MAIN LOBE RADIATION PATTERN AND ANTENNA COVERAGE
OF THE EARTH CAUSED BY A HOLE IN
THE SURFACE OF A SATELLITE'S
PARABOLIC DISH ANTENNA

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MAIN LOBE RADIATION PATTERN AND ANTENNA COVERAGE OF THE EARTH CAUSED BY A HOLE IN THE SURFACE OF A SATELLITE'S PARABOLIC DISH ANTENNA

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ABSTRACT

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The changes in the main lobe radiation pattern and the earth coverage from the impact of a single object striking and fully penetrating the center reflecting surface of a satellite's parabolic dish antenna are investigated. The antenna under consideration has a half-power beamwidth so as to approximately provide coverage to the earth within its main lobe. An object (such as a space particle) strikes the dish antenna surface and penetrates the reflector, leaving a single hole. This hole is characterized (for the purposes of this analysis) by a perfect circle centered symmetrically about the axis of the dish. The radiation pattern (and the corresponding directive gain pattern) of the antenna will change after being struck by the debris. Only first order changes in the main lobe directive gain are of interest (since the entire earth is within the main lobe). The size of the hole resulting in a 3 dB loss in directive gain towards Washington, DC is desired as well as the resulting change in the antenna's directive gain pattern and flux density coverage within the main lobe. Analysis of the hole size and corresponding changes is performed for various aperture distribution functions.
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I would also like to express my thanks to several people for their support without which I would not have been able to achieve this work. I would like to express my appreciation to my parents, Jim and Rene', for their support and advice on continuing my education. I thank Mr. and Mrs. Robert Adams for the use of a personal computer without which I would not have been able to complete the preparation of this text at home. Lastly, I would like to express my sincere thanks to my wife, Wendy, for her support, understanding, and patience during the preparation of this text and for her review of it.
PREFACE

I am currently a government-employed engineering analyst examining a variety of space and ground based dish antenna systems. A need has arisen to evaluate questions dealing with the operations of space-based parabolic dish antennas whose main reflecting surface is degraded. Such degradations could be caused by space particles or asteroid fragments on space based antennas. The coverage of earth provided by such a degraded antenna on a satellite is of interest. My research of the literature to answer questions on this topic revealed no information directly answering these questions. Therefore, this thesis represents an initial effort aimed at exploring this area to fulfill that need.
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### LIST OF SYMBOLS

- $(r, \theta, \phi)$: spherical coordinate system
- $(r', \theta', \phi')$: spherical coordinate of integration
- $(x, y, z)$: rectangular coordinate system
- $(x', y', z')$: rectangular coordinate of integration
- $\lambda$: wavelength
- $\text{dB}$: decibels
- $\text{eqn}$: equation
- $\exp\{x\}$: exponential ($e$ raised to the power of $x$)
- $\text{GHz}$: gigahertz (unit)
- $j = \sqrt{-1}$
- $k$: wavenumber, $2\pi/\lambda$
- $\ln$: natural logarithm (log base $e$)
- $\log$: logarithm (base 10)
- $m$: meter (unit)
- ref.: reference
- W: watts (unit)
CHAPTER I

INTRODUCTION

The problem of evaluating the degradation in earth coverage of a satellite's communication antenna due to a hole in the dish's surface is important and has many applications. Knowledge of how much of the aperture area of the dish can be lost while still maintaining adequate gain coverage is important in evaluating satellite link margins and operational coverage should a portion of the reflector surface be lost. Applications of this knowledge include cases where satellite antennas are struck by space debris or asteroids and space warfare.

The antenna under analysis is a 1.2 m diameter, front fed parabolic dish antenna mounted on a communications satellite orbiting the earth at an altitude of approximately 35786 km. The satellite transmitter power will be assumed to be 100 W (10 dB). The satellite revolves around the earth once every 24 hours and thus, seems stationary with respect to the earth. (Such an orbit is known as a geostationary orbit.) The antenna's reflector is made of a cloth-like material with a high concentration of copper embedded within it. Thus, the reflector is electrically equivalent to a solid surface but is physically susceptible to tears and holes. The antenna operates at a frequency of 1 GHz and is of such a design so that the earth is within the antenna's half power beamwidth. (This type of antenna is known as a global beam antenna). The reflector feed antenna is supported by four support arms which are connected to the edge of the dish. The blockage of the
reflector by the support arms and feed antenna will be considered small as compared to the area of the reflector. Figure 1 shows the satellite location. Figure 2 shows the satellite antenna.

It is desired to find the size of a hole located in the center of the reflector surface (made by a strike from an object in space) which can be made which will result in a half power loss as compared to the directive gain of the antenna in a specific direction before the hole occurs. The satellite antenna before the hole is made will be referred to as the normal dish and afterwards as the degraded dish. It is further desired to find the change in the directivity of the antenna after degradation in directions corresponding to various points on the earth's surface. Of specific interest is the size of the hole required to cause a half-power (3 dB) loss in directive gain towards Washington, DC. It is also desired to evaluate these changes using different aperture distribution patterns on the reflector to evaluate those distributions which make the reflector's directive characteristics less vulnerable to such strikes. Knowledge of a particular reflector illumination which will allow a larger hole in the dish with less changes in the earth coverage would be important in minimizing the antenna's vulnerability to such strikes. The resulting knowledge will characterize the changes in directive gain and flux density at specific locations on the earth's surface before and after the strike occurs on the antenna.

The results of the analysis can be used in evaluating the vulnerability of communication services from geosynchronous satellites employing global beam antennas to interruptions caused by reflector antenna surface strikes. An increasing number of space debris orbiting the earth increases the likelihood of such strikes. Only five percent of the 7000 space objects which are currently tracked represent satellites. Many thousands of space objects exist which cannot be tracked because of their small size [ref. 1]. Building
Figure 1.
Satellite Position
Figure 2.
Satellite Antenna
survivability against such degradation into the antenna design could potentially be very attractive to customers requiring fully reliable services in the future, especially since the number of space debris is likely to increase (with an increasing likelihood of debris strikes in spacecraft).
CHAPTER II
APERTURE INTEGRATION AND DIRECTIVE GAIN

An example of the problem to be solved is as follows. Assume the satellite antenna has a directivity of 20 dB. At \( \theta = \theta_o \), the relative pattern intensity is -2 dB. Therefore, the directive gain at \( \theta = \theta_o \) is 20 dB - 2 dB = 18 dB. We want to find the size of the circular hole required to reduce the directive gain at \( \theta = \theta_o \) to 15 dB. We want to find the difference in hole sizes required to do this using various aperture distributions. This will indicate if some aperture distributions are less vulnerable (a larger hole size required to make a 3 dB loss) than others. As an additional result, we want to plot the resulting earth contours depicting the directive gain on earth from the satellite to the point corresponding to \( \theta = \theta_o \). The information required to answer these questions are:

- Directivity of antenna
- Directive gain of antenna
- The relative pattern of the main lobe of the antenna.

This information can be found using the principles of Aperture Integration.

Aperture Integration

Aperture integration is the projection of the surface current density (on the surface of the reflector antenna) onto the aperture plane (the plane creating by the intersection of the projection of the dish and the x-y plane passing through the focus (see Figure 3) [ref. 2].
Figure 3.
Coordinate System
Note: \( p \) is the sum of the distance from the focal point to the reflector surface to the aperture plane.

