Approximate Motion Synthesis of Four-Bar Linkages Using Poles: A Bi-Invariant Approach

Tianze Xu

Advisors: Andrew Murray, David Myszka

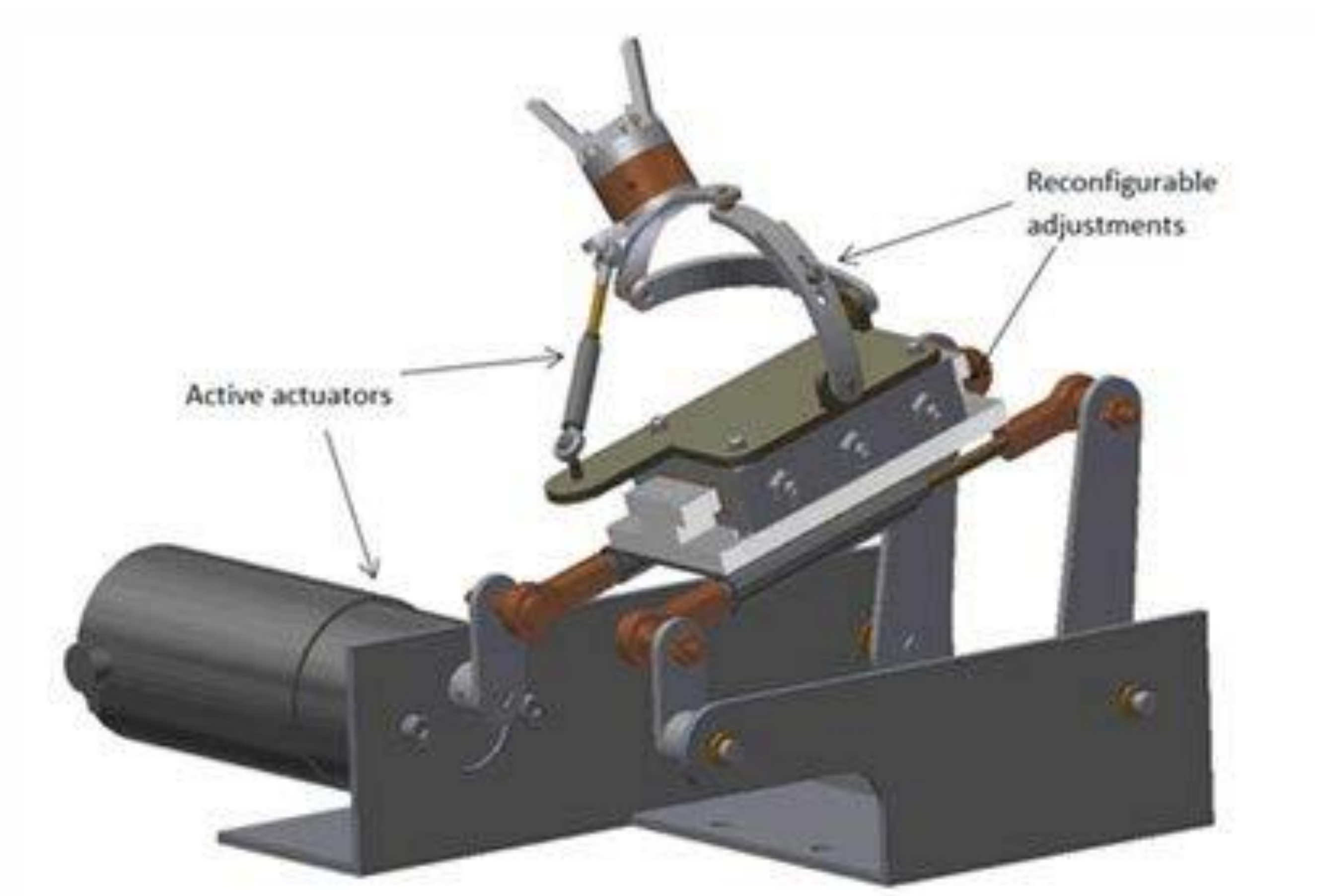
Department of Mechanical & Aerospace Engineering

Objective: This work generates a new approach to reformulate and solve approximate motion synthesis problems

Introduction

- Approximate motion synthesis generates a mechanism that can move near a given set of task positions.
- Previous methods that address approximate synthesis problems involve an optimization that uses multiple measures within the objective.
- This project proposes a technique based solely on the locations of displacement poles, resulting in a bi-invariant metric.
- The focus is on planar four-bar mechanisms due to their ability to provide efficient solutions to manufacturing or assembly problems
- The motion can then be refined with the addition of cascading mechanisms to create a new class of low DOF machines known as metamachines.

