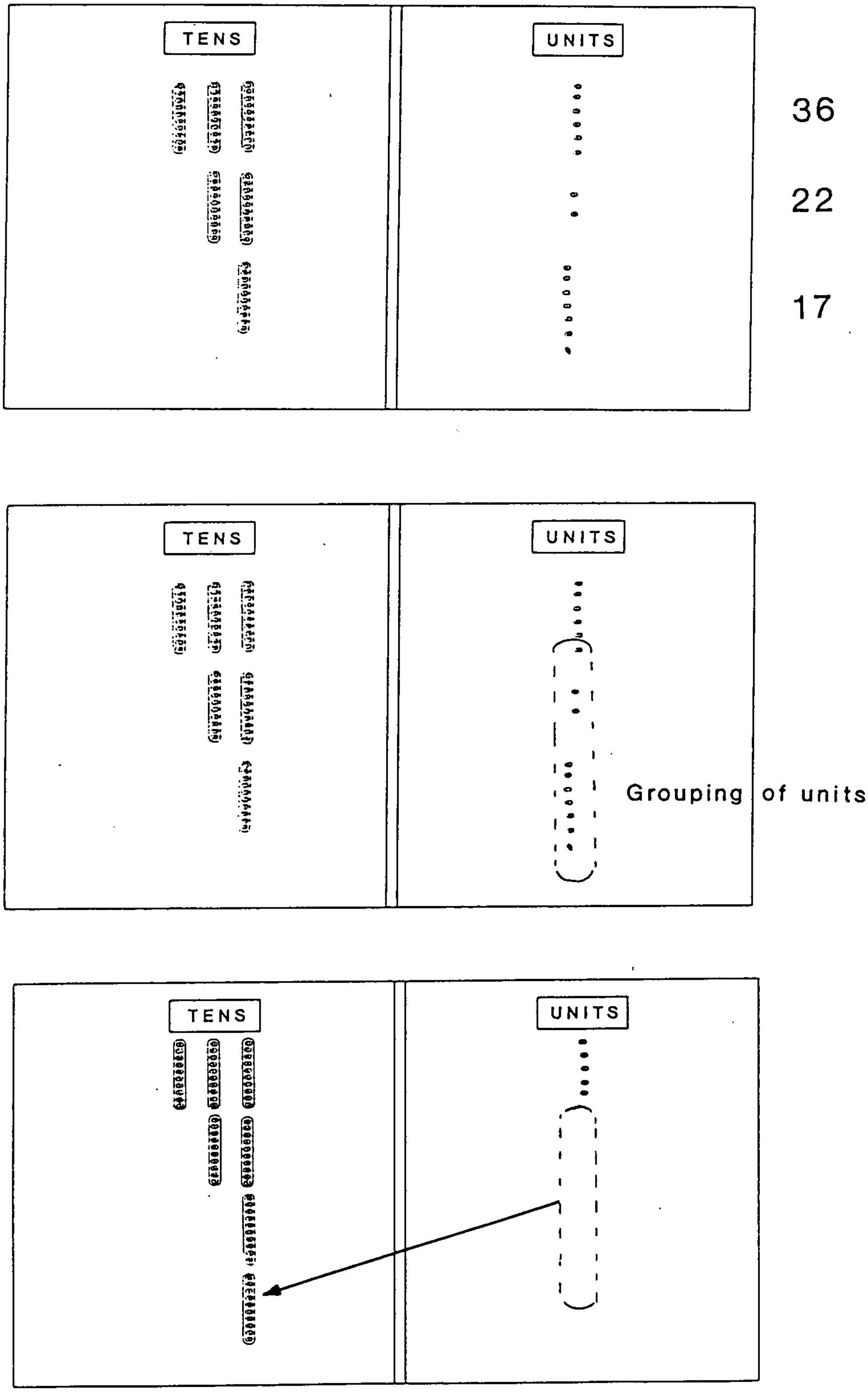
Directions:

a. To create a "tens" stick, glue 10 small washers to a craft stick or popsicle stick. (Each "unit" will be represented by a single velcro-backed washer.)

b. Place a strip of velcro hook tape down the middle of the carpet board. Over the left-hand column, place a 3" x 5" velcro-backed index card on which the word TENS has been written. Over the right-hand column, place a card with the word UNITS on it.

c. Name a two-digit number and ask a student to use the washer and washer sticks to represent that number on the carpet board. (The teacher may reverse this procedure by placing the sticks and washers on the board and asking the students to identify the number that is represented.)

Variation: Present 2 or more three-digit numbers to the class. Ask the students to show each of these numbers on the carpet board. Next, ask the children to add up these numbers by exchanging units for tens, where possible.



70 + 5 = 75

4. Graphing:

MATERIALS	
2	cloth measuring tapes
1-5	buttons
	glue or thread

Directions: Glue a small piece of velcro hook tape to the back of each button. Glue or sew velcro hook tape to the back side of each measuring tape. Place each measuring tape perpendicular to each other on the carpet board $\uparrow_{\rightarrow}$. Substituting the terms "over" and "up" for x, y, ask the students to plot points on the board by placing a velcro-backed button on the coordinates.

MATHEMATICAL BALANCE SCALE

A home crafted balance scale is practical, inexpensive and EASY to construct using conventional PVC piping and fittings. Through its classroom use, children can better visualize the concept of the term "equals" and the symbol "=." The pivot point on the cross beam of the scale becomes the "=" in equations. When the weighted cross beam is horizontal, the weight on one side is equal to the weight on the other. Activities in addition, subtraction, multiplication and division can be used to demonstrate equality.

The balance scale consists of four major assembly parts: leg supports, tie bar, vertical support, and cross beam.