Figure 4.
Dish Antenna Geometry
the radius vector within the system can then be expressed as

\[ r' = r'\cos(\phi')x + r'\sin(\phi')y \tag{7} \]

The vector expressing the location within the coordinate system is

\[ r = \sin(\theta)\cos(\phi)x + \sin(\theta)\cos(\phi)y + \cos(\theta)z \tag{8} \]

Forming the dot product of the two vectors \( r \) and \( r' \),

\[ r \cdot r' = r'\sin(\theta)\cos(\phi - \phi') \tag{9} \]

Thus,

\[ P = \int_{0}^{\pi} \int_{0}^{2\pi} E_a(r') \exp\{jkr\sin(\theta)\cos(\phi - \phi')\} \, d\phi' \, r' \, dr' \tag{10} \]

Eqn.(10) is the radiation integral describing the power pattern of the aperture distribution \( E_a(r') \) across the reflector through the Fourier transform of the incident wave on the reflector [ref. 6]. It is important to note eqn.(10) assumes a lossless system, implying the surface current density on the reflector (caused by the incident wave from the feed) is directly projected onto the aperture plane. It is also important to note once again eqn.(10) assumes a uniform phase across the aperture amplitude distribution \( E_a(r') \).

The radiation integral of eqn.(10) is a formidable problem to solve. Typically, sampling of \( r' \) is approximately 0.1 \( \lambda \). Additionally, increments of \( \theta \) and \( \phi \) must be small
with respect to beamwidth so as to adequately define the pattern. For each unique θ and φ at which the integral is to be solved, the integration must be repeated.

To a first order approximation, the radiation integral can be solved using Bessel functions [ref. 7]. The relative pattern of P from eqn.(10) may be described as [ref. 8]

\[
I(\theta, \phi) = \int_0^\infty \int_0^{2\pi} E_\alpha(r') \exp\{jk'r\sin(\theta)\cos(\phi - \phi')\} \, d\phi' \, r' \, dr'
\] (11)

Since the reflector is symmetric with respect to the z-axis (axially symmetric), the integration may be performed over \( \phi' \). The Bessel function identity [ref. 9]

\[
J_n(x) = j^{-n/2} \int_0^{2\pi} \exp\{jx\cos(\alpha)\} \exp\{jn\phi\} \, d\alpha
\] (12)

may be used, where

- \( J_n(x) \) is the Bessel function of the first kind and order \( n \),
- \( x = kr'\sin(\theta) \)
- \( \alpha = (\phi - \phi') \)
- \( d\alpha = d\phi' \)

Making the substitutions,

\[
J_0(kr'\sin(\theta)) = 1/2\pi \int_0^{2\pi} \exp\{jk'r\sin(\theta)\cos(\phi - \phi')\} \, d\phi
\] (13)

The relative pattern of the radiation integral of eqn.(11) may now be expressed as
\[ f(\theta) = 2\pi \int_0^r E_a(r') J_0(kr'\sin(\theta)) r' \, dr' \]  

(14)

where the integration over \( \phi' \) has been evaluated through the Bessel function substitution. This integral is much less numerically intensive as compared to the radiation integral yet provides good results [ref. 10]. Integration is now done only over the radius of the aperture since, by the methods of aperture integration, no currents exist outside the region of the projected aperture [ref. 11]. Integration over \( \phi' \) is no longer necessary since the antenna is symmetric with respect to the z-axis.

An expression which allows the representation of the unnormalized radiation pattern of the reflector for different types of aperture distributions has now been found. It is now desired to find the pattern of the antenna which has had a piece of its reflector surface removed. For this specific problem, a hole has been made which is axially symmetric. The hole is assumed to be circular. While a hole caused by a small asteroid is not expected to be perfectly circular, the hole will be represented as circular with a mean radius of the hole caused by the asteroid. This is depicted in Figure 5. This characterization of the hole will be sufficient for our analysis. Any jagged edges on the hole will be assumed to be small. Of importance is the fact that the circular hole model is of the same area as that of the real hole. Secondly, the analysis is concerned only with the resulting main lobe of the pattern (since the earth falls within the main lobe). Other more rigorous techniques, such as the Geometrical Theory of Diffraction, could be used to find the pattern in regions away from the main lobe. However, since the only interest is the effect in earth coverage, GTD is not necessary. [ref. 12].

Superposition will be used to calculate the pattern of the antenna with a hole. The basic theory behind aperture integration is that a group of point sources are defined on
Figure 5.
Circular Hole Approximation
\[
\begin{align*}
\int_{h} \int E_a(r') J_0(kr' \sin(\theta)) r' \, dr' & = \\
\int_{0}^{h} E_a(r') J_0(kr' \sin(\theta)) r' \, dr' - \\
\int_{0}^{h} E_a(r') J_0(kr' \sin(\theta)) r' \, dr' & \quad (16)
\end{align*}
\]

This represents the pattern from aperture integration with a hole of radius \( h \) (the characteristics of which have been previously discussed). By using aperture integration we assume all currents outside the aperture are zero.

**Directive Gain**

It is now desired to define directivity and directive gain of the antenna. For the purposes of this analysis, the only change in the antenna to be considered is that of the removal of a circular, axially symmetric hole from the reflector surface.

Directive gain is the measurement of how well an antenna concentrates its energy in a particular direction expressed in angular coordinates \((\theta, \phi)\). It compares how well the antenna concentrates its energy in a particular direction compared to a standard, usually an isotropic source. Directivity is the maximum directive gain [ref. 16]. Directive gain is expressed as

\[
D(\theta, \phi) = \frac{4\pi}{\Omega_a} \left| F(\theta, \phi) \right|^2
\]

where
\[
\Omega_a = 0^\pi \int_{0}^{2\pi} \left| F(\theta, \phi) \right|^2 \sin(\theta) \, d\theta \, d\phi
\]

This expression relates the measure of how concentrated the radiation pattern \( F(\theta, \phi) \) is over a sphere.

In the case of an aperture antenna, the directivity can be expressed as a function of frequency and the effective area of the antenna [ref. 17]. In the case of a reflector antenna,
the effective area is equal to its physical area, or the area projected upon the aperture plane. For the circular parabolic reflector under analysis,

\[
D = \frac{4\pi}{\lambda^2} \pi a^2 \epsilon_t
\]  

(18)

where \( a = \) radius of the reflector (aperture)

\( \epsilon_t = \) efficiency of the taper (distribution)

The efficiency of the antenna is usually taken into consideration when calculating its gain. However, since this analysis will be examining cases of aperture distributions other than the uniform case, the efficiency of the taper (\( \epsilon_t \)) must be considered since \( \epsilon_t \) changes with the type of distribution [ref. 18].

The directive gain of the antenna is expressed as the antenna directivity multiplied by the normalized radiation intensity,

\[
D(\theta,\phi) = D F(\theta,\phi)
\]  

(19)

Since the directivity is partially a function of aperture area, it will be assumed the directivity of a dish with a hole can be calculated by removing the area of the hole,

\[
D = \epsilon_t \frac{4\pi}{\lambda^2} (\pi a^2 - \pi h^2)
\]  

(20)

This assumption is based strictly on the aperture area and is valid for the case of a uniform distribution. The taper efficiency term \( \epsilon_t \) will not change since the taper does not change over the radius of the dish or the hole. The loss caused by the hole will be accounted for
by subtracting its area.

