Stander Symposium 2021

Domination and F-Domination

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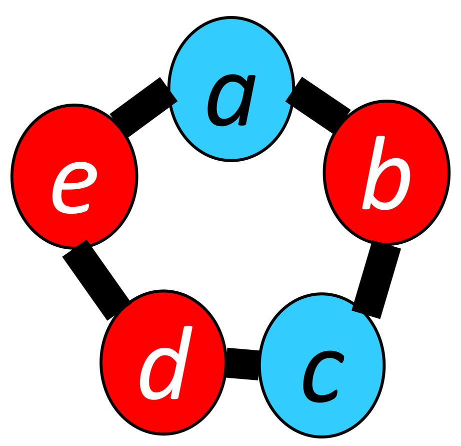
Introduction

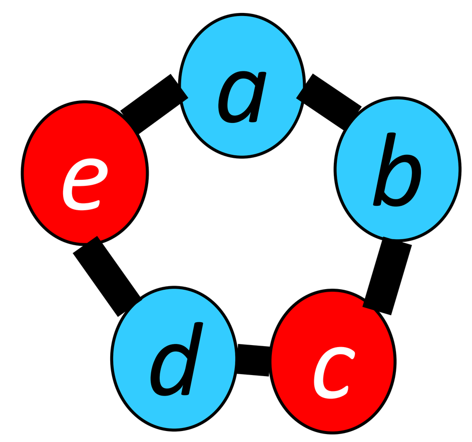
A **dominating set** of a graph G is a set of vertices S such that every vertex of G is a neighbor of some vertex in S or is in S. The **domination number** $\gamma(G)$ is the minimum number of vertices in a dominating set S.

Example 1:

G:



G:



$S_1 = \{e, d, b\}$ $S_2 = \{e, c\}$

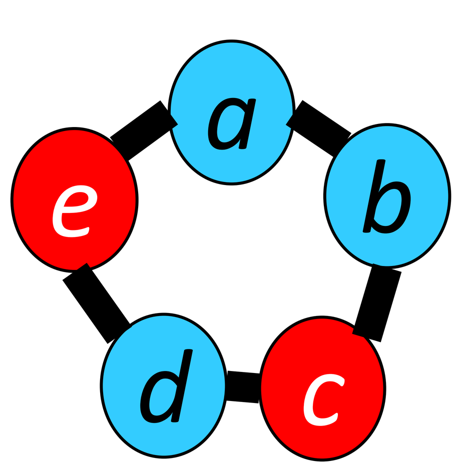
$\gamma(G) = 2$

Why not $\gamma(G) = 1$? A single vertex can only dominate 3 out of 5 vertices in G.

Let F be a graph which has all its vertices colored either red or blue. Let v be a designated blue vertex of F. An ***F*-coloring** of a graph G is a red-blue coloring of the vertices of G in which every blue vertex u belongs to a copy of the subgraph F of G rooted at v. The ***F*-domination number** $\gamma_F(G)$ is the minimum number of red vertices in an F-coloring of G.

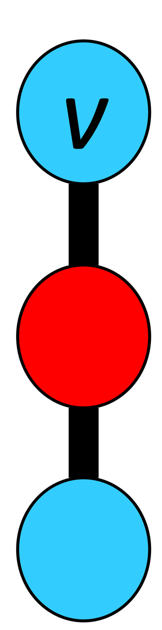
Example 2:

G:



$\gamma_F(G) = 2$

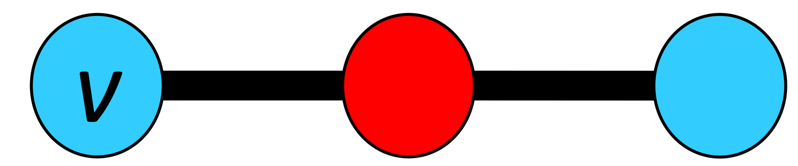
F:



Theorem

Let G be a connected graph of order 3 or more. Let F be the graph shown below. Then $\gamma_F(G) = \gamma(G)$.