MATERIALS

10'	3/4" PVC pipe
25"	3/4" x 3/4" woodstock (or one 3/4" wooden dowel)
4	3/4" PVC elbows (els)
3	3/4" PVC tee fittings
1	plastic cement (small tube)
1	4" #10 screw
4	#10 nuts
4	#10 washers
1	flexible soda straw
24	½" cup hooks
2 each	number decals (1-12) [Optional]
12	½ oz. bass fishing weights
½ pint	paint (any color)

DIRECTIONS

1. Using a hacksaw, cut the following lengths of 3/4" PVC pipe:

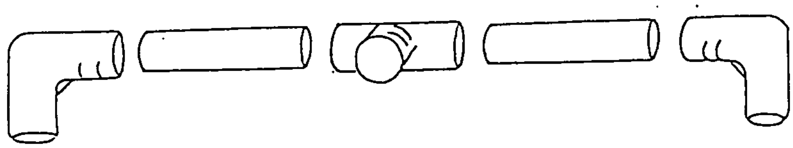
    4 - 4" pieces
    2 - 8" pieces
    1 - 18" piece

    NOTE: PVC piping is sold in 10' lengths at a very reasonable price. Since only a total of 50" is needed to assemble this project, it is possible to construct two of these scales using the purchased length of PVC.

2. **Assembly:**

    a. **Leg supports (2 required):**

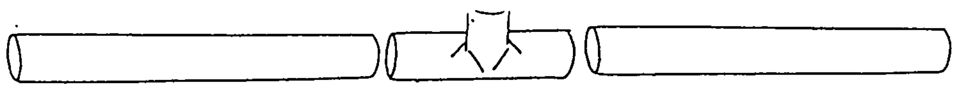
    1) Insert two elbow (els) fittings, two 4" sections of PVC pipe, and one tee connector into each other as illustrated below.



    2) Repeat Step 1) to assemble second leg support.

    b. **Tie bar assembly (1 required):**

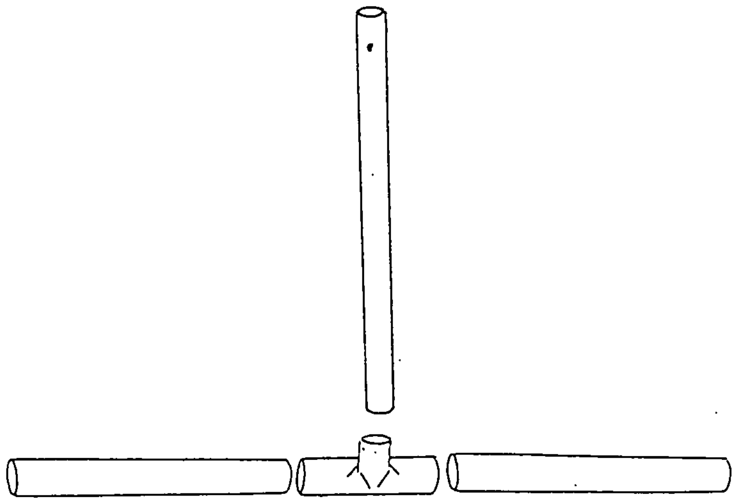
    Insert two 8" sections of PVC pipe into two "throats" of a tee connector as depicted in the following diagram.



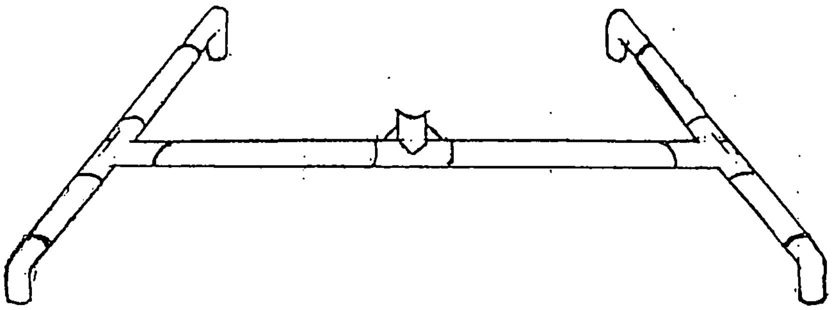
    c. **Vertical support:**

    1) Drill a 7/32" hole through the 18" long piece of PVC pipe at a location 2½" from one end of the pipe. (This now becomes the top end of the pipe.)

2) Insert the bottom (undrilled) end of the 18" pipe into the tee fitting of the tie bar assembly. (See diagram)



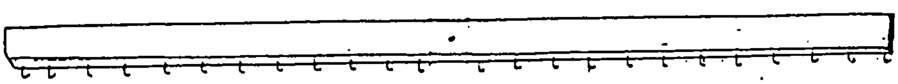
4. **Base assembly:** The base is formed by inserting each end of the tie bar into the remaining "throat" of each tee connector on the two leg supports. (See diagram)



5. **Cross beam assembly:**

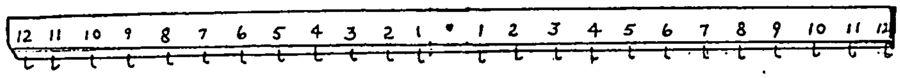
a. In the 25" wood stock (or 3/4" wood dowel) section, drill a 7/32" hole through the wood at a position 12½" from one end (center of the piece). This hole becomes the pivot hole.

b. On the lower surface of the wood (see diagram), mark location for hooks at 1" intervals. **NOTE:** Spacing for each side MUST be measured from the pivot hole outward!