To this point, the following information relevant to the problem has been found:

Normalized radiation pattern of the normal dish:

\[ \int_{r'} E_a(r')I_0(kr'sin(\theta))r' dr' \]  \hspace{1cm} (21)

Directivity of the normal dish:

\[ D = \varepsilon_i \frac{4\pi}{\lambda^2} (\pi a^2) \]  \hspace{1cm} (22)

Radiation pattern of the degraded dish:

\[ \int_{r'} E_a(r')I_0(kr'sin(\theta))r' dr' \]  \hspace{1cm} (23)

Directivity of the degraded dish:

\[ D = \varepsilon_i \frac{4\pi}{\lambda^2} (\pi a^2 - \pi h^2) \]  \hspace{1cm} (24)

Aperture Illumination Functions

To this point in the analysis the aperture distribution function used in calculation of the radiation integral of eqn.(10) has been expressed as \( E_a(r') \). Several classes of aperture distributions have been considered in the literature [ref. 10]. For the purposes of this
The Aperture Amplitude Distribution is given by

\[ Ea(r) = C + (1 - C) \left( 1 - \frac{r}{a} \right)^n \]

where
- \( C \) is the level of illumination present at the edge of the aperture,
- \( r \) is the radial distance of the point on the aperture,
- \( 0 < r < a \),
- \( a \) is the radius of the aperture,
- \( n \) is the taper of the distribution

<table>
<thead>
<tr>
<th>( n )</th>
<th>Edge Illumination</th>
<th>( C )</th>
<th>( C ) (dB)</th>
<th>( \epsilon_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000</td>
<td>0</td>
<td>0</td>
<td>1.000</td>
</tr>
<tr>
<td>1</td>
<td>0.000</td>
<td>0</td>
<td>0</td>
<td>0.750</td>
</tr>
<tr>
<td>2</td>
<td>0.000</td>
<td>0</td>
<td>0</td>
<td>0.550</td>
</tr>
<tr>
<td>1</td>
<td>0.100</td>
<td>-10</td>
<td>0.917</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.100</td>
<td>-10</td>
<td>0.877</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.316</td>
<td>-20</td>
<td>0.817</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.316</td>
<td>-20</td>
<td>0.690</td>
<td></td>
</tr>
</tbody>
</table>

The following illustration depicts the values \( n \) and \( C \) in relation to the aperture amplitude distribution.

Figure 6.
Aperture Distributions
is chosen \((n = 0, C = 0)\). The characteristics of this distribution are listed in Figure 6.

From eqn.(22), the directivity of the normal antenna is (for this distribution)

\[
D = 1.000 \frac{4\pi}{(\pi 0.6^2)} = 22.0 \text{ dB}
\]

From eqn.(21), the normalized relative intensity of the radiation pattern of the normal antenna is

\[
F(\theta) = \frac{\int_0^{0.6} 1.000 J_0(20.943 r' \sin(\theta)) r' \, dr'}{\int_0^{0.6} 1.000 J_0(20.943 r' \sin(0)) r' \, dr'}
\]

From eqn.(19), the directive gain associated with Washington, DC is

\[
D(7.40) = 22.0 - 10\log(F(7.40)) = 20.4 \text{ dB}
\]

It is now desired to find the size of the hole to be made in the dish which will make \(D(7.40)\) decrease by 3 dB (17.4 dB). It is important to realize that not only does the directive gain of the antenna decrease with decreasing aperture area, but the normalized relative intensity also changes since the integration is performed over a different size of aperture. To solve this problem, a computer program was developed to assist in the numerical solution of the problem. In general, we require a unique solution of \(h\) such that
Specific to this example, it is required

\[
10 \log \frac{4\pi}{0.3^2} \left( \pi 0.6^2 \right) \frac{\int_{0}^{0.6} E_a(r')J_0(20.943r'sin(\theta))r' \, dr'}{\int_{0}^{0.6} E_a(r')J_0(20.943r'sin(0))r' \, dr'}
\]

\[
- 10 \log \frac{4\pi}{0.3^2} \left( \pi 0.6^2 \right) \frac{\int_{0}^{b} E_a(r')J_0(20.943r'sin(\theta))r' \, dr'}{\int_{0}^{b} E_a(r')J_0(20.943r'sin(0))r' \, dr'}
\]

\[= 3 \text{ dB}. \]

A computer program was written to assist in the numerical solution (through error comparison) of the problem. The listing of the program is contained in Appendix B.

For this specific example, it is found a hole of radius 0.3821 m will degrade the directive gain of the antenna towards Washington, DC by 3 dB. Using the same method, the results using the aperture distributions of Figure 6 are listed in Table 1. A graphical comparison of the final results will be presented in Chapter V.
<table>
<thead>
<tr>
<th>n</th>
<th>C</th>
<th>$\epsilon t$</th>
<th>Hole radius h (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000</td>
<td>1.000</td>
<td>0.3821</td>
</tr>
<tr>
<td>1</td>
<td>0.000</td>
<td>0.750</td>
<td>0.3904</td>
</tr>
<tr>
<td>2</td>
<td>0.000</td>
<td>0.550</td>
<td>0.3906</td>
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<td>0.817</td>
<td>0.3898</td>
</tr>
<tr>
<td>2</td>
<td>0.100</td>
<td>0.690</td>
<td>0.3900</td>
</tr>
<tr>
<td>1</td>
<td>0.316</td>
<td>0.917</td>
<td>0.3884</td>
</tr>
<tr>
<td>2</td>
<td>0.316</td>
<td>0.877</td>
<td>0.3884</td>
</tr>
</tbody>
</table>

Table 1.
Hole Size Required For Aperture Distributions
CHAPTER III
ASSOCIATION OF ANTENNA PATTERN ANGLE WITH EARTH COORDINATES

The problem of locating the geographic position (latitude and longitude) corresponding to an angular coordinate $\theta, \phi$ within an antenna pattern is an involved one, primarily consisting of derivation and manipulation of several spherical trigonometric relationships [ref. 21]. Additionally, the curvature of the earth with respect to the location of the satellite in space introduces another factor in projecting the antenna pattern onto the earth. Solution of such a problem is numerically intensive when the directive gain plots for several different intensities at several locations is desired [ref. 22].

Figure 7 shows the geometrical problem of a satellite orbiting the earth and the parameters of interest. It is desired to find the angle $\theta$ within the antenna pattern of the satellite corresponding to an earth station given by its latitude and longitude coordinates. Since the antenna's pattern is circularly symmetric, its directive gain pattern is a function of $\theta$ only and does not vary with $\phi$. The basic spherical trigonometric relationships required for the analysis are contained in Appendix A. The great majority of the equations presented in this chapter are contained in [ref. 23].

It is helpful to represent specific levels of directive gain and flux density as the locus of points on earth [ref. 24]. Such contour maps indicate the maximum level of directive gain or flux density and contours of 1, 2, and 3 dB below the maximum. Representation of this information can be made on several different types of earth maps. However, the hodocentric
Figure 7. Satellite - Earth Geometry
projection is typically used [ref. 25]. The satellite antenna pattern can be directly projected onto this type of map. This is helpful since no skewing or elongation of the true antenna pattern is necessary. The hodocentric projection represents the earth as seen by the satellite [ref. 26].