Metamachines

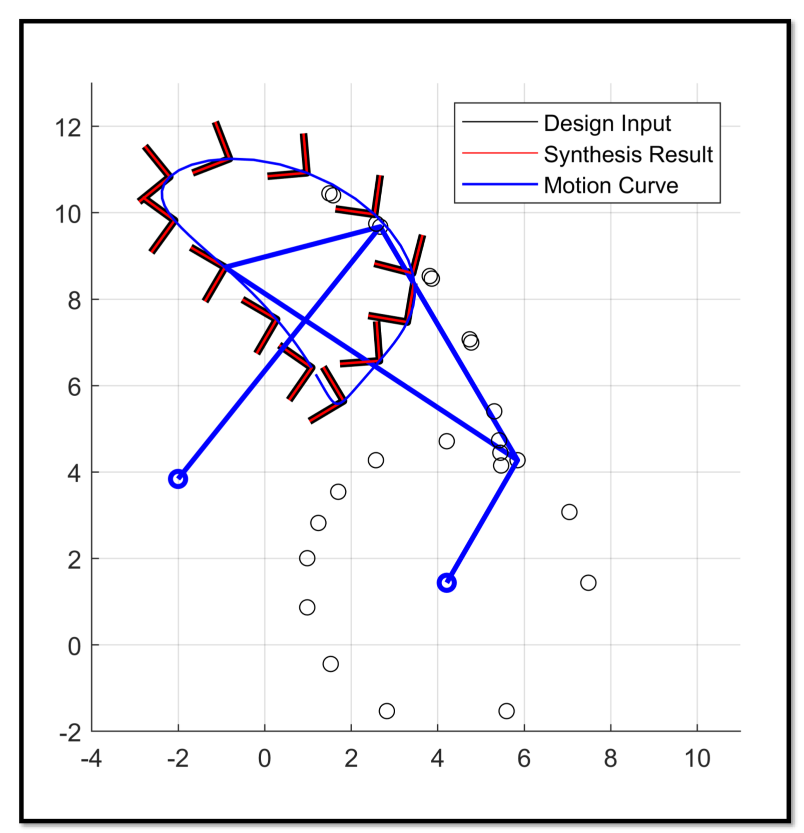


A metamachine

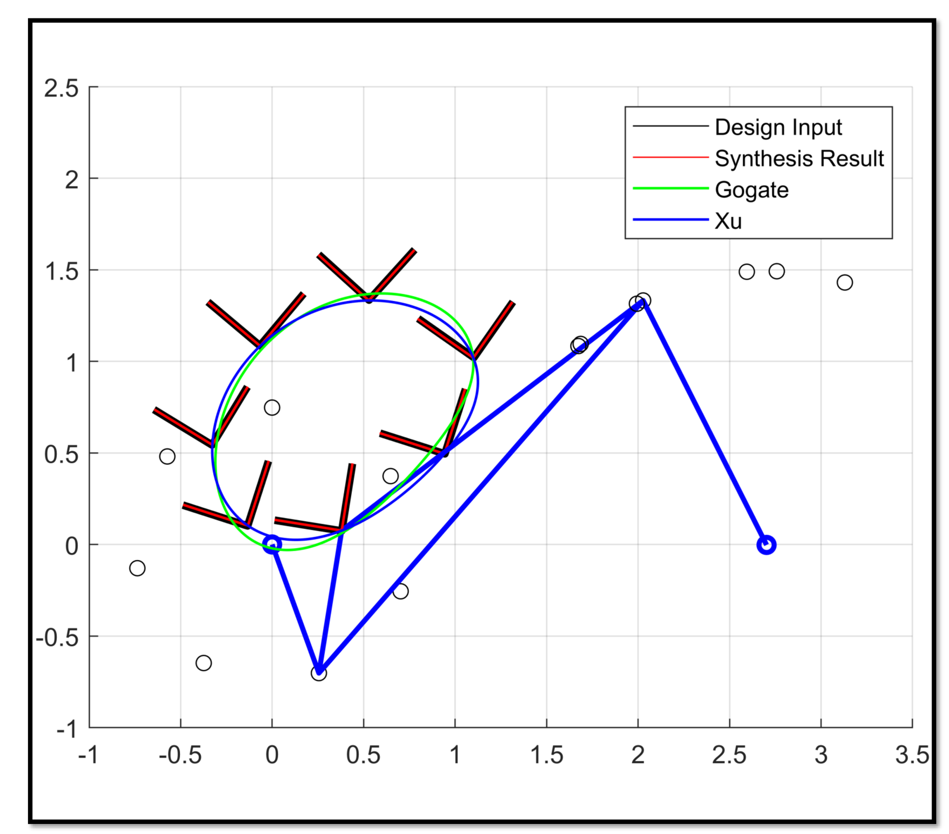
This research assists the design of a new class of low degree of freedom machines called metamachines:

- Significant impact on automated manufacturing by providing the Spatial pick & place motions required to perform spatial assembly tasks.
- Energy efficient when compared to traditional industrial robots.
- Reconfigurable to accommodate a variety of subassemblies or part families.

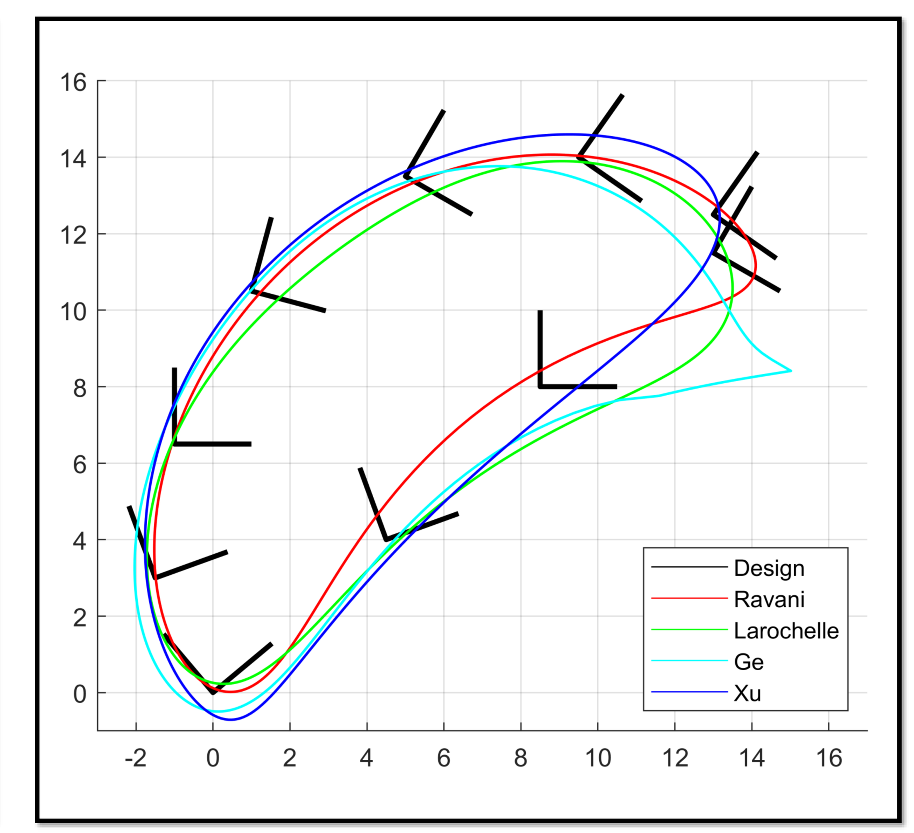
The Project Case Result Display



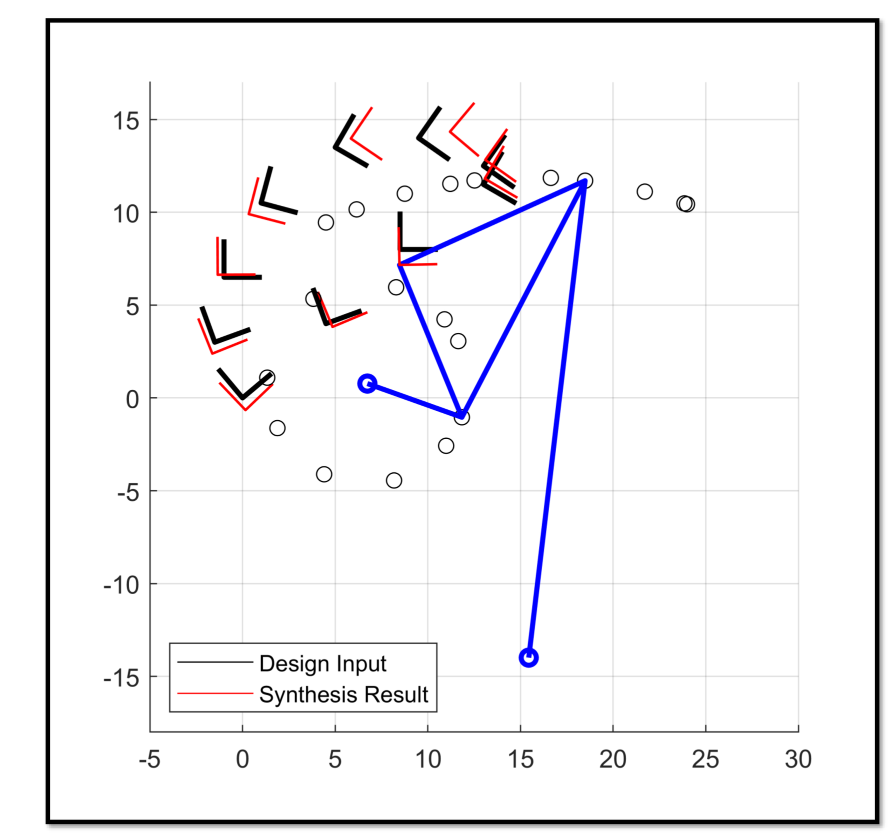
12 positions of an exact solution



Different solutions for 7 positions



Different solutions for 10 positions



Optimized solution using fewer poles

Poles:

Any displacement of a rigid body from position i to j can be accomplished by pure rotation about the displacement pole.

$$\vec{P}_{ij} = A_i(A_i - A_j)^{-1}\left(\vec{d}_j - \vec{d}_i\right) + \vec{d}_i$$

