Exploring a Mathematical Model of Crime Dynamics Including Media Coverage and the Police Force

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Abstract

In this project, MATLAB was used to analyze the model presented in the paper *Mathematical Model Analysis of Crime Dynamics Incorporating Media Coverage and Police Force*. The Runge Kutta methods of order two and four are implemented. We used the built in functions Ode45 and Ode23 to approximate the solution of the model, as well as compare the accuracy. As a pathway to discovery, the initial conditions and differential equations were altered to further improve the accuracy of the model by more accurately reflecting real world conditions. Euler's Method was utilized and compared to the Runge Kutta methods used in 'ode' functions to further demonstrate the accuracy of the Runge Kutta methods.

Model

(1) $$\frac{dS_u}{dt} = \sigma\Lambda - \beta S_u C - \eta S_u - \mu S_u,$$

(2) $$\frac{dS_a}{dt} = (1-\sigma)\Lambda + \eta S_u + (1-\theta)\nu R - \tau S_a - \left(1 - \frac{\rho C}{m+C}\right)\delta S_a C - \mu S_a,$$

(3) $$\frac{dC}{dt} = \beta S_u C + \left(1 - \frac{\rho C}{m+C}\right)\delta S_a C - \gamma C P - (\alpha + \mu)C,$$

(4) $$\frac{dR}{dt} = \gamma C P - (\nu + \mu)R,$$

(5) $$\frac{dQ}{dt} = \tau S_a + \theta \nu R - \mu Q,$$

(6) $$\frac{dP}{dt} = \phi C - \phi_0 (P - P_0),$$

S_u is the unaware, susceptible class.
S_a is the aware, susceptible class.
C is the criminal class
R is the prisoner class
Q is the non-criminal class
P is the police force