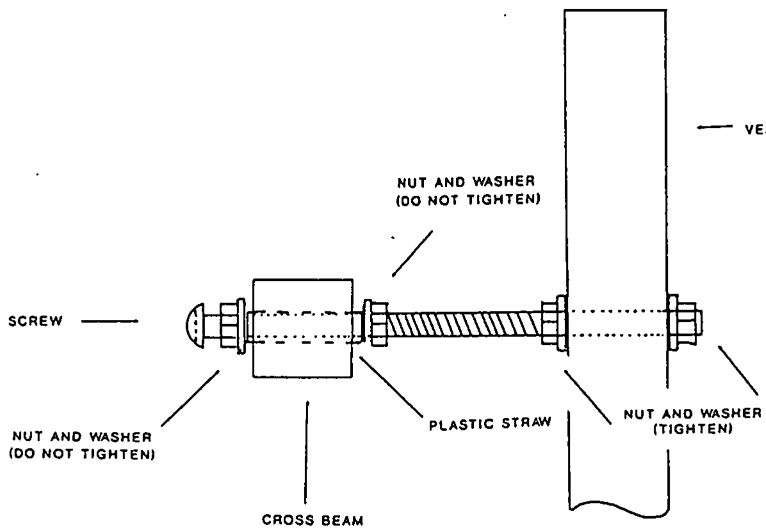
c. At each interval marking, screw a ½" cup hook, making sure that all hooks face forward. HINT: At each mark, start a small hole by tapping in and them removing a small nail.

d. Using small number decals or a marker, number each hook on both sides of the pivot (1-12) starting from the pivot hole and extending to the extremities of the cross beam. (See diagram)



e. Cut a piece of straw to be slightly longer (approximately 1/16") than the pivot hole in the cross beam. This will eventually become a sleeve through which the mounting hardware is placed.

f. Mount the cross beam to the vertical support, using the hardware as shown in the diagram.

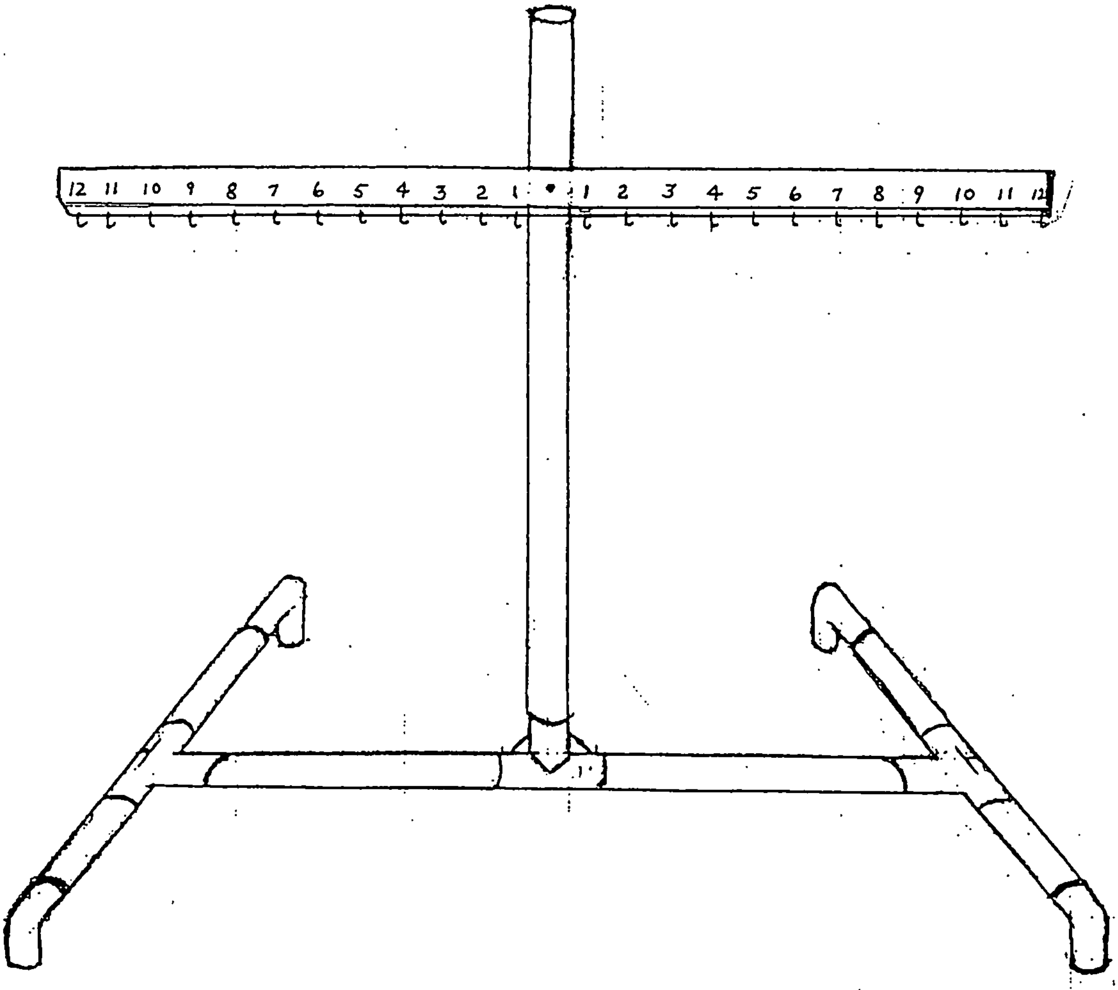


6. **Finishing touches:**

a. After the scale is assembled, check for proper alignment of all pieces. When alignment is satisfactory, place two pencil lines at each connection (extending from the pipe across to the fitting). These marks will be used to ensure that alignment is maintained during the gluing operation.

b. Remove the vertical support and place it to the side. With remaining PVC pieces, disassemble a joint at a time and apply plastic cement to joints. Reassemble each piece, making sure the alignment marks are straight.

c. After all joints are cemented, set aside to dry according to cement manufacturer's directions.