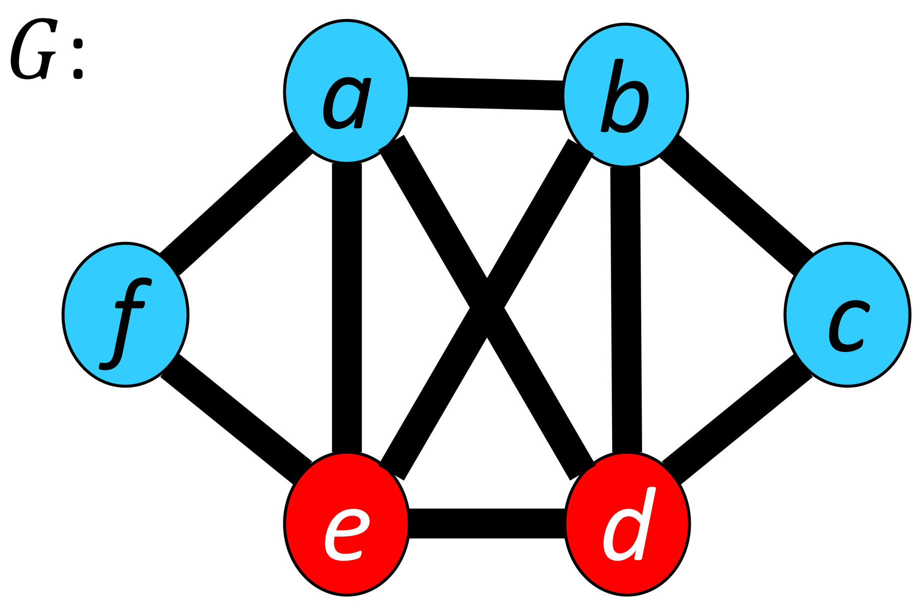
F:



Note that $\gamma(F) = 1$, which is the smallest possible domination number.

Example 3:

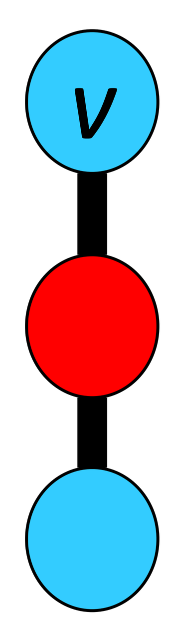
G:



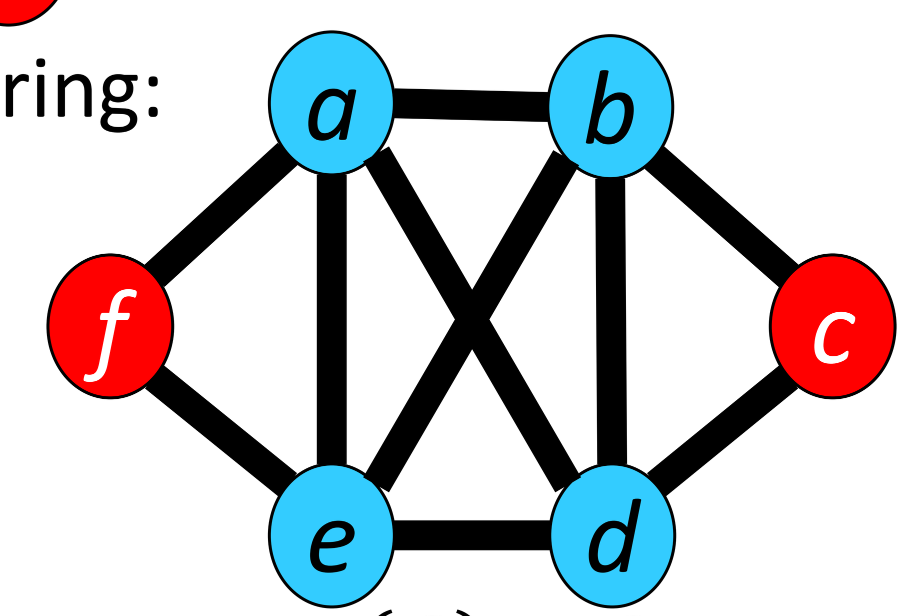
$S = \{e, d\}$

$\gamma(G) = 2$

F:



F-coloring:



$\gamma_F(G) = 2$

Since there are 4 blue vertices, there are 4 copies of F.

Why not $\gamma_F(G) = 1$? This implies a red vertex is adjacent to every blue vertex. Meaning that the red vertex would have *deg5* since $\Delta(G) \leq 6 - 1$, but $\Delta(G) = 4$.

References

[1] Chartrand, Gary, and Ping Zhang. *A First Course in Graph Theory*. Dover Publications, 2012.

[2] Gera, Ralucca M. *Stratification and Domination in Graphs and Digraphs*. Ralucca M. Gera, 2005.