Several parameters are required for the calculation of the directive gain and flux density contours. These include the relative latitude of point D with respect to point P (β), the relative longitude of point D with respect to arc PG (Δℓ), and the elevation angle between points P and G. First, let an intermediate angle β₀ be defined (using [ref. 27] as

$$\beta_0 = \cos^{-1} \left[ \sin(\alpha_1)\sin(\alpha_2) + \cos(\alpha_1)\cos(\alpha_2)\cos(\ell - \ell_1) \right]$$

(27)

The angle between points G and D is then found as

$$g = \cos^{-1} \left[ \sin(\alpha_2)\sin(\alpha) + \cos(\alpha_2)\cos(\alpha)\cos(\ell - \ell_2) \right]$$

(28)

Using these results, the relative latitude of point D with respect to point P is

$$\beta = \cos^{-1} \left[ \cos(\beta_0)\cos(g) + \cos(\beta_0)\sin(g)\cos(w_3) \right]$$

(29)

where w₃ represents the distinction between relative longitudinal locations of P, G, and D.
From [ref. 28],

\[ w_3 = \cos^{-1}\left(\frac{\cos(w_1 + w_2)}{\cos(w_1 - w_2)}\right) \quad \text{or} \quad \ell_1 \leq \ell_2 \leq \ell \]

\[ w_3 = \cos^{-1}\left(\frac{\cos(w_1 - w_2)}{\cos(w_1 - w_2)}\right) \quad \text{or} \quad \ell_2 > \ell_1 \text{ and } \ell_2 > \ell \]

\[ \ell_2 < \ell_1 \text{ and } \ell_2 < \ell \]

where

\[ w_1 = \cos^{-1}\left(\frac{\sin(\alpha) - \sin(\alpha_2)\cos(\beta_o)}{\cos(\alpha_2)\sin(\beta_o)}\right) \]

\[ w_2 = \cos^{-1}\left(\frac{\sin(\alpha) - \sin(\alpha_2)\cos(g)}{\cos(\alpha_2)\sin(g)}\right) \]

Let \( \Delta \ell \) be the relative longitude of point D with respect to the arc PG. From Figures 7 and 8 and [ref. 29],

\[ \Delta \ell = \cos^{-1}\left(\frac{\cos(g) - \cos(\beta_o)\cos(\beta)}{\sin(\beta_o)\sin(\beta)}\right) \]

Similarly, the angle \( \Psi \) representing the elevation of the satellite with respect to point D is required. Figures 7 and 8 show the geometry. From [ref. 30],

\[ \Psi = \sin^{-1}\left(\frac{R_p\sin(\beta)}{(4R_p d + r^2)\sin^2(\beta/2) + d^2)}\right) \]

Figure 8 shows the geometrical relationship between points P, D, G, and the angles \( \beta, \beta_o, g, w_1, w_2, \) and \( w_3. \)
Figure 8.
Earth Points Geometry
The angle $\theta$, the angle within the antenna pattern corresponding to the location of point D, can be calculated as

$$
\theta = \sin^{-1}\left\{ \frac{\cos(\Psi_0)(d/R) + 2\sin^2(\beta/2) + \tan(\Psi_0)\sin(\beta)\cos(\Delta \ell)}{(4(Rd + r^2)\sin^2(\beta/2) + d^2)} \right\}
$$

(33)

The above equation allows the particular angle $\theta$ within the antenna pattern to be found as a function of the points P, D, and G.

A computer program was written to calculate the angle $\theta$ corresponding to various locations on the earth. The program is contained in Appendix B. Using the program, the angle $\theta$ corresponding to Washington, DC for the specific case being considered was found to be 7.40 degrees.
CHAPTER IV
APPLICATION OF SRIVANSON’S METHOD

Srivanson’s method [ref. 31] which allows the determination of a locus of earth locations toward which a specific flux density (up to 4 dB below maximum) is present for the specific case of a satellite at geosynchronous orbit. Instead of performing the usual process of finding an angle \( \theta \) for every point on earth and associating a value of directive gain based on the antenna pattern, the points on earth associated with a particular flux density can be calculated. This information is required by earth terminal operators who need to know the flux density present at their antenna. For a 100 W transmitter the flux density present at an earth terminal is given by [ref. 32]

\[
p = 10 \log \left\{ 100 \frac{D(\theta)}{4\pi R_D} \right\}
\]  

(34)

where \( R_D \) is the range between the satellite and earth point corresponding to \( \theta \). Referring to Figure 7, the difference in distance of line AD and AP introduce an additional loss of gain.

Defining the relative gain as

\[
RG = 10 \log \left\{ \frac{R_P}{R_D} \frac{D(0,0) F(\theta,\phi)}{D(0,0) F(0,0)} \right\}
\]  

(35)

A value \( S \) is now defined which represents a contour ring of the locus of earth points along which the flux density is constant [ref. 33]. This value can be expressed as
\[ S = RG - 10 \log D(0,0) \]  

(36)

To utilize Srivanson's method, the main lobe of the antenna pattern must be modeled as a sinc function [ref. 34],

\[ F(\theta, \phi) = \left( \frac{\sin(k(\phi)\theta)}{k(\phi)\theta} \right)^2 \]  

(37)

where \( k(\phi) \) is a constant term denoting any ellipticity of the pattern. In the case of the antenna under consideration, the pattern is circular symmetric, so \( k(\phi) = k \). Eqn.(37) above is defined

\[ F(\theta, \phi) = \left( \frac{\sin(k\theta)}{k\theta} \right)^2 = 0.5 \]  

(38)

at the half power point (3 db down). From tables [ref. 35],

\[ \left( \frac{\sin(k\theta)}{k\theta} \right)^2 = 0.5 \text{ for } k\theta = 1.3916 \]  

(39)

Therefore,

\[ k = \frac{1.3916}{\theta_{3dB}} \]  

(40)

where \( \theta_{3dB} \) is expressed in radians.

Using Srivanson's method, the calculations of \( \theta \) for specific levels of relative gain can be found via the solution to a quadratic equation. This method takes advantage of several approximations possible since the satellite is in a geostationary orbit.

Referring to eqn.(37), the series expansion for the sinc function is (from [ref. 36]),

\[ \frac{\sin(k\theta)}{k\theta} = 1 + \frac{(k\theta)^2}{3!} - \frac{(k\theta)^4}{5!} + \ldots \]
The terms after the second term of this expansion is less than 0.1 of the second term for \( k\theta > 1.414 \) and will be considered adequate for a first order solution (since \( k\theta = 1.3916 < 1.414 \)). Using this approximation [ref. 37],

\[
\frac{\sin(k\theta)}{k\theta} = 1 - \frac{(k\theta)^2}{3!}
\]

and

\[
F(\theta) = 20 \log \left\{ 1 - \frac{k^2\theta^2}{6} \right\}
\]

\[
= \frac{20}{2.3026} \ln \left\{ 1 - \frac{k^2\theta^2}{6} \right\}
\]

\[
= -\frac{20}{2.3026} \frac{1}{6} k^2\theta^2
\]

\[
= -1.45k^2\theta^2 \tag{41}
\]

The relationship between the distances to the satellite subpoint \( P \) and the earth station point \( D \) is given by [ref. 38]

\[
\frac{R_P}{R_D} = \frac{R_P/6378}{(1 + R_P/6378)\cos(\Psi) - 1 - (1 + R_P/6378)^2\sin(\Psi)} \tag{42}
\]

Since the maximum value \( \Psi \) can take to relate to a point on the earth’s surface is 8.7 degrees [ref. 39], and since \( \sin \theta \sim \theta \) for \( \theta < 10^\circ \),
\[
\cos(\Psi) = \sqrt{1 - \sin^2(\Psi)}
\]

\[= 1 - 0.5\sin^2(\Psi) \quad (43)\]