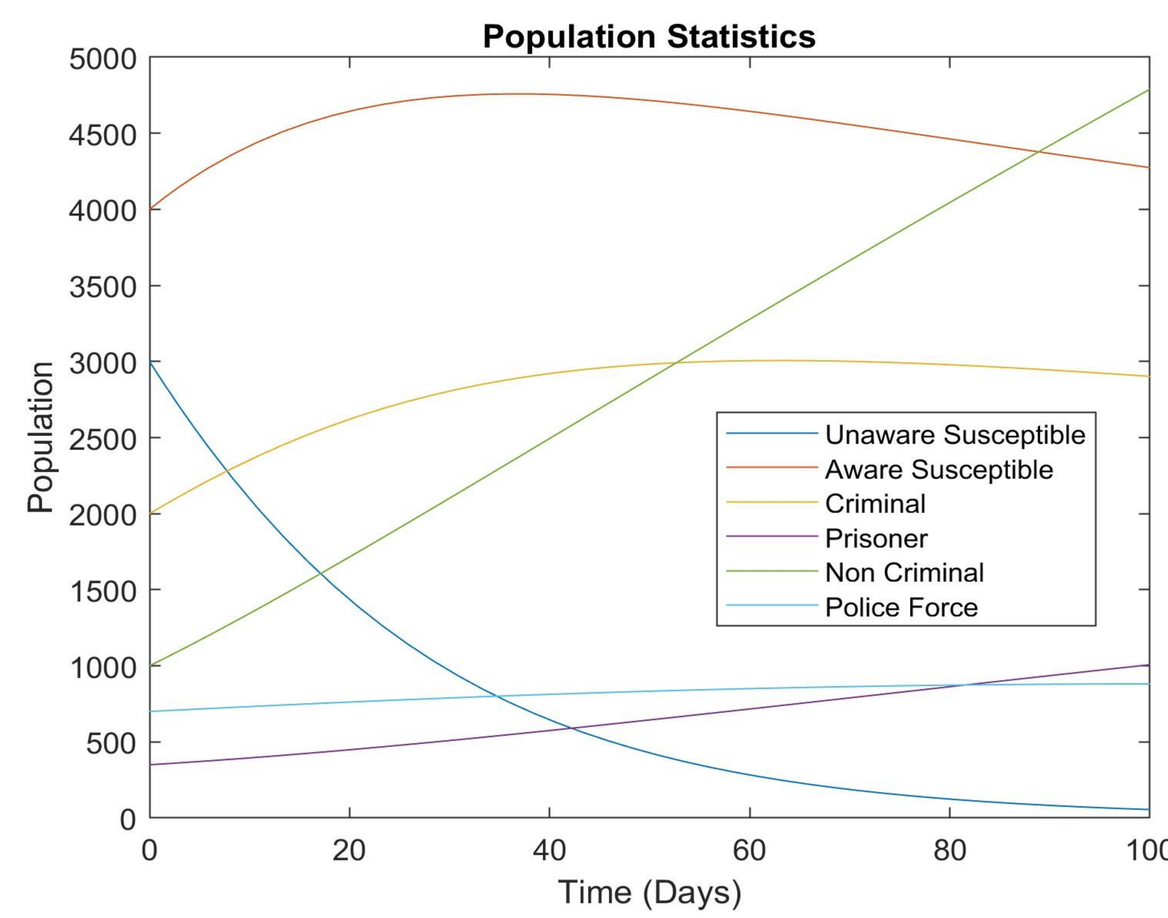

Figure 1: Crime data from [1]

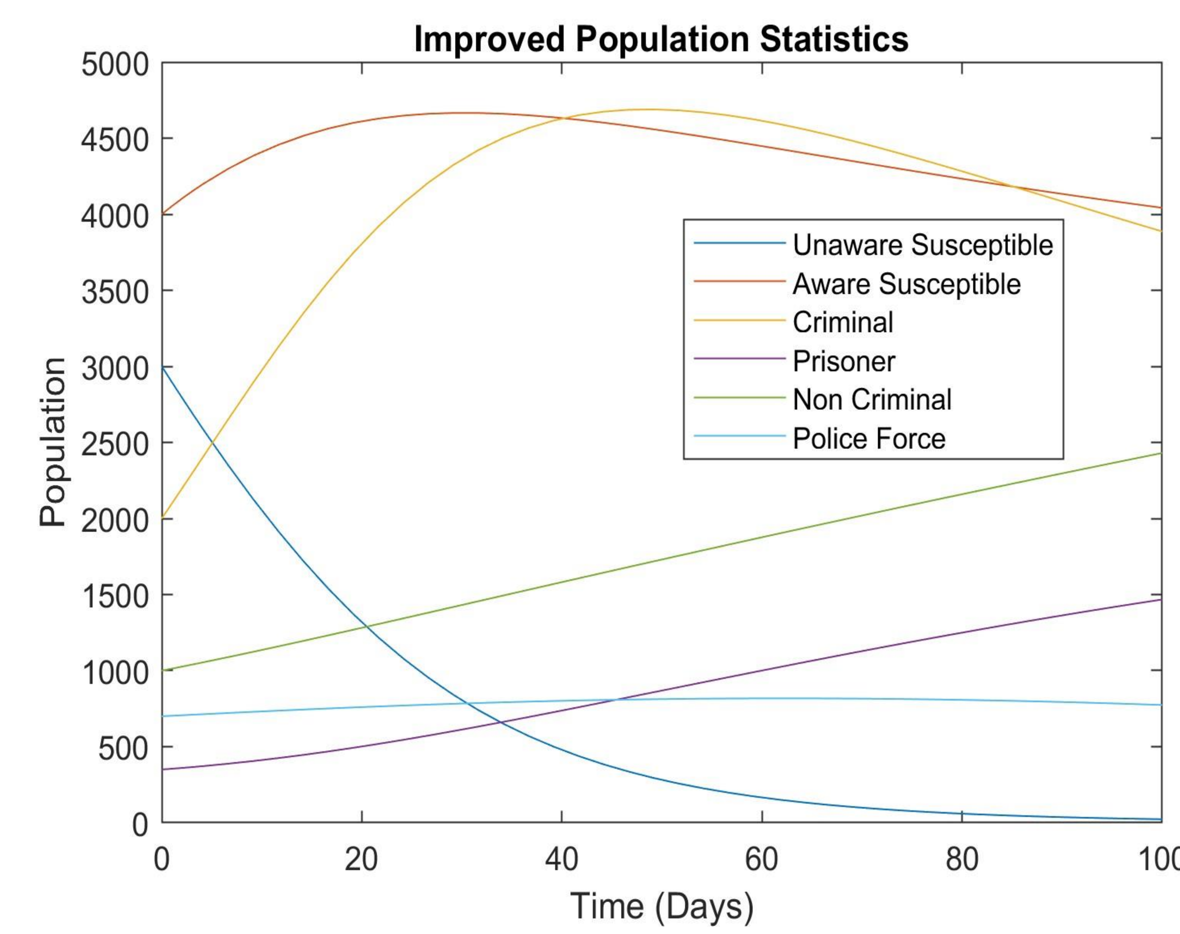

Figure 2: Crime Data after accounting for extended prison sentences, as well as wealth inequality.