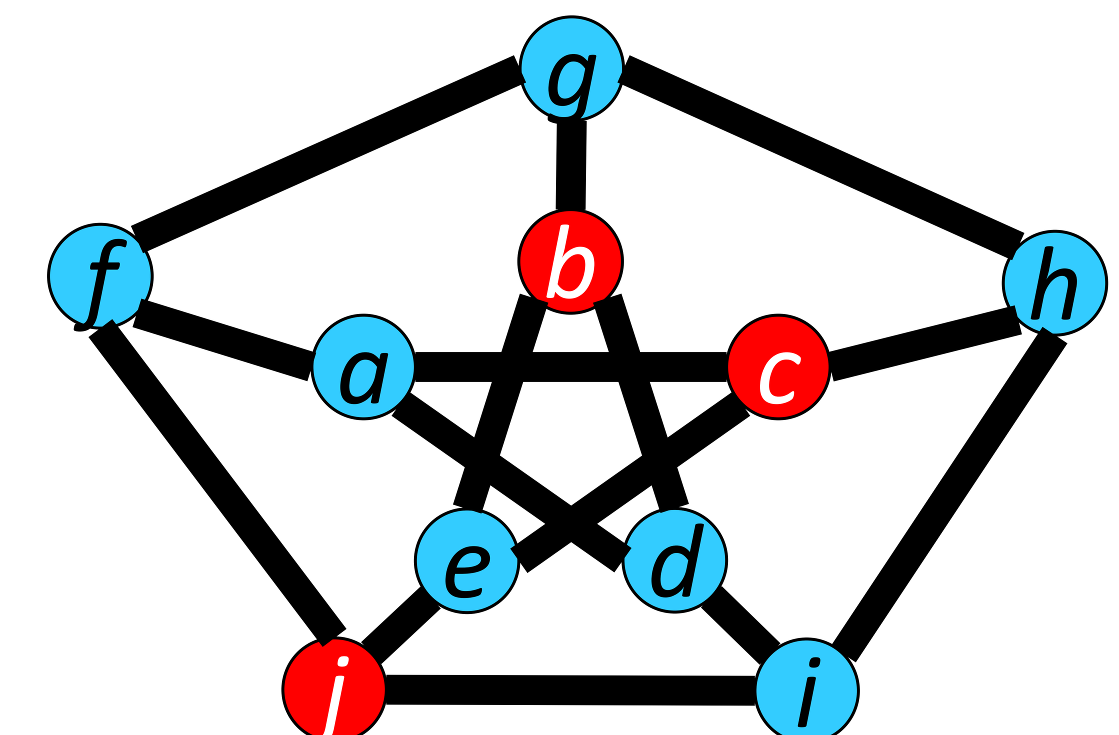
Peterson Graph and F-Domination

Gera presents a corollary that says if H is a connected graph and G is a connected spanning subgraph of H, then

$$\gamma_F(H) \leq \gamma_F(G).$$

How big can $\gamma_F(G)$ be for a particular graph G? Observe the parameters for the Peterson Graph and one of its subgraphs G

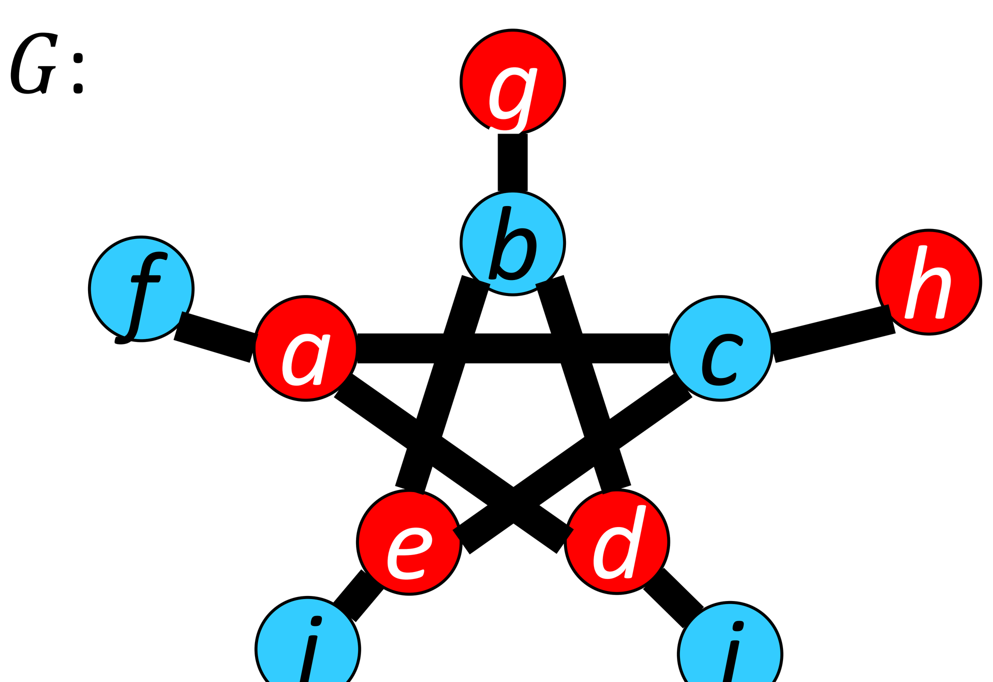
H:



$\gamma_F(H) = 3$

Why not $\gamma_F(H) = 2$? H is 3-regular, so at most 2 red vertices can dominate 8 vertices, but H has order 10.

G:



$\gamma_F(G) = 5$

Thus, $\gamma_F(H) \leq \gamma_F(G)$. Increasing edges in a graph correlates to a smaller F-domination number.