7. **Balancing the cross beam:**

   a. Insert vertical support into the tie bar. (To facilitate storage, DO NOT apply cement to this joint.)

   b. Ideally, with no weights on the cross beam, it should remain in a horizontal position. If, however, additional balancing is required, lay a thumb tack on the top surface of the cross beam; slide thumb tack along top surface until balance is obtained. Remove tack from top surface and gently stick it into the back surface of the cross beam at the location where balance was achieved.

8. **Weights:**

   a. Tie 12" of sewing thread to each weight.

   b. Dip each weight in paint and hang to dry. [HINT: Ensure that paper (or some protective covering) is under each painted weight.]

   c. When paint is dry, remove thread.

ACTIVITIES

1. **ADDITION:** Present a problem in addition to the students (e.g., 5 + 2 = ?). Have a student place one weight on the 5 and another weight on the 2 on the left side of the balance scale. Ask the student to place a weight on the number on the right side of the scale which is equal to 5 + 2. If his/her answer is correct, the scale will be balanced.

   Present the preceding problem in another manner. Instruct the student to place one weight on 5 and another on 2. Now see if he/she can balance the scale by placing a weight on each of two numbers (other than 5 and 2) on the right hand of the scale whose sum equals 5 + 2.

2. **SUBTRACTION:** Direct the students to solve a problem (e.g., 5 + ? = 7, using the balance scale. Place a 5 on one side of the balance and a 7 on the other side. Ask where a weight must be added to balance the scale.

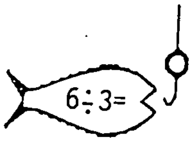
3. **MULTIPLICATION:** Give the students a problem in multiplication (e.g., 5 x 2 = ?). The designated "scale balancing person" places five weights on the 2 on one side of the scale. On what number on the other side of the scale must he/she place a weight which will balance the scale?

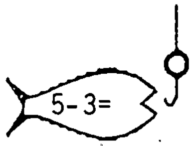
   Next, have the student remove all the weights from the scale. Then, have him/her place two weights on the 5 on one side of the scale. On what number on the other side of the scale must he/she place a weight which will balance the scale?

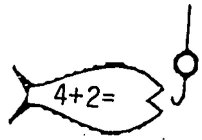
   Again, ask the student to remove all the weights from the scale. Now, direct him/her to place two weights on the 5 on one side of the scale and five weights on the 2 on the other side of the scale. What happens? What conclusions can be drawn from this? (Commutative property)

4. **DIVISION:** Using the same problem as above for a sample, offer this problem to the class: 5 x ? = 10. If one weight must be placed on the 10 on one side of the scale, on what number on the other side of the scale must five weights be placed to result in a balanced scale?

MATHEMATICAL GO FISH







Help kids from FLOUNDERing in arithmetic by exposing them to a REELy upSCALE game which can be adapted for addition, subtraction, multiplication and division. ForTUNAte students will have the opportunity to enjoy this math skills game from start to FINish!

The fish are crafted from felt, can lids, and vinyl. Arithmetic problems can be written on the exterior clear vinyl covering with a grease pencil. Want to change the problems or mathematical operations? Then simply rub off the old with a dry rag and begin anew!

A magNETic fishing poll is constructed from a wooden dowel, some string and a simple magnet.

<u>MATERIALS</u>

FISH:

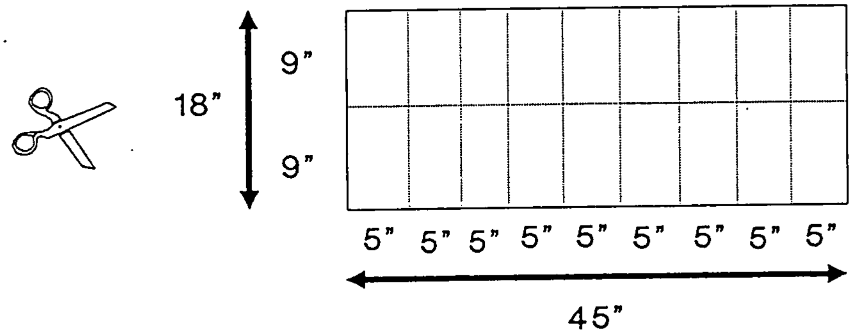
1	Fish pattern (provided)
18	Soup can lids and bottoms
1/2 yd **each**	Green, yellow, red, orange felt (72" wide)
1 yd	4-Gauge clear vinyl fabric (54" wide)
1 sheet	carbon paper
1 sheet	6" x 10" cardboard/tag board
36 sheets	loose leaf paper or tissue paper (9" x 5")
	thread
1	grease pencil

ROD:

4	48" x 5/8" wooden dowels
4	36" strings
4	magnets

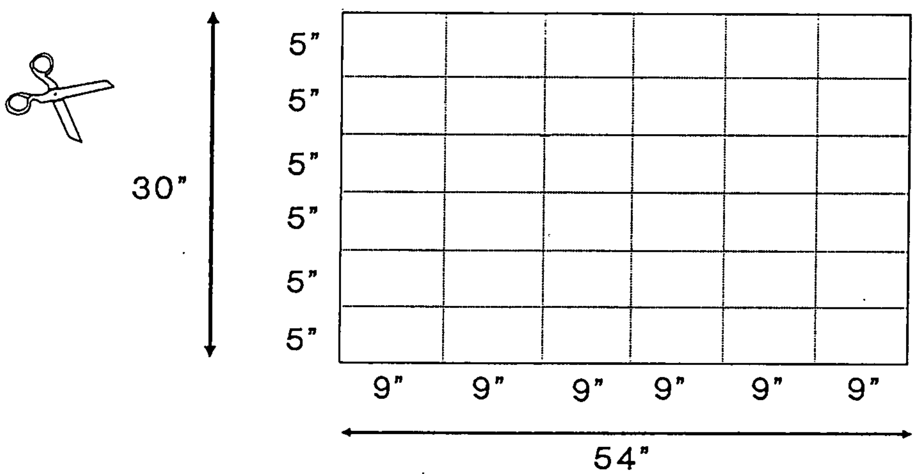
DIRECTIONS

1. From each ½ yard of felt, cut 9" x 5" rectangles. (Yield = 18 rectangles per color) [See Felt Cutout Figure]



Felt Cutout

From 1 yard of clear plastic vinyl, cut 9" x 5" rectangles. (Yield = 36 rectangles) [See Vinyl Cutout Figure]