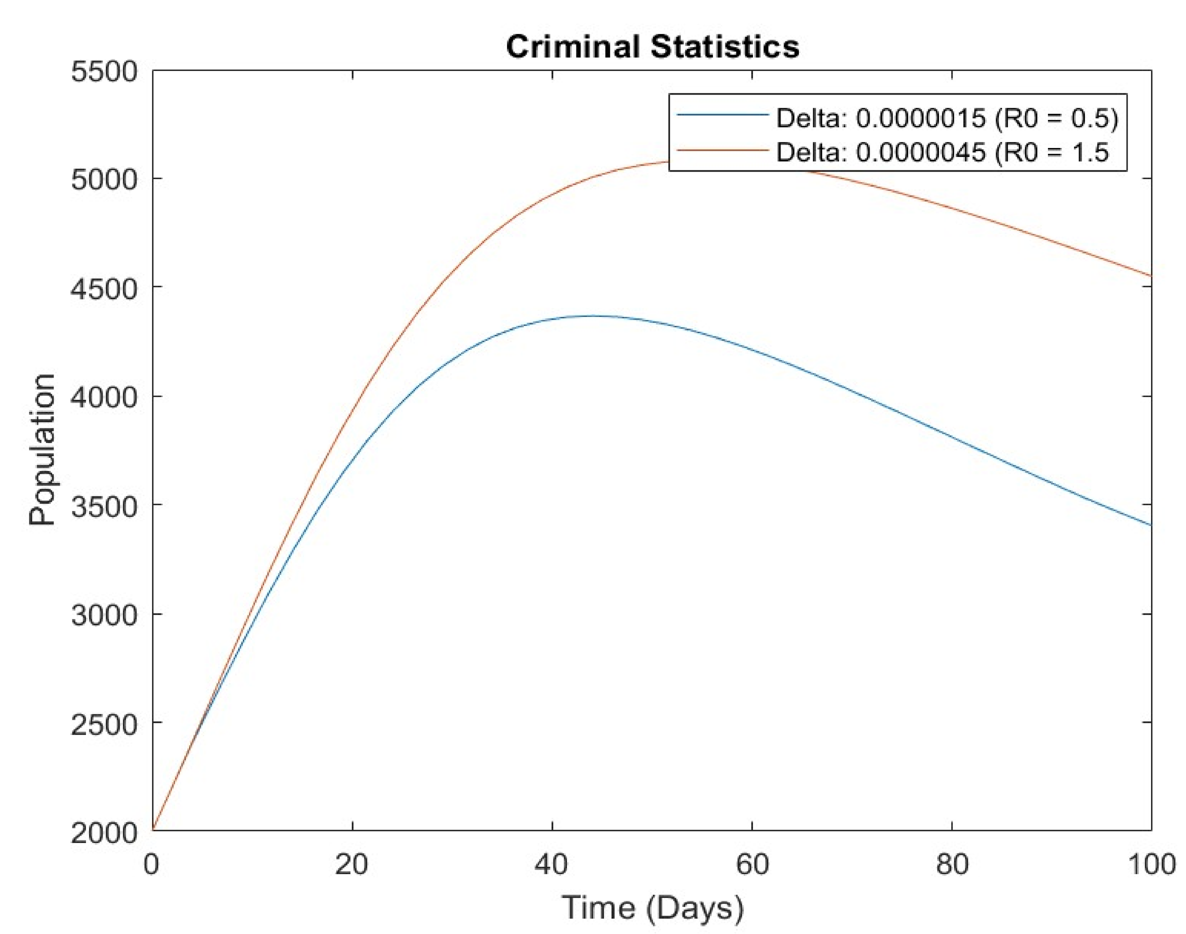

Figure 3: Analysis of criminal population with a greater rate of criminal activity spread