Since the term \((1 + \frac{R_p}{6378})\sin^2(\Psi)\) in eqn.(42) is always less than 1 [ref. 43],

\[
\frac{R_p}{R_d} = \frac{\frac{R_p}{6378}}{1 + \frac{R_p}{6378} - 1 + (0.5 + \frac{R_p}{3189})\sin^2(\Psi)\left(\frac{R_p}{6378}\right)}
\]

\[= \frac{1}{1 + (0.5 + \frac{R_p}{3189})\sin^2(\Psi)} \quad (44)\]

Therefore,

\[
\ln \frac{R_p}{R_d} = -\ln \left(1 + (0.5 + \frac{R_p}{3189})\sin^2(\Psi)\right) \quad (45)\]

Recalling the relative gain RG eqn.(35) and substituting eqns.(41) and (44),

\[
RG = -1.45k^2\theta^2 + 10\log\left(\frac{R_p}{R_d}\right)^2 \quad (46)\]

Recalling \(\Psi\) is always less than 10 degrees [ref. 41], eqn.(45) can be expressed as

\[
\ln \left(\frac{R_p}{R_d}\right)^2 = -28.71\Psi^2 \quad (47)\]

Substituting eqn.(47) into eqn.(36) and solving for \(\Psi\),

\[
\Psi^2 = \frac{-1.45k^2\theta^2 - S}{28.71} \quad (48)\]
Thus for a desired level of $S$ (flux density below the maximum), the corresponding $\theta$ can be solved [ref. 42].

A computer program was written to numerically model the degraded dish antenna and find the resulting flux density levels. The program is included in Appendix B. The resulting patterns differ from those found by projecting the antenna’s directive gain pattern onto the earth since Srivanson’s method takes into account slant range variations and solves for flux density. This information is valuable to satellite earth stations since the operators are interested in the level of flux density present at their antenna. The resulting flux density contour plots are presented in Chapter V for the aperture distributions shown in Figure 6 of Chapter II.
CHAPTER V
RESULTS AND FINDINGS

This section consolidates the results of the analysis performed by the methods presented in the preceding chapters. The results are presented concisely in this chapter since comparison between aperture distribution cases is much easier.

The results of the analysis answer the questions initially asked, namely the effects on the main lobe directive gain pattern and corresponding earth coverage contours as a result of a hole introduced into the center of the satellite antenna’s reflecting surface. In addition, characteristics of the aperture distribution have been examined to determine if the net effect of the hole can be minimized via a wise choice of distribution.

Size of Hole

The size of the hole corresponding to a 3 dB loss in the angular direction of Washington, DC (7.40 degrees) was found to vary only slightly depending upon the taper and edge level of the antenna aperture distribution. The hole radius required is on the order of 0.38 meters, ranging from 0.3821 to 0.3906 m. The results of the required hole sizes are listed in Table 2. It is interesting to note the hole size remains relatively constant (to within 0.005 m) as the aperture distribution is varied. Upon initial examination of this result it seems incorrect. Since the concentration of the aperture amplitude distribution is more towards the center of the aperture plane (in most distribution cases) it would seem a smaller
hole would be required to remove an equivalent portion of the distribution as compared to the uniform distribution case. However, since the taper efficiency of the distribution must also be considered in the analysis a net cancellation occurs, thus requiring a hole size relatively equal across all cases considered.

Changes in Directive Gain

Figures 9-14 show a comparison of the changes in the directive gain and flux density levels for the normal and degraded dish for the uniform distribution. Figures 9 and 10 show the directive gain pattern of the normal and degraded antennas (respectively) with respect to the angle $\theta$. As can be seen in Figure 9, the normal dish provides coverage of the entire earth within 3 dB of maximum. (As stated in Chapter 4, the earth subtends a half-angle from geostationary orbit of 8.7 degrees.) Figure 10 shows the effects of the degradation of the dish by introducing a hole of radius 0.3821 m. In the case of the normal dish, the directive gain towards Washington, DC was 20.4 dB. The hole makes the directive gain towards Washington, DC 3 dB less, or 17.4 dB. However, the introduction of the hole has drastically changed the pattern. The directivity of the antenna is now 18.1 dB, almost 4 dB less than the normal case. However, the degraded antenna now provides coverage of the earth within 1 dB. Figures 11 and 12 show the directive gain contours plotted on a generic hodocentric projection of the earth’s surface. These two figures, taken with Figures 9 and 10, respectively, indicate the fact that, while the directive gain towards Washington, DC ($\theta = 7.40$ degrees) has been degraded by 3 dB, a larger degradation actually occurs for angles less than 7.40 degrees. Almost 4 dB in directive gain has been lost at $\theta = 0$. The directive gain pattern has, in essence, been spread out and lowered in intensity.
If what has been done to the dish is pondered and the results described above kept in mind, they make sense. A 1.2 meter diameter dish antenna has had a 0.7641 m diameter hole made in it. This equates to a 40 percent loss in the aperture area. For reflector antennas, the directivity is a function of aperture area. Degradation of the antenna in this manner has comparatively made the operational system a smaller diameter dish antenna. (For the same aperture distribution, smaller dishes have lower directivity and wider beamwidths than larger dishes.) The results produced via application of aperture integration agree with these well known intuitive relationships of basic antenna theory.

**Changes in Flux Density**

The results of the degradation of the dish on the flux density present at earth station is shown in Table 3. While the directive gain patterns are important to satellite designers in calculating link characteristics, the flux density contours are important to earth terminal designers in choosing earth station antenna sizes to meet the appropriate specifications to ensure a successful communications link [ref. 43]. Figures 13 and 14 present the contour levels of flux density present at the earth station based upon the normal and degraded dish, respectively. A 100 watt transmitter is assumed on the spacecraft. Since both the directive gain and range between the satellite and earth station are considered in the calculation of flux density the plots in Figures 13 and 14 are not the same as Figures 11 and 12. The angles within the antenna pattern corresponding to a 1, 2, and 3 dBW/m down in flux density changes only by approximately 0.4 - 0.8 degrees for the normal and degraded cases, respectively. The degradation did not result in a greatly wider beamwidth with lower intensity as in Figures 11 and 12 since slant range differences were included. However, due to degradation, the flux density at $\theta = 0$ was reduced by approximately 4 dBW/m in all cases.
The Figures listed below are for the distributions listed:

<table>
<thead>
<tr>
<th>Figures</th>
<th>Taper level n</th>
<th>Edge Level C</th>
</tr>
</thead>
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<tr>
<td>15-20</td>
<td>1</td>
<td>0.000</td>
</tr>
<tr>
<td>21-26</td>
<td>2</td>
<td>0.000</td>
</tr>
<tr>
<td>27-32</td>
<td>1</td>
<td>0.100</td>
</tr>
<tr>
<td>33-38</td>
<td>2</td>
<td>0.100</td>
</tr>
<tr>
<td>39-44</td>
<td>1</td>
<td>0.316</td>
</tr>
<tr>
<td>45-50</td>
<td>2</td>
<td>0.316</td>
</tr>
</tbody>
</table>

**Observations**

Examination performed of the individual aperture distribution cases (as was done in the previous paragraphs) yield the same conclusion: given a specific parameter which sets the directive gain level in the direction of Washington, DC, the hole size is not significantly altered based on the choice of distribution. This indicates, for the case studied in this analysis, the size of the hole radius can not be made larger than approximately 0.38 m for a 3 dB loss. This indicates a wiser choice of aperture distribution (based on the antenna feed pattern) cannot be made which will allow a larger hole in the antenna (based on the distributions considered in this analysis). Unfortunately, due to the nature of the problem, no comparison of the theoretical results was possible with actual data.