$$\vec{Q}_{ij} = B_i(B_i - B_j)^{-1}(\vec{e}_j - \vec{e}_i) + \vec{e}_i$$

Objective function …

$$\text{Min } J = \Sigma\left(\vec{P}_{ij} - \vec{Q}_{ij}\right)^{\top}\left(\vec{P}_{ij} - \vec{Q}_{ij}\right)$$

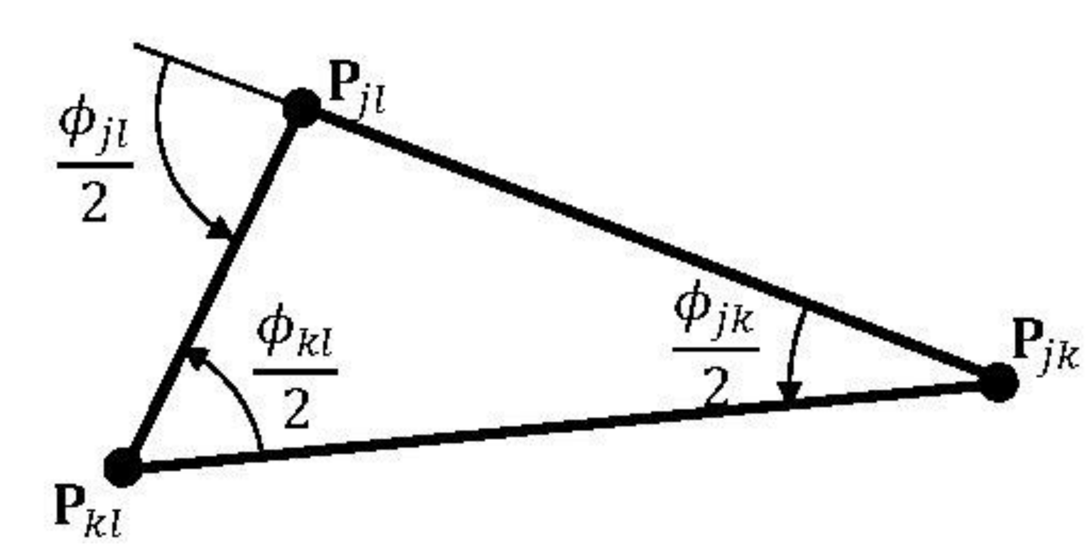
Subject to the Crank Constraint

$$\left|\vec{Z}_i - \vec{G}\right| = \left|\vec{Z}_1 - \vec{G}\right|$$

$$\left|\vec{Y}_i - \vec{G}\right| = \left|\vec{Y}_1 - \vec{G}\right|$$

Pole Triangle

"The Pole Triangle Principle" is an important theory that helps reduce the number of position poles needed in optimization synthesis.



The pole Triangle made by three Poles

Number of positions	Number of poles	Minimum poles needed
6	15	9
7	21	11
8	28	13
9	36	15
10	45	17

The poles generated by corresponding positions and the minimum poles for optimization