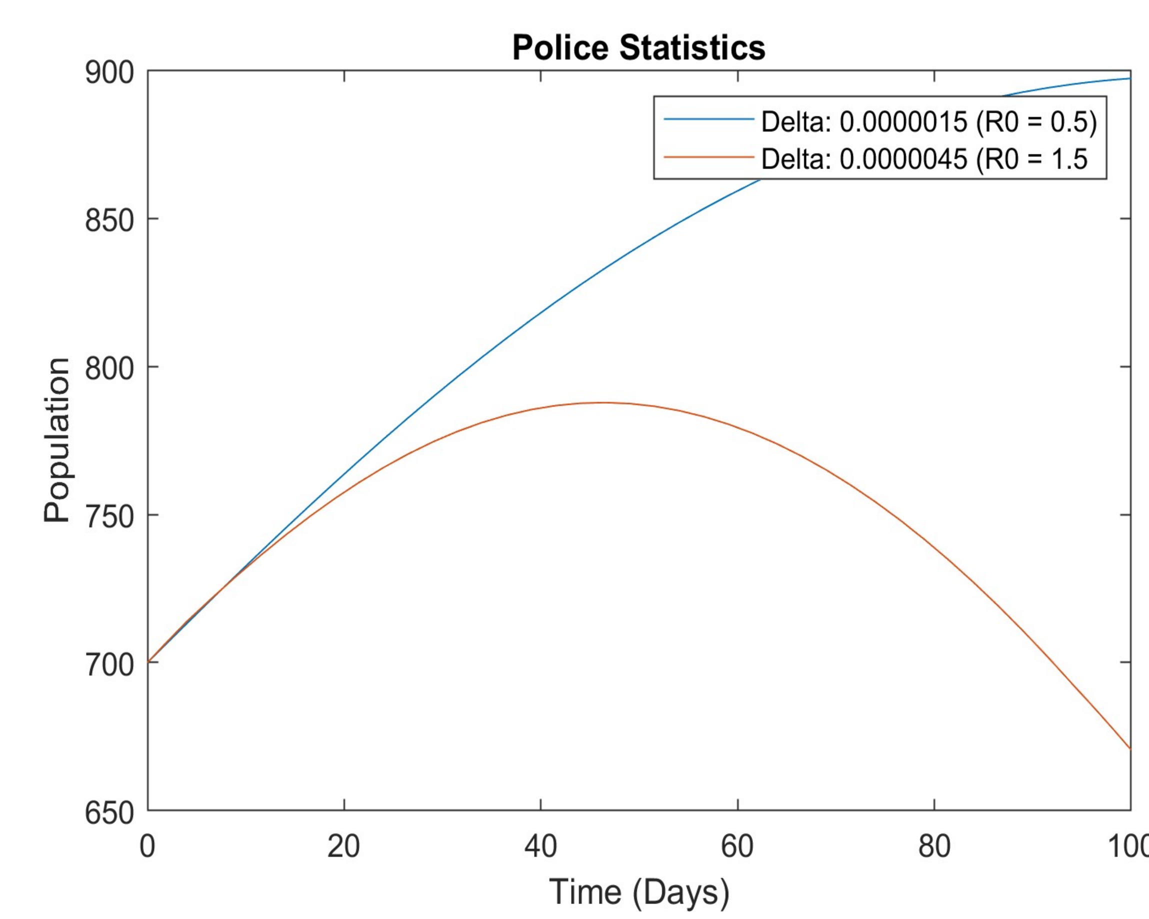

Figure 4: Analysis of police population with a greater criminal activity spread

Runge Kutta 2nd Order:

$$y_{i+1} = y_i + (h/2)(k_1 + k_2)$$
$$k_1 = f(t_i, y_i) \;\; ; \;\; k_2 = f(t_i + h, y_i + h\,k_1)$$

Runge Kutta 4th Order:

$$y_{i+1} = y_i + (h/6)(k_1 + 2k_2 + 2k_3 + k_4)$$
$$k_1 = f(t_i, y_i) \;\; ; \;\; k_2 = f(t_i + h/2, y_i + h/2\,k_1)$$
$$k_3 = f(t_i + h/2, y_i + h/2\,k_2) \;\; ; \; k_4 = f(t_i + h, y_i + h\,k_3)$$

Conclusion

Using the Ode23 and Ode45 functions in MATLAB, we were able to recreate and examine the data provided in the paper [1]. To further explore predicting crime in a complex environment, we altered the initial conditions. Utilizing the methods from [1], we adapted their data to more accurately reflect an American society. In order to achieve this, we lengthened prison sentences and added a variable to account for wealth inequality. Furthermore, as seen in figures 3 and 4, we analyzed the consequences of a higher reproduction rate of criminal activity. As the spread of crime increases, the criminal population increases when the police force population heavily decreases.

References

[1] Meskerem Abebaw Mebratie, Mohammed Yiha Dawed, (2021). *Mathematical model analysis of crime dynamics incorporating media coverage and police force*, J. Math. Comput. Sci., 11, 125-148

[2] Nagle, R., Saff, E., & Snider, A. (2018). *Fundamentals of Differential Equations* (9th ed.). Pearson.

[3] Class notes for Applied Differential Equations by Dr. Muhammad Usman.