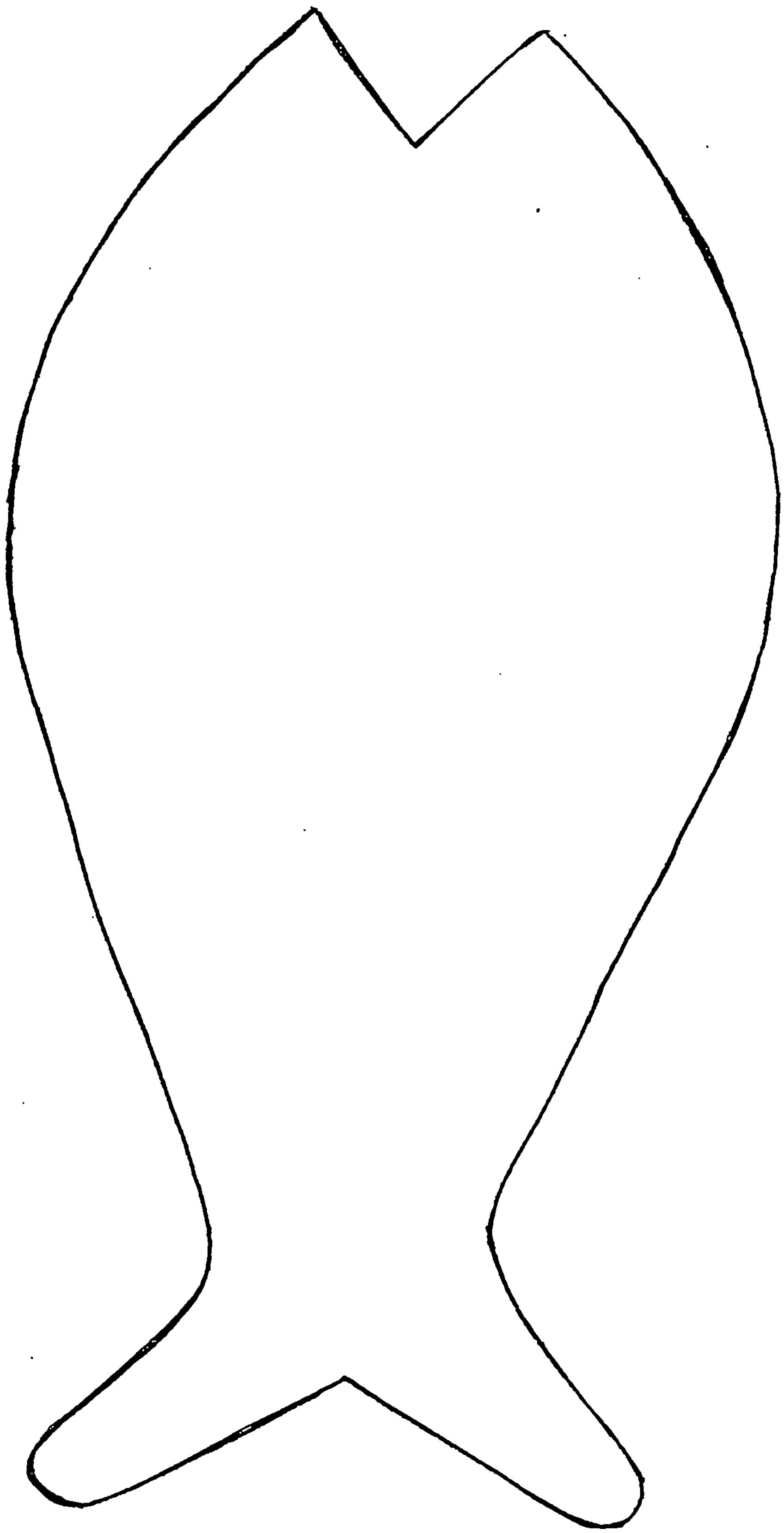


Vinyl Cutout

2. With a can opener, remove the tops and bottoms from 18 soup cans. Save. Discard remainder of cans (or save for another project).

3. Using the pattern provided, perform the following operations:

   a. Trace the outline of the fish on the 6" x 10" piece of cardboard.



FISH PATTERN

b. Next, cut out the traced cardboard fish. This will serve as your "working" pattern.

c. On each of the 36 paper rectangles, transfer the outline of the fish by tracing along the edges of your working pattern.

4. Assembly:

Felt Rectangle	Can Top/Bottom	Felt Rectangle	Clear vinyl Rectangle	Paper Rectangle

Piece #1	Piece #2	Piece #3	Piece #4	Piece #5

a. Place a 9" x 5" felt rectangle on a flat surface. (Piece #1)

b. Next, position a can top/bottom (Piece #2) on Piece #1.

c. Proceed by placing another felt rectangle (Piece #3) of the same color directly on top of Pieces #1 and #2.

d. Place a clear plastic vinyl rectangle (Piece #4) atop Pieces #1, #2, and #3.

e. Cover all pieces with 9" x 5" paper (Piece #5) on which the outline of the fish has been drawn.

f. Using straight pins, pin all four rectangular pieces together, making sure that the can top is positioned <u>inside</u> the outline of the fish.