**Limitations of the Method**

The specific case examined here points to some limitations of the methods used. First, the focus of the analysis was on changes in the main lobe of the satellite antenna pattern (since the earth subtends an angle less than the 3 dB beamwidth of the antenna). The radiation integral in its Bessel function approximation gives good results for this area of the pattern. Where information about more of the pattern is required, GTD analysis would be necessary. Second, the definition of the hole in the dish being circularly symmetric
allowed the Bessel function approximation to be made. If the hole was not circularly symmetric or located off-axis, the radiation integral of eqn.(10) in Chapter II would need to be solved, requiring much more computer storage capacity and time. Third, the application of Srivanson's method is valid only for geosynchronous satellites. Satellites in other than this unique orbit would require much more intense geometrical calculations to formulate the flux density contours. Additionally, the directivity calculations of eqn.(20) are based on the assumption the area of the hole may be subtracted directly from the normal aperture area. This assumption is strictly based on the area of a uniformly illuminated circular aperture. For the hole sizes considered in this analysis, the assumption is valid. The methods used in ref.[46] (which take into account the tapered illumination of the aperture) must be used for larger hole sizes and for a more exact calculation of non-uniform aperture distributions.
<table>
<thead>
<tr>
<th>n</th>
<th>C</th>
<th>εr</th>
<th>D(0), Normal Dish (dBi)</th>
<th>D(0), Degraded Dish (dBi)</th>
<th>Hole Radius (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000</td>
<td>1.000</td>
<td>22.0</td>
<td>18.1</td>
<td>0.3821</td>
</tr>
<tr>
<td>1</td>
<td>0.000</td>
<td>0.750</td>
<td>20.7</td>
<td>17.0</td>
<td>0.3904</td>
</tr>
<tr>
<td>2</td>
<td>0.000</td>
<td>0.550</td>
<td>19.4</td>
<td>15.7</td>
<td>0.3906</td>
</tr>
<tr>
<td>0</td>
<td>0.100</td>
<td>0.817</td>
<td>21.1</td>
<td>17.4</td>
<td>0.3898</td>
</tr>
<tr>
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<td>21.6</td>
<td>17.8</td>
<td>0.3884</td>
</tr>
<tr>
<td>1</td>
<td>0.316</td>
<td>0.877</td>
<td>21.4</td>
<td>17.6</td>
<td>0.3884</td>
</tr>
</tbody>
</table>

Table 2.
Hole Size and Directive Gain Characteristics For Aperture Distributions
<table>
<thead>
<tr>
<th>n</th>
<th>C</th>
<th>( \epsilon t )</th>
<th>( p(0) ), Normal Dish (dBW/m)</th>
<th>( p(0) ), Degraded Dish (dBW/m)</th>
<th>Hole Radius (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000</td>
<td>1.000</td>
<td>-44.6</td>
<td>-48.2</td>
<td>0.3821</td>
</tr>
<tr>
<td>1</td>
<td>0.000</td>
<td>0.750</td>
<td>-45.9</td>
<td>-49.6</td>
<td>0.3904</td>
</tr>
<tr>
<td>2</td>
<td>0.000</td>
<td>0.550</td>
<td>-47.2</td>
<td>-50.9</td>
<td>0.3906</td>
</tr>
<tr>
<td>0</td>
<td>0.100</td>
<td>0.817</td>
<td>-45.5</td>
<td>-49.2</td>
<td>0.3898</td>
</tr>
<tr>
<td>1</td>
<td>0.100</td>
<td>0.690</td>
<td>-46.2</td>
<td>-50.0</td>
<td>0.3900</td>
</tr>
<tr>
<td>0</td>
<td>0.316</td>
<td>0.917</td>
<td>-45.0</td>
<td>-48.8</td>
<td>0.3884</td>
</tr>
<tr>
<td>1</td>
<td>0.316</td>
<td>0.877</td>
<td>-45.2</td>
<td>-49.0</td>
<td>0.3884</td>
</tr>
</tbody>
</table>

Table 3.
Flux Density of Normal and Degraded Dish Antenna
Figure 9.
Normalized Radiation Intensity Pattern (Normal Dish, n=0, C=0)

Figure 10.
Normalized Radiation Intensity Pattern (Degraded Dish, n=0, C=0)
Figure 11.
Directive Gain Contour Map (Normal Dish, n=0, C=0)
Figure 12.
Directive Gain Contour Map (Degraded Dish, n=0, C=0)
Figure 13.
Flux Density Contour Map (Normal Dish, n=0, C=0)
Figure 14.
Flux Density Contour Map (Degraded Dish, n=0, C=0)
Figure 15.
Normalized Radiation Intensity Pattern (Normal Dish, n=1, C=0)

Figure 16.
Normalized Radiation Intensity Pattern (Degraded Dish, n=2, C=0)
Figure 17.
Directive Gain Contour Map (Normal Dish, n=1, C=0)
Figure 18.
Directive Gain Contour Map (Degraded Dish, n=1, C=0)
Figure 19.
Flux Density Contour Map (Normal Dish, $n=1$, $C=0$)
Figure 20.
Flux Density Contour Map (Degraded Dish, n=1, C=0)
Figure 21.
Normalized Radiation Intensity Pattern (Normal Dish, n=2, C=0)

Figure 22.
Normalized Radiation Intensity Pattern (Degraded Dish, n=2, C=0)
Figure 23.
Directive Gain Contour Map (Normal Dish, n=2, C=0)
Figure 24.
Directive Gain Contour Map (Degraded Dish, n=2, C=0)
Figure 25.
Flux Density Contour Map (Normal Dish, n=2, C=0)
Figure 26.
Flux Density Contour Map (Degraded Dish, n=2, C=0)
Figure 27.
Normalized Radiation Intensity Pattern (Normal Dish, \( n=1, C=0.100 \))

Figure 28.
Normalized Radiation Intensity Pattern (Degraded Dish, \( n=1, C=0.100 \))
Figure 29.
Directive Gain Contour Map (Normal Dish, \( n=1, \ C=0.100 \))
Figure 30.
Directive Gain Contour Map (Degraded Dish, n=1, C=0.100)
Figure 31.
Flux Density Contour Map (Normal Dish, n=1, C=0.100)
Figure 32.
Flux Density Contour Map (Degraded Dish, n=1, C=0.100)
Figure 33.
Normalized Radiation Intensity Pattern (Normal Dish, n=2, C=0.100)

Figure 34.
Normalized Radiation Intensity Pattern (Degraded Dish, n=2, C=0.100)
Figure 35.
Directive Gain Contour Map (Normal Dish, n=2, C=0.100)
Figure 36.
Directive Gain Contour Map (Degraded Dish, n=2, C=0.100)
Figure 37.
Flux Density Contour Map (Normal Dish, n=2, C=0.100)
Figure 38.
Flux Density Contour Map (Degraded Dish, n=2, C=0.100)
Figure 39.
Normalized Radiation Intensity Pattern (Normal Dish, n=1, C=0.316)

Figure 40.
Normalized Radiation Intensity Pattern (Degraded Dish, n=1, C=0.316)
Figure 41.
Directive Gain Contour Map (Normal Dish, n=1, C=0.316)
Figure 42.
Directive Gain Contour Map (Degraded Dish, $n=1$, $C=0.316$)
Figure 43.
Flux Density Contour Map (Normal Dish, n=1, C=0.316)
Figure 44.
Flux Density Contour Map (Degraded Dish, \( n=1 \), C=0.316)
Figure 45.
Normalized Radiation Intensity Pattern (Normal Dish, n=2, C=0.316)

Figure 46.
Normalized Radiation Intensity Pattern (Degraded Dish, n=2, C=0.316)
Figure 47.
Directive Gain Contour Map (Normal Dish, n=2, C=0.316)
Figure 48.
Directive Gain Contour Map (Degraded Dish, n=2, C=0.316)
Figure 49.
Flux Density Contour Map (Normal Dish, n=2, C=0.316)
Figure 50.
Flux Density Contour Map (Degraded Dish, n=2, C=0.316)
APPENDIX A

SPHERICAL GEOMETRY RELATIONSHIPS

The following equations and relationships of spherical geometry are helpful in understanding the work in Chapters III and IV. More detailed information may be found in the literature [ref. 43 and 44].