With the paper rectangle face side up*, machine stitch (with long stitches), using the fish outline as the sewing line.

Trim fabric, leaving approximately 3/8" <u>outside</u> the stitched edge.

5. Assemble rods by securing one end of the 36" string to one end of the wooden dowel and the other end of the string to the magnet.

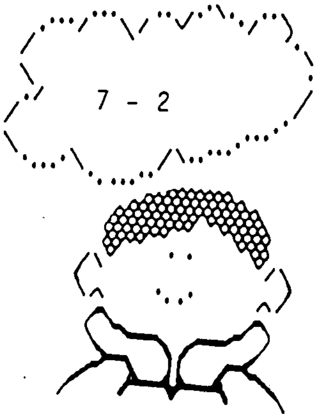
*Paper prevents the vinyl from dragging on the sewing machine foot.

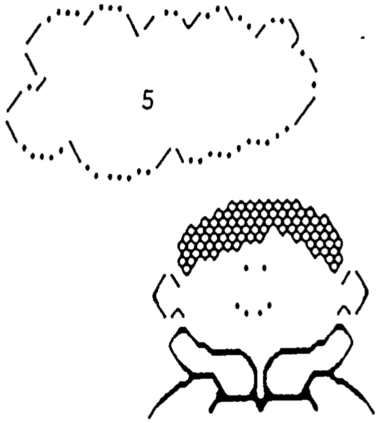
ACTIVITIES

1. **Catch Fish of the Same Variety/Species:** Label each of the fish with an addition/subtraction/multiplication or division equation. Place fish on the floor with the vinyl side up. Instruct fishermen that they may catch only those fish whose sum/difference, etc. equals a certain number. For older students, all four mathematical operations may be combined in one game. If desirable, four teams (each assigned a different color fish) may play at one time.

2. **Game Warden:** Write an equation on the side of each fish. Instruct fishermen that **only** the fish whose sums/differences, etc. are EVEN are big enough to keep. Odd answered fish must be left on the floor. For variation, all the fish may be piled in a basket. As the student "catches" each fish he/she must decide whether to keep it or throw it on the floor. Older students may enjoy a timed version of this game. As a brain teaser, select only one color fish whose answer is always odd. See how long it takes for the students to "catch" on to this ploy.

MEMORY CARDS/HOLDERS





Memory is a popular children's game which challenges its players to match pairs of objects by remembering where each of the objects appears on a surface. The old television version of this game was called Concentration because it required mental organization to play the game successfully.

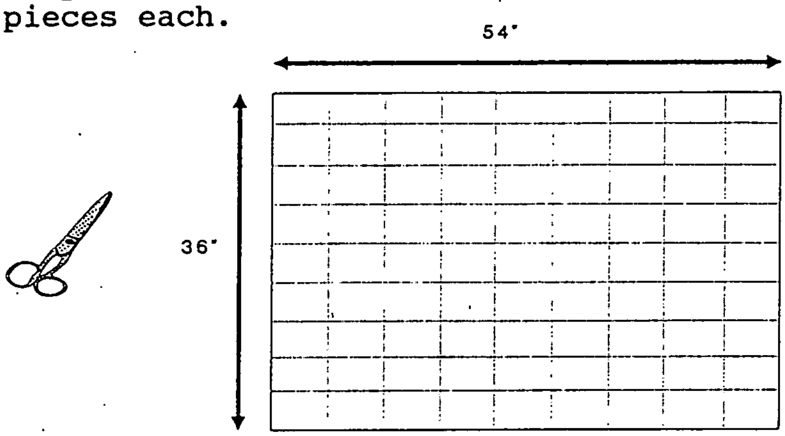
Now it is possible to create math memory games for the classroom, using inexpensive materials. The clear vinyl surface can be written on with a grease pencil and, later, wiped clean with a dry rag. Additionally, since only three sides of the vinyl covering are stitched (forming a pocket through which unlined 3" x 5" index cards can be placed), the teacher can design card games whose playing cards can be inserted and preserved behind the clear vinyl covering. The use of these handcrafted memory cards is limited only by the teacher's imagination. They can be adapted for instruction in other subject areas as well.

MATERIALS

1 yd	54" solid yellow vinyl fabric
1 yd	54" 4-gauge clear vinyl
	newspaper
	thread
	index cards OR grease pencil

DIRECTIONS:

1. Cut 1 yard of the yellow vinyl and 1 yard of the clear vinyl into 4" x 6" pieces. (See diagram) Yield = 81 pieces each.



Yellow Vinyl/Clear Vinyl Cutouts

2. From old newspaper (or tissue paper), cut 81 4" x 6" rectangles.

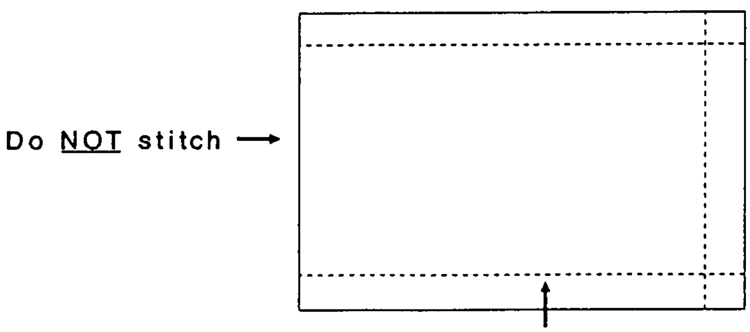
3. **Assembly:**

Yellow Vinyl Piece #1 | Clear Vinyl Piece #2 | Newspaper Piece #3

a. Place a clear vinyl rectangle (Piece #2) on top of the <u>wrong</u> side of a yellow vinyl rectangle (Piece #1).

b. Cover Pieces 1 & 2 with a 4" x 6" newspaper cutout (Piece #3).

c. Using a long sewing stitch, sew through all three pieces along a 3/8" sewing line. (NOTE: Only one edge of the shorter side is sewn.)



3/8" stitching line

d. Gently remove newspaper cutout (Piece #3) from the clear vinyl surface. (Newspaper/tissue paper is used to facilitate sewing on vinyl. Without this covering, the vinyl tends to drag under the sewing foot.)

e. If necessary, trim outside edges of vinyl so that fabric is even.

4. Using a grease pencil, print math problems on the clear vinyl side of assembled "memory" cards. Print the solutions to these problems on the vinyl side of other memory cards <u>OR</u> print problems/solutions on the unlined sides of 3" x 5" index cards and insert cards into the "pockets" of these cards so that the print is visible through the clear vinyl.

<u>ACTIVITIES</u>

1. Math Memory:

On one 3" x 5" unlined index card, write an arithmetic problem (e.g., 7 - 2). On another card, write the solution to the problem (i.e., 5). [NOTE: More than one problem with the same solution may be used. Be sure to pair all problems with an appropriate solution. For example,

7 - 2	5	3 + 2	5

Create an equal number of problems and solutions.]

Directions for Play:

a. Shuffle or mix all the cards and place them, face down, on a flat surface (table or floor).

b. Each player takes a turn by turning over any two cards of his/her choice so that the numbers are visible to all players. Players try to memorize the position of the cards.

c. If the two cards DO NOT equal each other mathematically, the player turns both cards, face side down, and ends his/her turn.

d. If the two cards DO equal each other, the player takes these two cards and places them in front of him/her. This player's turn continues until he/she fails to match pairs.

Possible combinations follow:

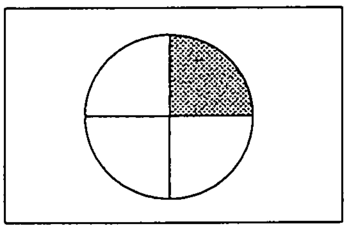
7 - 2	&	5
3 + 2	&	5
3 + 2	&	7 - 2
5	&	5

e. The player who accumulates the most pairs WINS.

Variations:

Fractions:

On one card, draw a circle and shade in a fractional area of the circle. On another card, write the shaded area's fractional equivalence. For example,



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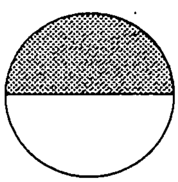
[NOTE: A fractional equivalence can be represented in a variety of ways. For example,



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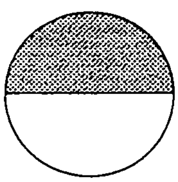

= 1/2

Be sure to pair each circle with a fractional equivalent.]

Shapes (for younger children):

On one set of 3" x 5" unlined index cards, draw various shapes (e.g., square, circle, etc.). On another set of cards, write the word which identifies each shape (e.g., SQUARE, CIRCLE, etc.). Play the memory game by asking the students to pair shapes with words.

2. Fill In the Missing Number:

Write arithmetic problems on unlined 3" x 5" index cards (e.g., 7 + $\boxed{}$ = 9, 3 + 6 = $\boxed{}$). Insert each card in a memory card pocket. Quiz the students by having them write the missing number in the empty box, using a grease pencil.

3. "Oh-Oh!" Old Maid:

MATERIALS
31 unlined 3" x 5" index cards marker (or color coding circles)

a. Divide the index cards into three sets of ten. On the remaining index card, print the phrase Oh-Oh!

b. On the first set of cards, print the numbers 1, 2,... 10.

   On the second set of cards, print the words ONE, TWO, ...TEN.

   On the third set of cards, place dots representing the numbers. Use color coding dots or marker.

c. Place the 31 cards into the memory card holders so that numbers/words/dots are visible through the clear vinyl.

Directions for Playing: (3 Players)

a. Deal all 31 cards to the players.

b. Each player reviews his/her hand and attempts to form "sets" of numbers, words, dots representing a single number. (NOTE: One "set" is the collection of three cards representing the same number.) Players discard their set(s) and play with the cards remaining in their hands.

c. The player to the left of the dealer begins play by drawing a card from the hand of the player to his/her left. Each player draws from the player to his/her left, discarding sets as they are formed.

d. A player is eliminated when all the cards in his/her hand have been played.

e. The losing player is the one who is stuck with the Oh-Oh! card in his/her hand when all possible plays have been completed.

APPENDICES

APPENDIX A
BILL OF MATERIALS

MATHEMATICAL BALANCE SCALE

ITEM		SOURCE	COST
10'	3/4" PVC pipe	Carter's Lumber (Beavercreek, Fairborn)	3.19
48"	3/4" wooden dowel (or 3/4" x 3/4" wood moulding)	Wickes Lumber (Beavercreek)	1.09
4	3/4" PVC 90° elbows	Carter's Lumber (Beavercreek, Fairborn)	1.20
3	3/4" PVC Tees	Carter's Lumber (Beavercreek, Fairborn)	1.35
1	Plastic cement (6 oz.)	Ballweg Hardware (Beavercreek)	2.69
1	4" #10 screw	Ballweg Hardware (Beavercreek)	.14
4	#10 nuts	Ballweg Hardware (Beavercreek)	.32
4	#10 flat washers	Ballweg Hardware (Beavercreek)	.24
1	Flexible soda straw	-----	-----
24	1/2" cup hooks	K-Mart (Many Dayton Locations)	1.36
2 ea	Number decals (1-12) [Optional]	Mendelson's Electronics (Downtown Dayton)	1.20
12	1/2 oz bass fishing weights	K-Mart (Many Dayton Locations)	1.80
1/2 pt	Paint (any color)	Odd Lots (Many Dayton Locations)	.99 =======
		TOTAL:	$15.57