A spherical triangle is one bounded by arc of great circles (of a sphere), formed by the intersection of a sphere and planes passing through the sphere's center.

Equations A-1 through A-6 refer to Figure 51. Equations A-7 through A-8 refer to Figure 52.

Figure 51.
Spherical Trigonometry Relationships
76
From the Law of Cosines for sides:

\[
\begin{align*}
\cos(a) &= \cos(b) \cos(c) + \sin(b) \sin(c) \cos(A) & (A-1) \\
\cos(b) &= \cos(c) \cos(a) + \sin(c) \sin(a) \cos(B) & (A-2) \\
\cos(c) &= \cos(a) \cos(b) + \sin(a) \sin(b) \cos(C) & (A-3)
\end{align*}
\]

From the Law of Cosines for Angles:

\[
\begin{align*}
\cos(A) &= -\cos(B) \cos(C) + \sin(B) \sin(C) \cos(a) & (A-4) \\
\cos(B) &= -\cos(C) \cos(A) + \sin(C) \sin(A) \cos(b) & (A-5) \\
\cos(C) &= -\cos(A) \cos(B) + \sin(A) \sin(B) \cos(c) & (A-6)
\end{align*}
\]

The range of the satellite to point D is given by

\[
d = \sqrt{r^2 + (r + h)^2 - 2r(r + h)\cos(u)} = \sqrt{h^2 + 2r(h + r)(1 - \cos(f)\cos(g)}
\]
Figure 52.
Geometry of a Geosynchronous Satellite
APPENDIX B

The following computer programs were written to aid in the analysis of the work performed. The programs were written using MathCad 2.5.

Computer Program 1

This program calculates the angle within a satellite antenna pattern corresponding to a particular earth location given by its latitude and longitude.

.MCD 25000 0
.CMD ACE MAT rot 10 tilt 35 v cale 20 si e 15 30
.CMD ETC MAT mag 1 0 0 0 0 0 1 0 0 0 0 0
.center 0.500000 0.500000 si e 15 30 box y
.CMD PT MAT logs 0 0 subdiv 1 1 si e 5 15 type l
.CMD MAT rd d ct 10 im i et 3 t 15 pr 6 mass 1 length 1 X
.charge
.CMD ET I IN 0
.CMD ET T 0.001000
.CMD MA IN 0
.CMD INE EN T 78
.CMD ET P NC IDT 8
.CMD ET P NP ECI I N 4
.TXT 0 0 4 80
.a79 78 192
.MAT CAD 2.5
C I
.C IDE ^Q
^Q
This program finds the required hole size RB based on the
required gain at a
specified angle within the antenna pattern"
.EQN 7 0 1 9
.r.:6^Q
.EQN 0 10 1 9
.1:3^Q
.EQN 0 12 1 8
.n:0^Q
.EQN 0 9 1 8
.C:0^Q
.EQN 0 14 1 26
.RB:.3821:.3822:.383^Q
.EQN 1 -45 1 11
.et:1.0^Q
.EQN 1 64 3 10
\[ \frac{\pi}{180^Q} \]

\[ \theta:0,1;9^Q \]

\[ \text{fun} \text{ is the unnormalized pattern of the perfect reflector} \]

\[ \text{fun}(\theta) = 2\pi*(0.6*(1-(a/r)^2)^n)*a*J_0(2\pi/|a*sin(\theta*\pi/180)|^Q \]

\[ \text{gun} \text{ is the unnormalized pattern of a dish RB in radius} \]

\[ \text{gun}(RB,\theta) = 2\pi*(0\&RB*(1-(a/r)^2)^n)*a*J_0(2\pi/|a*sin(\theta*\pi/180)|^Q \]

\[ \text{DO:10*log(} \epsilon^*4*\pi/|2\pi*r^2+10*log(\text{fun}(7.4)/\text{fun}(0)) \]

\[ \text{DRB}(RB):10*log(} \epsilon^*4*\pi/|2\pi*(RB^2)+10*log(\text{gun}(RB,7.4)/\text{gun}(RB,0)) \]

\[ \text{RESULTS FOR PATTERN:} \]

\[ n=23 \]

\[ \text{C=3} \]

\[ \text{RB}=\text{GAIN}(RB) \]

\[ \text{ERROR}(RB): \text{GAIN}(RB)-3)/(\text{GAIN}(RB)+3) \]

\[ \text{where RB is the hole radius (m)} \]

\[ \text{GAIN}(RB) \text{ is the difference in the boresight gain and the gain in the specified angular direction} \]

\[ \text{ERROR}(RB) \text{ is the error comparison of GAIN}(RB) \text{ and 3 dB, the specified value} \]

\[ \text{DO1}(\phi):10*log(} \epsilon^*4*\pi/|2\pi*(xrb^2)+10*log(\text{gun}(xrb,\phi)/\text{gun}(xrb,0)) \]

\[ \text{DRB1}(\phi):10*log(} \epsilon^*4*\pi/|2\pi*(xrb^2)+10*log(\text{gun}(xrb,\phi)/\text{gun}(xrb,0)) \]

\[ \phi=\text{spec} \]

\[ \text{DO1}(\phi)-\text{DRB1}(\phi) = \text{spec} \]

\[ \text{EQN 0 22 12 17} \]
D 1(0) D 1(\phi) = \phi
\text{EQN} 21 -27 11 69
&14&DO1(\phi)(8,10,10,60,1) \& \& \phi
\text{EQN} 12 10 1 22
DO1(0) = ? \phi
\text{EQN} 0 26 1 24
DO1(8.5) = ? \phi
\text{EQN} 2 -26 1 24
DO1(6.1) = ? \phi
\text{EQN} 0 26 1 25
DO1(10.2) = ? \phi
\text{EQN} 5 -36 11 70
22&14&DRB1(\phi)(8,10,10,60,1) \& \& \phi
\text{EQN} 12 10 1 23
DRB1(0) = ? \phi
\text{EQN} 0 26 1 24
DO1(7.4) = ? \phi
\text{EQN} 1 -26 1 25
DRB1(9.7) = ? \phi
\text{EQN} 0 25 1 25
DRB1(7.4) = ? \phi

\textbf{Computer Program 2}

This computer program was written to numerically solve for the hole radius required
(see eqn.(26) of Chapter II).