MATHEMATICAL GO FISH

	ITEM	SOURCE	COST
FISH:			
18	Soup can lids and bottoms	------	-----
1/2 yd ea	Green, red, yellow, orange felt (72" wide)	Jo-Ann Fabrics (Many Dayton Locations)	10.98
1 yd	4-gauge clear vinyl fabric (54" wide)	Jo-Ann Fabrics (Many Dayton Locations)	1.98
1 sheet	Carbon paper	------	-----
1 sheet	6" x 10" cardboard/tagboard	------	-----
36 sheets	looseleaf paper OR tissue paper OR newspaper (9" x 5")	------	-----
	thread	------	-----
1	Grease pencil	Geyer's Office Supply (Fairborn)	.98
RODS:			
4	48" x 5/8" wooden dowels	Carter's Lumber (Beavercreek, Fairborn)	2.36
4	36" strings	------	-----
4	Hook magnets	Handyman (Fairborn)	1.79 =======
		TOTAL:	$18.09

THE GORGEOUS GEOBOARD

	ITEM	SOURCE		COST
GEOBOARD # 1				
3	1/4" x 48" wooden dowels	Carter's Lumber (Beavercreek, Fairborn)		.87
1	2' x 4' x 1/4" pegboard sheet	Furrow's Lumber (Kettering)		2.99
1	10 oz. Elmer's wood glue	Odd Lots (Many Dayton Locations)		.99
1 pkg.	Rubber bands (assorted colors/ sizes)	Geyer's Office Supplies (Fairborn)		.87 ======
			TOTAL:	$5.72
GEOBOARD #2				
3	1/4" x 48" wooden dowels	Carter's Lumber (Beavercreek, Fairborn)		.87
1	2' x 4' x 1/4" pegboard sheet	Furrow's Lumber (Kettering)		2.99
1	10 oz. Elmer's wood glue	Odd Lots (Many Dayton Locations)		.99
1 pkg.	Rubber bands (assorted colors/ sizes)	Geyer's Office Supplies (Fairborn)		.87 ======
			TOTAL:	$5.72
GEOBOARD #3				
3	1/4" x 48" wooden dowels	Carter's Lumber (Beavercreek, Fairborn)		.87
1	2' x 4' x 1/4" pegboard sheet	Furrow's Lumber (Kettering)		2.99
4 yds	Velcro hook and loop tape	Jo-Ann Fabrics (Many Dayton Locations)		5.76
1 ft	String (heavy)	------		-----
1	10 oz. Elmer's wood glue	Odd Lots (Many Dayton Locations)		.99 ======
			TOTAL:	$10.61

MAGIC VELCRO CARPET BOARD

	ITEM	SOURCE	COST
1	4' x 8' sheet of panel board	Furrow's Lumber (Kettering)	3.79
1	3' x 6' automobile carpet	Odd Lots (Many Dayton Locations)	5.99
12"	String (heavy)	------	-----
1 qt.	Carpet cement	Furrow's Lumber (Kettering)	4.97
1 pkg.	Tongue depressors/craft sticks	Jo-Ann Fabrics (Many Dayton Locations)	2.49
2	Cloth measuring tapes	Jo-Ann Fabrics (Many Dayton Locations)	.66
5	Buttons	Jo-Ann Fabrics (Many Dayton Locations)	.10
1 lb.	#4 flat washers	Mendelson's Electronics (Downtown Dayton)	1.25
			=======
		TOTAL:	$19.25

MEMORY CARDS/HOLDERS

	ITEM	SOURCE	COST
1 yd	Solid yellow vinyl fabric (54" wide)	Jo-Ann Fabrics (Many Dayton Locations)	6.99
1 yd	4-Gauge clear vinyl fabric (54" wide)	Jo-Ann Fabrics (Many Dayton Locations)	1.99
	Several sheets of newspaper	------	-----
	Thread	------	-----
1 pkg.	3" x 5" index cards (unlined)	Geyer's Office Supplies (Fairborn)	.85
1	Grease pencil	Geyer's Office Supplies (Fairborn)	.98 =======
		TOTAL:	$10.81

APPENDIX B
SUGGESTED READINGS

SUGGESTED READINGS

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CHAPTER IV

CONCLUSION AND RECOMMENDATIONS

Modern research in the areas of cognitive and learning development suggests that children from the ages of seven to eleven cogitate on a concrete operational level which is limited to physical reality. In other words, most children are limited to logically reasoning **only** about ideas which can be represented in the real world.

In light of these findings, many educators have attempted to incorporate manipulative teaching devices into their mathematics instruction. However, to the dismay of some teachers, the costs of the commercially-available manipulative aids exceed the normal classroom budget.

The purpose of this handbook is to offer math instructors a low cost, do-it-yourself alternative to store-bought manipulative products.

It is recommended that teachers integrate these manipulatives into the classroom mathematics curriculum for the purpose of providing concrete hands-on learning experiences for their elementary school-aged pupils.

Furthermore, it is suggested that teachers supplement their stocks of manipulative equipment through resourceful planning. For example, many of the projects contained in this handbook can be mass-produced with the assistance of

local PTO members, class parents, and junior- and senior-high school home economics classes and shop classes. Additionally, material costs can be substantially reduced by purchasing the construction materials at sale prices from dealers who stock the necessary items in large quantities.

The author of this handbook hopes that its use will introduce an increasing number of teachers to the advantages of instructing students in mathematics through a concrete link with the physical world which surrounds them.

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