.MCD 25000 0
.CMD ACE MAT rot 10 tilt 35 v cale 20 si e 15 30
.CMD ETC MAT mag 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 center 0.500000 0.500000 si e 15 30 box y
.CMD P T MAT logs 0 0 subdivs 1 1 si e 5 15 type l
.CMD MAT rd d ct 10 im i et 3 t 15 pr 3 mass length time charge
.CMD ET I IN 0
.CMD ET T 0.001000
.CMD MA IN 0
.CMD INE EN T 78
.CMD ET P NC IDT 8
.CMD ET P NP ECI I N 4
.TXT 0 0 5 77
a5 76 78 179
MAT CAD2.5
C IVE \phi

\text{THIS PROGRAM CALCULATES THE ANGLE WITHIN AN ANTENNA PATTERN}
\text{CORRESPONDING TO A SPECIFIC EARTH LOCATION}"

.EQN 6 0 3 10
p: \pi /180
.EQN 1 14 1 11
r: 6378
.EQN 0 14 1 12
d: 35786
.TXT 3 -28 1 18
a1, 17, 78, 16
ATE ITE P INT
.E N 0 23 1 14
lat sat 0^Q
.EQN 0 22 1 16
lon sat:105^Q
.TXT 3 -45 1 20
a1,19,78,18
ANTENNA BORESIGHT
.EQN 0 24 1 14
lat bor:0^Q
.EQN 0 22 1 16
lon bor:105^Q
.EQN 1 -21 1 11
elev:0^Q
.TXT 2 -25 1 23
a1,22,78,21
LOCATION OF INTEREST
.EQN 0 25 1 15
lat ear:39^Q
.EQN 0 21 1 15
lon ear:77^Q
.EQN 2 -46 1 28
δ lon:lon sat-lon bor^Q
.EQN 2 0 1 41
Bo:acos(cos(lat_bor*π)*sin(δ_lon*π))^Q
.EQN 3 0 1 101
y:acos(sin(lat_bor*π)*sin(lat ear*π)+cos(lat_bor*π)*sin(lat_ear*π)*cos(lon ear*π-lon bor*π))^Q
.EQN 3 0 3 53
1:acos(((sin(lat_sat*π)-sin(lat_bor*π)*cos(Bo))/(cos(lat_bor*π)*sin(Bo)))^Q
.EQN 6 0 3 52
2:acos(((sin(lat ear*π)-sin(lat_bor*π)*cos(y))/(cos(lat_bor*π)*sin(Bo)))^Q
.EQN 5 0 1 43
3(lat ear,lon ear):acos(cos(1+2)^Q
.EQN 3 0 1 86
β(lat ear,lon ear):acos(cos(Bo)*cos(y)+sin(Bo)*sin(y)*cos(3
(lat ear,lon ear)))^Q
.EQN 3 0 3 70
1(lat ear,lon ear):acos((cos(y)-cos(Bo)*cos(β(lat ear,lon ear)))/(sin(Bo)*sin(β(lat ear,lon ear)))^Q
.EQN 5 0 10 100
θ(lat ear,lon ear):acos((cos(elev)*(d/r+2*(sin(β(lat ear,lon ear)/2))2+tan(elev)*sin(β)*cos(4))/(4*(1+(d/r))*(sin(β(lat ear,lon ear)/2))2+(d/r)^2)))^Q

Computer Program 3

This computer program was written to calculate the flux density contours (see Chapter IV).
.CMD ETC MAT mag 1.000000 1.000000
center 0.500000 0.500000 si e 15 30 box y
.CMD P T MAT logs 0 0 subdivs 1 1 si e 5 15 type 1
.CMD MAT rd d 'ct 10 im i et 3 t 15 pr 6 mass 'length time charge
.CMD ET I IN 0
.CMD ET T 0.001000
.CMD MAK 0
.CMD INE EN T 78
.CMD ET P NC IDT 8
.CMD ET P NP ECI I N 4
.TXT 0 0 4 77
a4 76 78 211
MAT CAD 2.5
C IVE ^Q
^Q
RELATIVE PATTERN INTENSITY WITH HOLE IN MIDDLE OF DISH CALCULATES^Q
SUPERPOSITION OF TWO PATTERNS AND ERROR, SINC FUNCTION MODEL
AND CONTOURS" .EQN 5 60 3 10
p: $\pi/180$^Q
.EQN 1 -60 1 9
r: .6^Q
.EQN 0 10 1 9
l: .3^Q
.EQN 0 12 1 8
n: 0^Q
.EQN 0 9 1 8
C: 0^Q
.EQN 0 14 1 13
RB: .3821^Q
.EQN 1 -45 1 11
c: t: 1.0^Q
.EQN 2 0 1 15
e: 1,2;10^Q
.TXT 1 0 1 59
a1,58,78,57
fun is the unnormalized pattern of the perfect reflector
.EQN 2 0 6 72
fun(0): 2$\pi*(0^&.6'(1+(1-C)*(1-(a/r)^2)^n)*a*J0(2$\pi/ [a*sin(0* 
\pi/180)]&a)^Q
.TXT 8 0 1 57
a1,56,78,55
gun is the unnormalized pattern of a dish RB in radius
.EQN 1 0 6 72
gun(0): 2$\pi*(0$RB'(1+(1-C)*(1-(a/r)^2)^n)*a*J0(2$\pi/ [a*sin(0* 
\pi/180)]&a)^Q
.EQN 6 0 5 30
fun1(0): 4$\pi^2*r^2/ [^2*fun(0)*e*t^Q
.EQN 2 3 0 5 43
gun1(0): (4$\pi^2*r^2/ [^2-4$\pi^2*RB^2/ [^2]*gun(0)*e*t^Q
.EQN 4 -30 3 27
F(0): (fun1(0) -gun1(0))/(fun1(0) -gun1(0))^Q
.EQN 4 0 1 22
kk: 8.4,8.45; 8.6^Q
.EQN 2 0 4 28
YY(kk,0): (sin(kk*0*p)/(kk*0*p))^2^Q
.EQN 8 0 3 32
ERROR(kk,0): (F(0) -YY(kk,0))/(F(0) +YY(kk,0))^Q
.EQN 4 0 11 6
0 =^Q
.E N 0 8 11 12
(θ)=^Q
.EQN 0 14 6 8
kk=^Q
.EQN 0 9 6 20
ERROR(kk, 7.40)*100=^Q
.EQN 6 22 1 22
F(7.40)=?^Q
.EQN 8 -53 1 21
i: 9.2, 9.25; 9.6^Q
.EQN 0 28 1 12
AA: 8.55^Q
.TXT 0 17 1 30
al, 29, 33, 28
AA is the choice from above
.EQN 2 -45 13 16
(sin(AA*i*p)/(AA*i*p))^2=^Q
.EQN 3 16 10 8
i=^Q
.EQN 3 14 1 11
BB: 9.3^Q
.TXT 0 16 1 33
al, 32, 32, 31
BB is the choice from the left
.EQN 2 -15 3 15
k: (1.3916/(p*(BB)))^Q
.EQN 10 -31 1 17
j: 0, -.5; -4^Q
.EQN 2 0 2 20
a: .5+.0252*k^2^Q
.EQN 3 0 1 19
c(j): 0.0174*(j)^Q
.EQN 2 0 6 23
θdeg(j): (-4*a*c(j))/(2*a)/p^Q
.TXT 12 0 1 10
al, 9, 78, 8
RESULTS
.EQN 0 13 1 9
n=?^Q
.EQN 0 11 1 9
C=?^Q
.EQN 0 13 1 15
RB=?^Q
.EQN 0 18 1 12
AA=?^Q
.EQN 3 -12 1 13
BB=?^Q
.EQN 0 18 1 12
j=?^Q
.EQN 11 -19 1 9
n=?^Q
.EQN 0 9 1 9
c=?^Q
.EQN 2 -17 11 65
0&-4&j(4, 10, 10, 60, 1)&&θdeg(j)^Q
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